ECE201 Laboratory – 3

Introduction To
Electric Circuit Transients

(Created by Prof. Walter Green, Edited by Prof. M. J. Roberts)

Objectives

The objective of this laboratory is to develop an understanding of circuits containing *R*, *L*, and *C* components. Specific goals are to;

- understand the transient of a series *RC* circuit and the first order differential equation for a step input,
- understand the transients of a series *RL* circuit and the first order differential equation for a step input,
- understand the transients of an *RLC* circuit for and the second order differential equation for overdamped, critically damped and underdamped conditions for a step input.

Series *RC* Circuit

Consider the series *RC* circuit shown in Figure 4.1. For this laboratory we will only be concerned with a switched input voltage as shown.

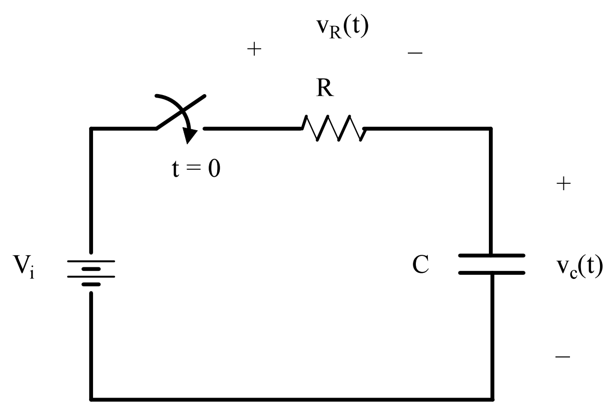

Figure 4.1: Series *RC* circuit with a switched voltage input.

Applying Kirchhoff's Voltage Law around the circuit $R\,\mathrm{i}(t) + \mathrm{v}_c(t) = V_\mathrm{i}\ \ ,\ t > 0$. The current and the voltage of the capacitor are related by $\mathrm{i}(t) = C\dfrac{d\,\mathrm{v}_c(t)}{dt}$. This current also flows through the resistor so one may write

$$RC\frac{d\,\mathrm{v}_c(t)}{dt}+\mathrm{v}_c(t)=V_\mathrm{i} \quad , \; t>0$$

or

$$\frac{d\,\mathrm{v}_c(t)}{dt}+\frac{\mathrm{v}_c(t)}{RC}=\frac{V_\mathrm{i}}{RC} \quad , \; t>0$$

This is a linear, constant-coefficient, time-invariant, first-order, inhomogeneous, ordinary differential equation. The solution is the sum of a decaying exponential and a constant

$$\mathrm{v}_c(t)=Ae^{-t/RC}+\mathrm{v}_f$$

A is determined by using the initial condition $\mathrm{v}_c(0)=0$. Then $\mathrm{v}_c(t)=V_\mathrm{i}\left(1-e^{-t/RC}\right) \; , \; t>0$

Two important things to remember about a capacitor are

- The voltage across a capacitor cannot change instantaneously
- The current through a capacitor can change instantaneously

The wave shape of $\mathrm{v}_c(t)$ for the case V_i = 10 volts and RC = 0.1 seconds can be determined and plotted using the MATLAB program of Figure 4.2.

```
%  Program Use: Lab 4, RC circuit response
%  Program Name: R_Ckt.m
%  History:  By WLG, July 3, 2003, office computer
%  Purpose:  Runs and plots the equation vc= Vi-Vi*e-t/RC

% Vi = 10; RC = 0.1

t = 0:.001:0.5;
vc = 10 - 10*exp(-10*t);

plot(t,vc)
title('Step Response of a Series RC Circuit')
ylabel('Vc (volts)')
xlabel(' t (sec) ')
grid
```

Figure 4.2 MATLAB program for graphing the step response of *RC* circuit

The response $\mathrm{v}_c(t)$ is shown in Figure 4.3.

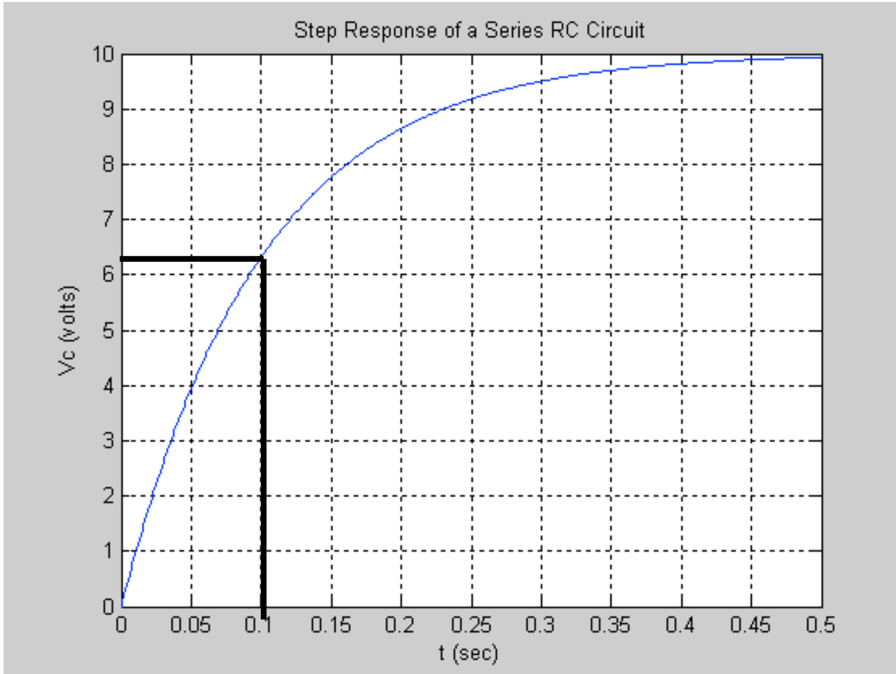

Figure 4.3 Step response of capacitor voltage in a series *RC* circuit.

The product of *R* and *C* is defined as the circuit *time constant*. This is usually written $\tau = RC$. We evaluate $\mathrm{v}_c(t) = V_{\mathrm{i}}\left(1 - e^{-t/RC}\right)$ at $t = \tau = RC = 0.1$ as

$$\mathrm{v}_c(0.1) = 10\left(1 - e^{-10\times 0.1}\right) = 10 \times 0.632 = 6.32$$

The significance of this result is that a capacitor in a series *RC* circuit, when subjected to a step input, will charge to 63% of the final capacitor voltage in one time constant. This is shown in the graph of Figure 4.3.

In this laboratory exercise you will construct the circuit shown in Figure 4.1 and apply a switched voltage and show this result.

Series *RL* Circuit

Consider the series *RL* circuit shown in Figure 4.4. For this laboratory we will only be concerned with the case for a switched input voltage as shown.

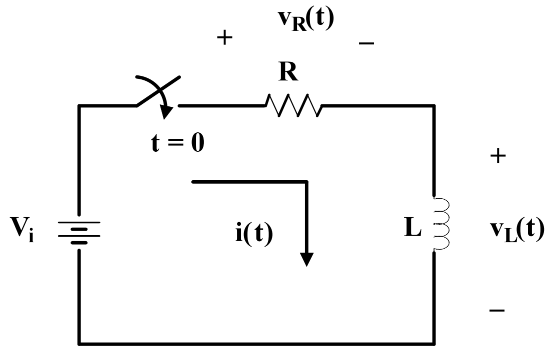

Figure 4.4 Series *RL* circuit.

Applying Kirchhoff's voltage law around the circuit

$$R\,\mathrm{i}(t) + L\frac{d\,\mathrm{i}(t)}{dt} = V_{\mathrm{i}} \quad , \; t > 0$$

or

$$\frac{d\,\mathrm{i}(t)}{dt} + \frac{R}{L}\mathrm{i}(t) = \frac{V_{\mathrm{i}}}{L} \quad , \; t > 0$$

The solution $\mathrm{i}(t)$ is $\mathrm{i}(t) = \frac{V_{\mathrm{i}}}{R}\left(1 - e^{-Rt/L}\right) \;,\; t > 0$. If we take the derivative of $\mathrm{i}(t)$ and multiply by *L* we will have $\mathrm{v}_L(t) = V_{\mathrm{i}} e^{-Rt/L} \;,\; t > 0$.

Two important things to remember about an inductor are

- The voltage across an inductor can change instantaneously
- The current through an inductor cannot change instantaneously

Similar to the definition of the time constant of the *RC* circuit, we define the time constant of the series *RL* circuit as $\tau = L / R$. The mathematical treatments of the *RC* and *RL* circuits are very similar. If we replace current with voltage, *L* with *C* and *R* with 1/*R*, we get the same results. This comes about because of duality of the two circuits.

If $V_{\mathrm{i}} = 10$ volts and the time constant for the circuit is again set to be 0.1 seconds, the voltage response across the inductor is shown in Figure 4.5. To graphically determine the time constant from the plot, we drop down by 63% of the original voltage (37% going up, 63% coming down) to find the time constant. This is illustrated in the graph.

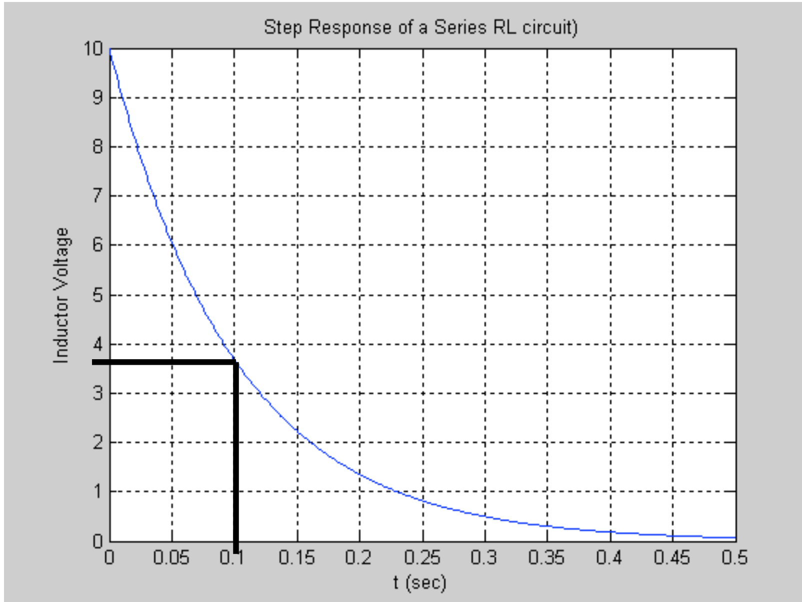

Figure 4.5 Voltage response across an inductor in an RL circuit for a step input.

In this laboratory exercise you will construct the circuit shown in Figure 4.4 and apply a step voltage and substantiate the above.

Series *RLC* Circuit

Consider the series *RLC* circuit shown in Figure 4.6. For this laboratory we will only be concerned with the case for a switched input voltage as shown.

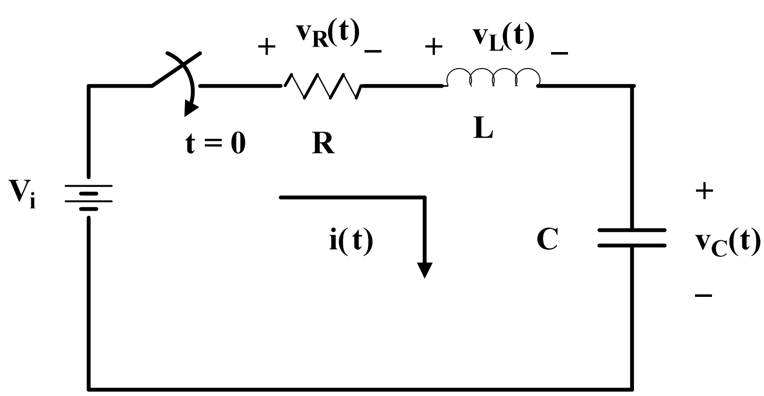

Figure 4.6 Series *RLC* circuit for laboratory 4.

Writing Kirchhoff's voltage law around the circuit $R\,\mathrm{i}(t)+L\dfrac{d\,\mathrm{i}(t)}{dt}+\mathrm{v}_c(t)=V_\mathrm{i}$

If we use $\mathrm{i}(t)=C\dfrac{d\,\mathrm{v}_c(t)}{dt}$ in this equation we get

$$LC\frac{d^2 v_c(t)}{dt^2} + RC\frac{d v_c(t)}{dt} + v_c(t) = V_i$$

or

$$\frac{d^2 v_c(t)}{dt^2} + \frac{R}{L}\frac{d v_c(t)}{dt} + \frac{v_c(t)}{LC} = \frac{V_i}{LC}$$

This is a second-order, linear, constant-coefficient, time-invariant, inhomogeneous, ordinary differential equation. The characteristic equation is $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ where s is the eigenvalue. The nature of the response of $v_c(t)$ will depend upon the roots of the above equation. The roots can be (a) real and unequal (overdamped), (b) real and equal (critically damped) and (c) complex (underdamped). A convenient way to examine the characteristic equation is to compare a given second-order characteristic equation with a standard form expressed as

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

ζ is called the damping factor; ω_n is called the undamped natural resonant frequency. If

$\zeta < 1$, the system response is underdamped
$\zeta = 1$, the system response is critically damped
$\zeta > 1$, the system response is overdamped

As an illustration on how to use the above, suppose R, L, and C have values such that the characteristic equation is $s^2 + 2s + 16 = 0$. Comparing coefficients $\omega_n = 4$ and $\zeta = 0.5$. Since $\zeta < 1$, we know the system response will be underdamped. This is an easy way of checking the system characteristic equation for the nature of the response. About the only thing left to do is explained what is meant by overdamped, underdamped and critically damped.

- If the system is overdamped, the step response will not have overshoot.
- If the system is critically damped, the step response will not have overshoot and it will exhibit the fastest response without having overshoot.
- If the system is underdamped, the step response will have overshoot. Sometimes we refer to the overshoot as “ringing”.

The three responses discussed here are illustrated in the diagram below.

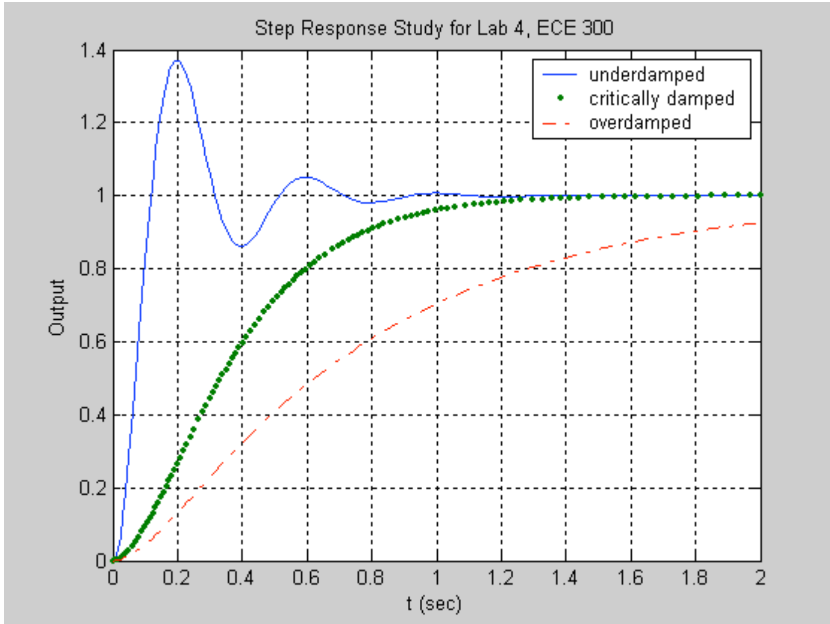

Figure 4.7 Graphs showing (i) underdamped, (ii) critically damped, (iii) overdamped Step response of a series *RLC* circuit.

In the laboratory you will build *RLC* circuits that have the above three responses.

Overdamped Response

When the circuit of Figure 4.6 is overdamped, the roots of the characteristic equation are real and unequal and can be factored in the form $(s+\alpha_1)(s+\alpha_2)=0$. The voltage across the capacitor is $\mathrm{v}_c(t)=V_\mathrm{i}+A_1e^{-\alpha_1 t}+A_2e^{-\alpha_2 t} \quad , \ t>0$. A_1 and A_2 are obtained from initial conditions. The values of α_1 and α_2 are obtained by solving $s^2+\dfrac{R}{L}s+\dfrac{1}{LC}=0$.

Critically Damped Response

When the circuit of Figure 4.6 is critically damped, the system characteristic equation is of the form $(s+\alpha)^2=0$. The expression for the capacitor voltage becomes

$$\mathrm{v}_c(t)=V_\mathrm{i}+(A_1+tA_2)e^{-\alpha t} \quad , \ t>0$$

As with the overdamped case, A_1 and A_2 are determined from initial conditions. Since we have two unknowns, we need two initial conditions to find them.

Underdamped Response

When the circuit of Figure 4.6 is underdamped, the system characteristic equation is of the form $(s+\alpha+j\omega_d)(s+\alpha-j\omega_d)=0$. The output voltage is

$$\mathrm{v}_c(t)=V_{\mathrm{i}}+e^{-\alpha t}\left[B_1\cos(\omega_d t)+B_2\sin(\omega_d t)\right] \;\;,\;\; t>0$$

B_1 and B_2 are determined from the circuit initial conditions. α and ω_d are determined in terms of the *R*, *L*, and *C* values of the circuit. It is easy to get bogged down in algebra when determining α, ω_d, B_1 and B_2 in terms of *R*, *L*, and *C* for the underdamped case.

In the laboratory, when you observe the voltage $\mathrm{v}_c(t)$ you will not actually see two distinct sinusoids. However, you will be able to observe the steady state value V_{i} and a "damped" sinusoidal. Another way of expressing

$$\mathrm{v}_c(t)=V_{\mathrm{i}}+e^{-\alpha t}\left[B_1\cos(\omega_d t)+B_2\sin(\omega_d t)\right] \;\;,\;\; t>0$$

is

$$\mathrm{v}_c(t)=V_{\mathrm{i}}+\sqrt{B_1^2+B_2^2}\cos(\omega_d t+\theta) \;.$$

This is a somewhat more direct expression for what you will observe in the lab.

A typical underdamped step response is shown in Figure 4.8.

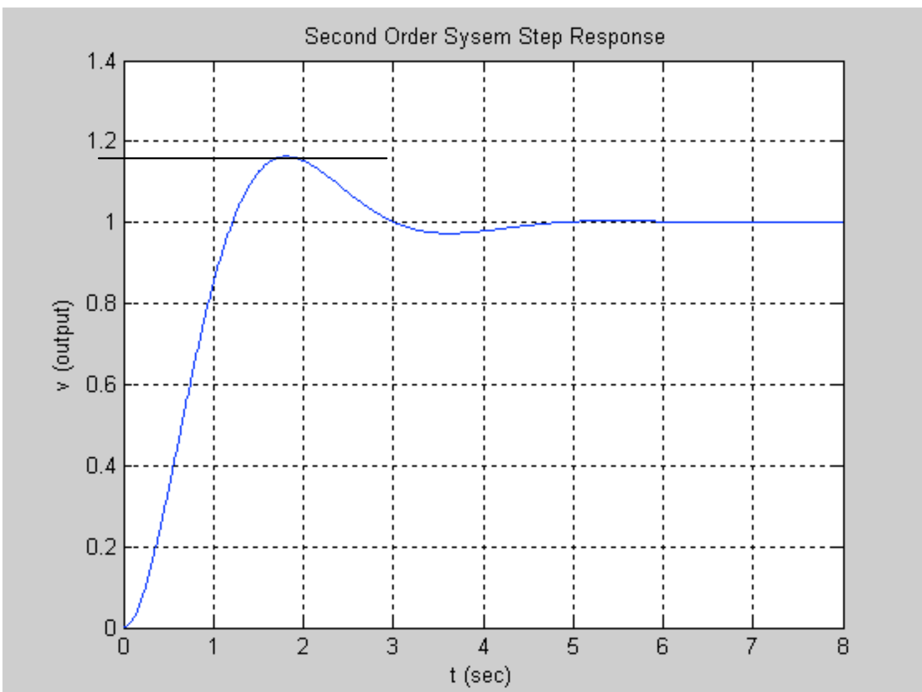

Figure 4.8 Second order system step response

The overshoot of the response is indicated by the horizontal line. In this case the overshoot is approximately 18%. The percent overshoot can be calculated by

$$\%OS = 100\frac{\text{Peak value - Final Value}}{\text{Final Value}}$$

It can be shown that the overshoot is related to the damping factor by

$$\%OS = 100e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

By taking the log (to the base *e*) of both sides one can solve for the value of ζ that produces a given value of overshoot. You will have an opportunity to do this during the Laboratory Exercises.

Prelab Exercises

Complete the following exercises prior to coming to the lab. As usual, turn-in your prelab work to the lab instructor before starting the Laboratory Exercises.

The following exercises are to be performed and checked by the laboratory instructor prior to performing the Laboratory Exercises. You should study the background material presented with this laboratory prior to performing the Prelab Exercises.

Part 1PE: Series *RC* Circuit

Finding equations for $\mathrm{i}(t)$ in the series *RC* circuit. [$R = 5000\Omega$ and $C = 1\mu\text{F}$]

(a) Start with

$$V_{\mathrm{i}} = R\,\mathrm{i}(t) + \frac{1}{C}\int_0^t \mathrm{i}(\lambda)\,d\lambda + \mathrm{v}_c(0)$$

Take the derivative on both sides of this equation to form the differential equation

$$\frac{d\,\mathrm{i}(t)}{dt} + \frac{1}{RC}\mathrm{i}(t) = 0$$

You are to solve this equation, with $\mathrm{v}_c(0) = 0$, to show that the expression for $\mathrm{i}(t)$ is given by

$$\mathrm{i}(t) = \frac{V_{\mathrm{i}}}{R}e^{-t/RC} \quad , \; t > 0$$

(b) Starting with $\mathrm{v}_c(t) = V_{\mathrm{i}}\left(1 - e^{-t/RC}\right)$, $t > 0$ and using $\mathrm{i}(t) = C\frac{d\,\mathrm{v}_c(t)}{dt}$ you are to show that $\mathrm{i}(t) = \frac{V_{\mathrm{i}}}{R}e^{-t/RC}$, $t > 0$.

(c) During the Laboratory Exercises you will use $R = 5000\Omega$ and $C = 1\mu\text{F}$ for the circuit of Figure 4.1. Determine the *RC* time constant.

(d) Explain how to determine the time constant for the circuit of Figure 4.1 from the step response of the capacitor voltage.

Part 2PE: Series *RL* Circuit

(a) Finding equations for $\mathrm{v}_L(t)$ for the series *RL* circuit.[$R = 1000\Omega$ and $L = 19.2\text{H}$]

Start with $V_\mathrm{i} = R\mathrm{i}(t) + \mathrm{v}_L(t)$ and substitute $\mathrm{i}(t) = \frac{1}{L}\int_0^t \mathrm{v}_L(\lambda)d\lambda + \mathrm{i}(0)$ to form

$$\frac{d\,\mathrm{v}_L(t)}{dt} + \frac{R}{L}\mathrm{v}_L(t) = 0$$

You are to solve the above differential equation with $\mathrm{i}(0) = 0$ and show that

$$\mathrm{v}_L(t) = V_\mathrm{i}e^{-Rt/L} \quad , \; t > 0$$

(b) Show that the voltage across the resistor of the circuit in Figure 4.4 is given by

$$\mathrm{v}_R(t) = V_\mathrm{i}\left(1 - e^{-Rt/L}\right) \quad , \; t > 0$$

(c) In the Laboratory Exercises you will use a $R = 1000\Omega$ and $L = 19.2\text{H}$ in the circuit of Figure 4.4. What is the circuit time constant?

(d) If a step input is applied to the *RL* circuit of Figure 4.4, explain how to find the circuit time constant from the inductor voltage.

Part 3PE: Series *RLC* Circuit

(a) Find B_1 and B_2 in terms of *R*, *L* and *C*

(b) Show that *R* for critical damping for the circuit of Figure 4.4 is $R = 2\sqrt{\frac{L}{C}}$.

Laboratory Exercises

Part 1LE: This part of the lab involves construction and measurement of capacitor voltage in an *RC* circuit.

(a) Connect the circuit in Figure 4.9 with the values of *R* and *C* as shown.

Note : Make sure the capacitor is discharged before closing the switch each time. You can discharge the capacitor by shorting the two ends.

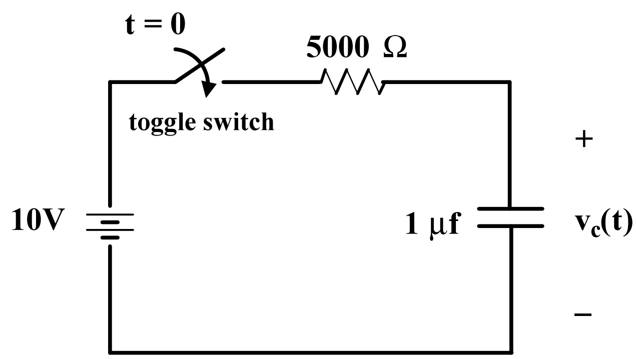

Figure 4.9 Circuit for Laboratory Exercise, Part 1LE (a).

Connect the oscilloscope across the capacitor. Depress the toggle switch and record the transient of $v_c(t)$.

(b) Using the cursors on the oscilloscope, find the time it takes $v_c(t)$ to change from 0 V to 63% of the final capacitor voltage. This will yield the measured time constant of the circuit. Obtain a printout of this transient.

Part 2LE: This part of the lab involves construction and measurement of an *RL* circuit.

It is not easy to find an inexpensive *L* component in the mH to H range unless you make your own inductor by winding a coil. To avoid this problem, the primary of the transformer in your parts kit is used for the *L*.

Diagrams of your transformer showing the configuration and the windings are given in Figure 4.10.

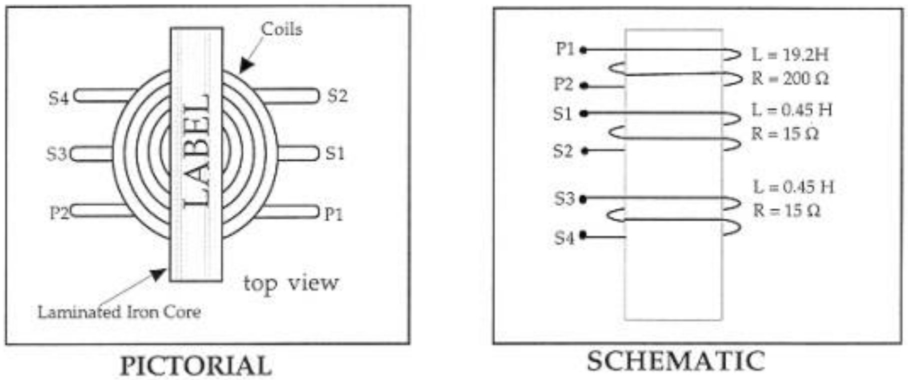

Figure 4.10 Diagram showing transformer used in Laboratory Exercise, Part 2LE (a).

The coil identified with terminal endings P1 and P2 will be used in connecting up your *RL* and *RLC* circuits. Be sure that all other coils of the transformer are left open as shown in the above schematic.

As the schematic of Figure 4.10 indicates, the P1-P2 winding has nominal inductance of 19.2 H and an inseparable resistance of 200 Ω . You might question why the coil has 200 Ω. Recall the resistance of wire is given by $R = \frac{\rho L}{A}$ where *L* is the length of the wire in meters, *A* is the cross sectional area in meters^2 and ρ is the resistivity of copper (in this case) in $\text{ohm} \cdot \text{meters}$. P1-P2 is a coil of several hundred turns made-up of very small diameter wire with 200 Ω of resistance.

Since the resistance is inherent with the *L* value, we are not able to read the inductor voltage directly. In this lab we will read either resistance voltage (a resistor separated from the coil) in the *RL* circuit or the capacitor voltage in the *RLC* circuit. Resistance voltage and capacitance voltage have the same characteristic equation.

(a) Construct the circuit of Figure 4.11. Connect the oscilloscope across the 3300 Ω resistor

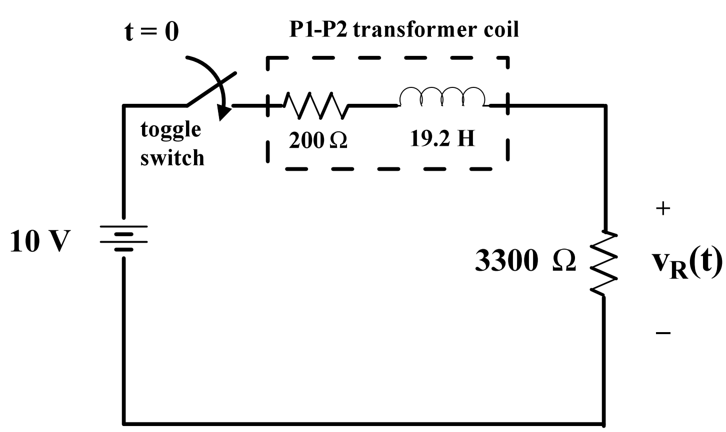

Figure 4.11 Circuit used for Laboratory Exercises, Part 2LE (a)

(b) Depress the toggle switch and capture the resistor step response voltage. Use the cursor keys to determine the time for the resistor voltage to reach 63% of its final value. Obtain a hardcopy of this response.

Part 3LE: This part of the lab involves construction and measurement of capacitor voltage in an *RLC* circuit.

(a) Critically Damped. Construct the circuit shown in Figure 4.12 using the circuit parameters indicated in diagram. Determine the value of R_{total} that causes the circuit to be critically damped. With $R_{\text{total}} = R + 200$ calculate R and use this value in the circuit.

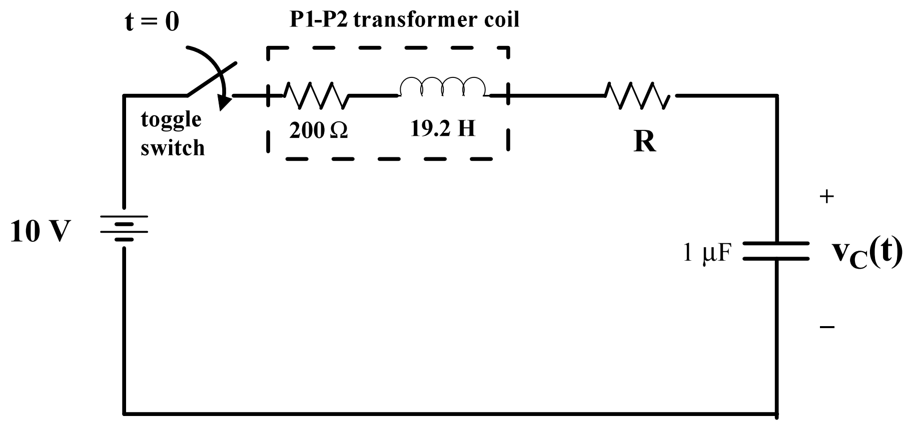

Figure 4.12 Circuit used for Laboratory Exercises, Part 3LE (a).

(b) Connect the oscilloscope across the capacitor. Depress the toggle switch and capture the step response. Use the cursor keys to measure the 10% to 90% rise time. Obtain a hardcopy of this response with rise time markings.

(c) Overdamped. Connect the circuit shown in Figure 4.11 with an R value of 39 kΩ. This R will cause the circuit responses to be overdamped. Connect the oscilloscope across the capacitor.

(d) Depress the toggle switch and capture the step response voltage across the capacitor. Use the cursor keys of the oscilloscope to measure the 10 % to 90 % rise time for the capacitor voltage. Obtain a hardcopy of this response showing the rise time markers.

(e) Underdamped. Connect the circuit shown in Figure 4.11 with an R value of 3300 Ω. This value of R should cause the circuit to be underdamped. Connect the oscilloscope across the capacitor.

(f) Depress the toggle switch and capture the capacitor step response voltage. Use the cursor keys of the oscilloscope to measure the 10% to 90% rise time of the capacitor voltage. Obtain a hardcopy of the response showing the rise time markers.

Before Leaving The Laboratory

Be sure the following is completed before you leave the laboratory.

(a) Make sure you have all the necessary readings and printouts.

(b) Have your readings checked off by the TA.

(c) Restore your lab station (equipment and chair) to the condition they were in when you arrived and remove any debris from the work area and floor.

Thank You for your cooperation

Questions, Comparisons and Discussions

The following should be completed and included with your laboratory report.

(a) Why is it necessary to discharge the capacitor every time you want to record another transient voltage across the capacitor?

(b) If the capacitor remains charged, what would you expect to see across the capacitor when you re-close the switch to try to record another transient?

(c) Compare the *RC* time constant calculated in Part 1PE(c) with the measured value found in Part 1LE(b).

(d) Compare the *L*/*R* time constant calculated in Part 2PE(c) with the measured value found in in Part 2LE(b).

(e) What conclusion did you reach regarding the 10% to 90% rise time for the overdamped, critically damped, and underdamped responses of the *RLC* circuit?

(f) Refering to the characteristic equation $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$. Use your value of R_{total}, C = 1μF and L = 19.2 H to determine the characteristic equation. Compare this equation with $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ and determine numerical values for ζ and ω_n.

(g) From your step response of Part 3LE(f), use Equation 4.29 to calculate the %OS for the step response.

(h) Use the overshoot calculated above and $\%OS = 100e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$ to determine ζ. Compare this value of ζ with that obtained in (f) above.

Laboratory Report

The following should be included in your laboratory report. If you have any questions be sure to contact the lab instructor.

(a) Give a short summary (50 to 100 words) of what is to be accomplished in the lab exercise.

(b) Write the procedure followed for each part of lab work.

(c) Present all the printout of the oscilloscope screen neatly labeled.

(d) Answer the questions listed above.

(e) Write a brief conclusion (approximately 200 words)

(f) Attach the graded prelab at the end of your report.