# THE STRONG PERFECT GRAPH THEOREM

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and
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joint work with

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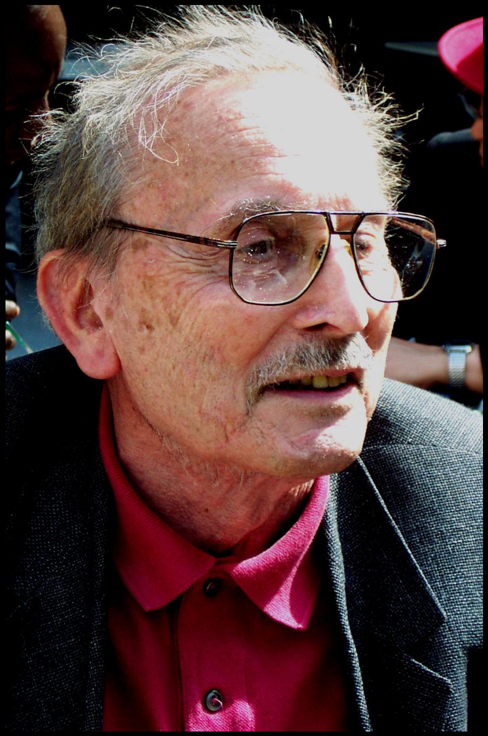
# PART I

History and relevance of perfect graphs

# PART II

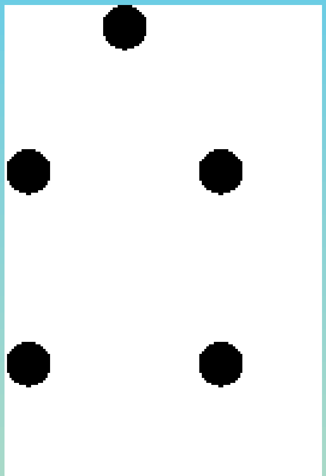
The strong perfect graph conjecture

# CLAUDE BERGE



1926–2002

# Graphs have vertices



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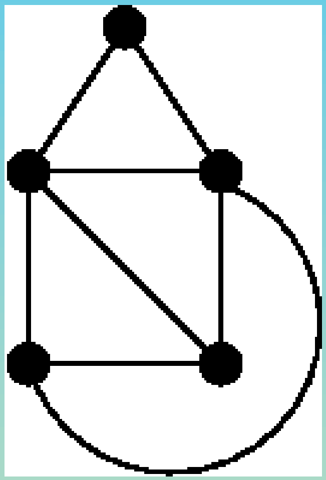

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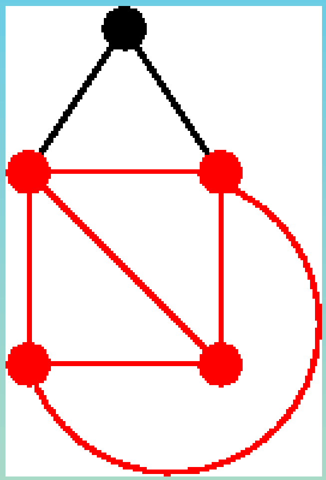

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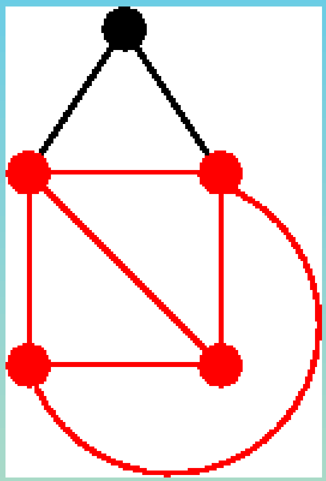

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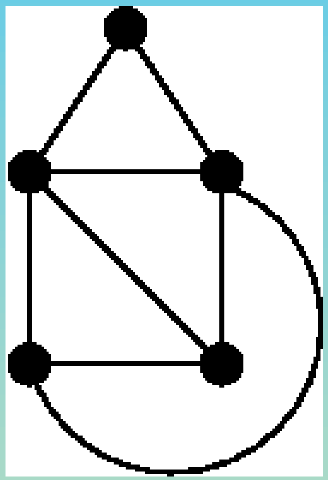

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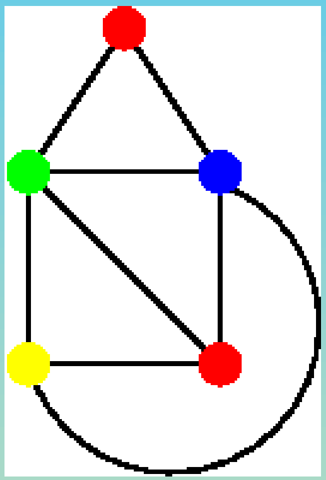

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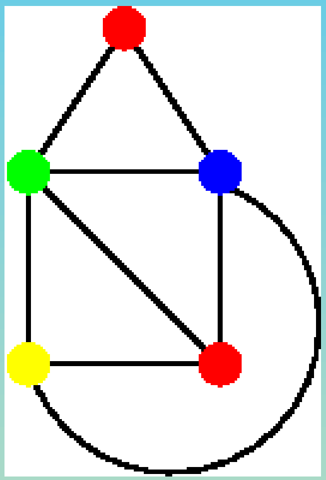

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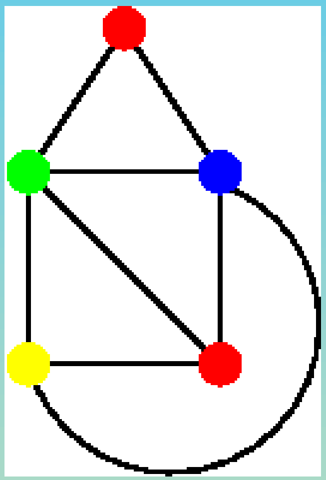

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Graphs that have an odd hole or odd antihole

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THE STRONG PERFECT GRAPH CONJECTURE (SPGC) (Berge 1960)
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- Fundamental and beautiful open problems

THEOREM (Lovász) Let $A$ be a $0,1$-matrix. For every non-negative objective function $c$ the LP

$$\max c^T x \text{ subject to } x \geq 0 \text{ and } Ax \leq 1$$

has integral optimum $\Leftrightarrow$ the undominated rows of $A$ form the vertex versus maximal cliques incidence matrix of some perfect graph.

# PART II

## The Strong Perfect Graph Conjecture

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MAIN THEOREM Every Berge graph is either basic, or has a certain decomposition.

# THE SPGC WAS KNOWN FOR

- planar graphs (Tucker)
- claw-free graphs (Parthasarathy, Ravindra)
- $K_4$-free graphs (Tucker)
- diamond-free graphs (Tucker)
- bull-free graphs (Chvátal, Sbihi)
- dart-free graphs (Sun)
- $C_4$-free graphs (Conforti, Cornuéjols, Vušković)
- "wheel-and-parachute-free" graphs (Conforti, Cornuéjols)

NOTE All of the above exclude specific graphs.

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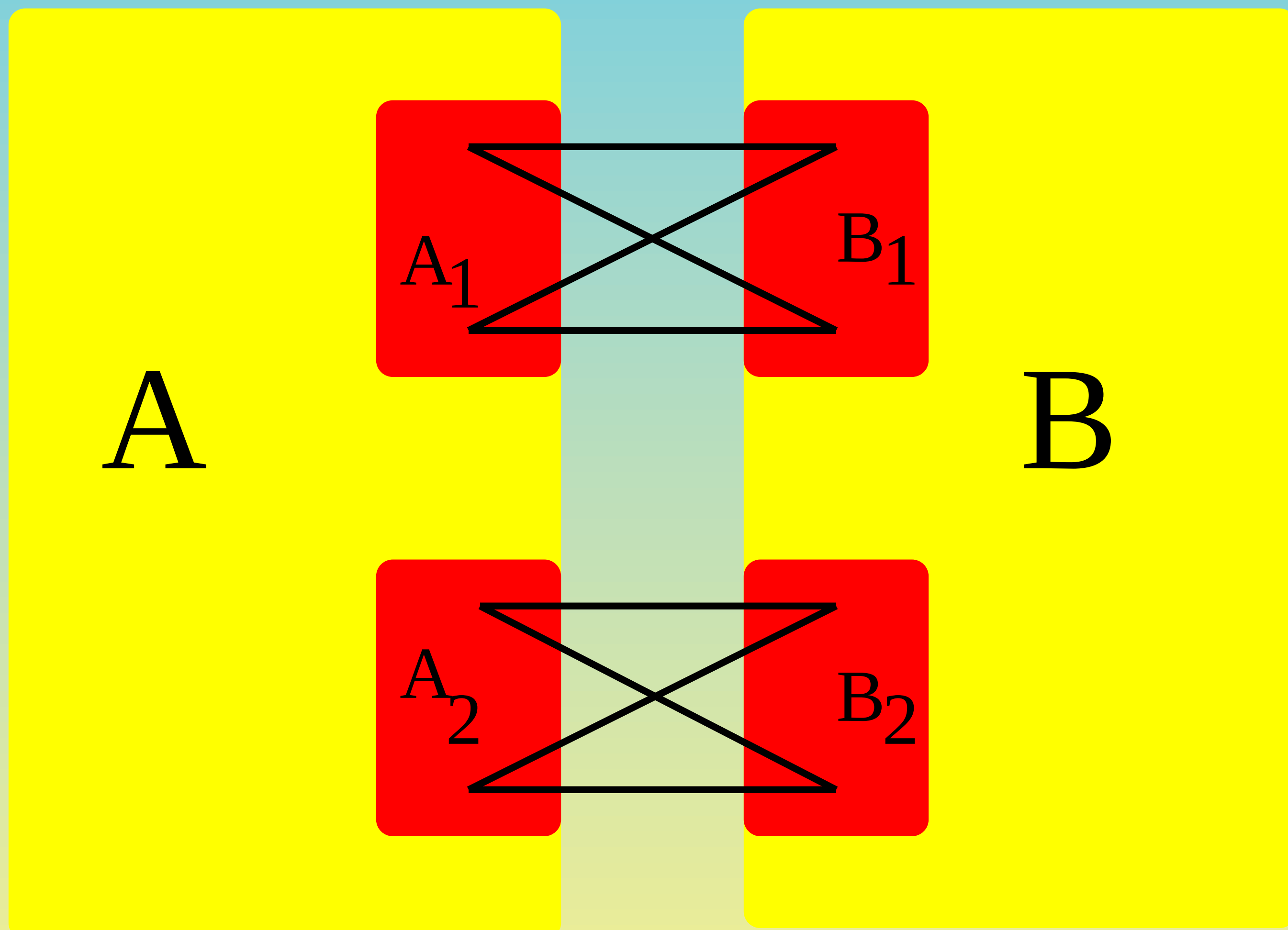

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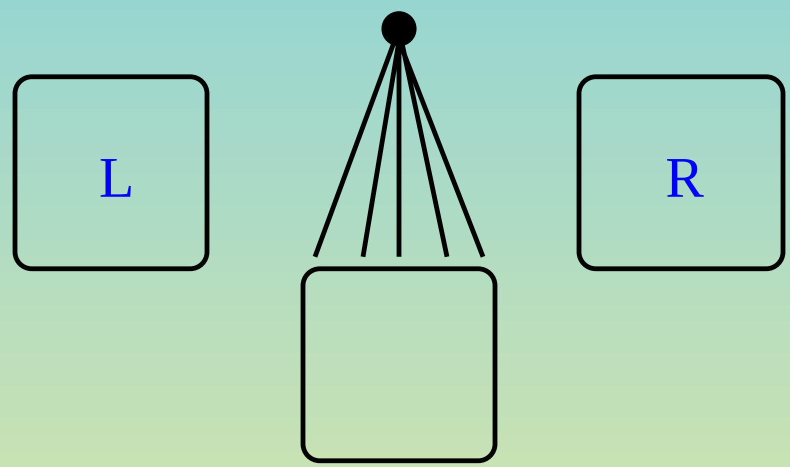

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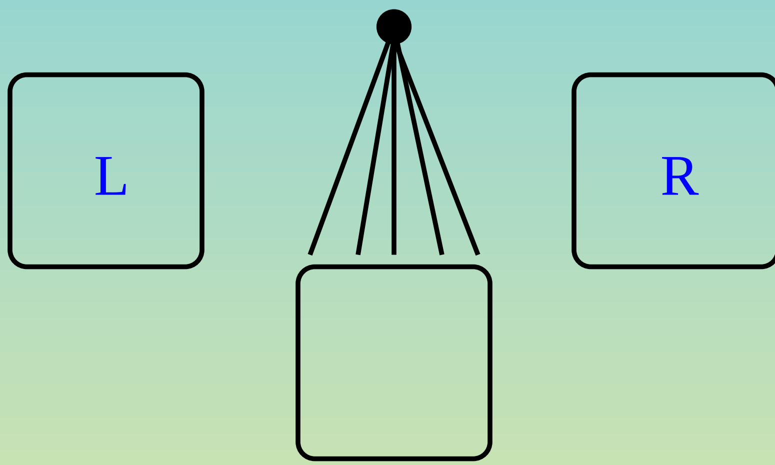


THEOREM (Chvátal) No minimally imperfect graph has a star cutset.

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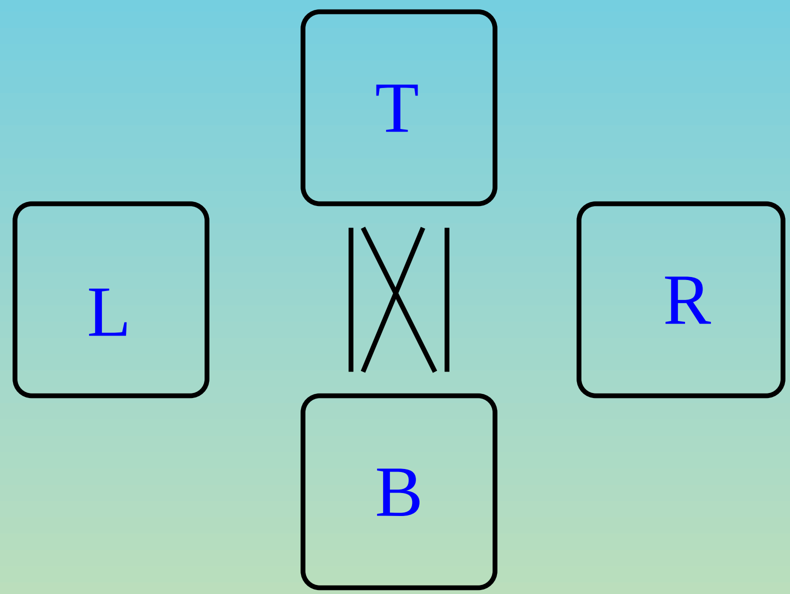

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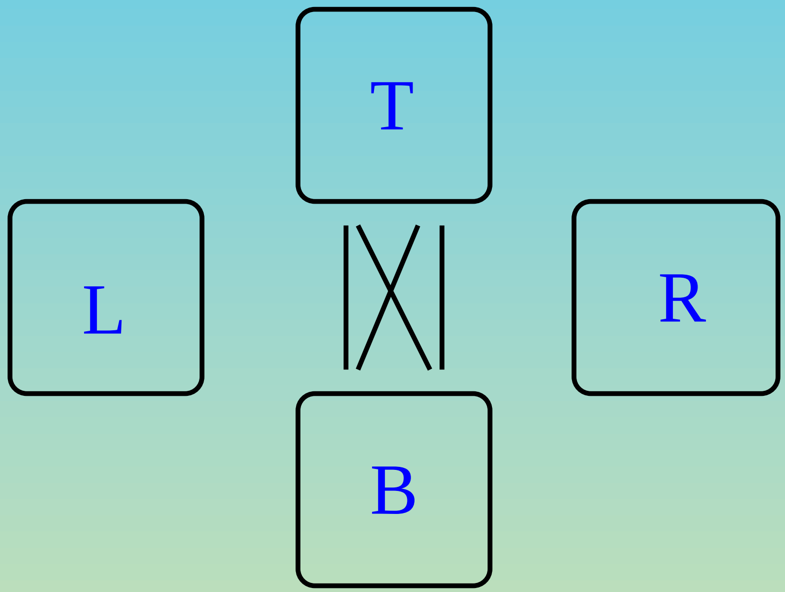


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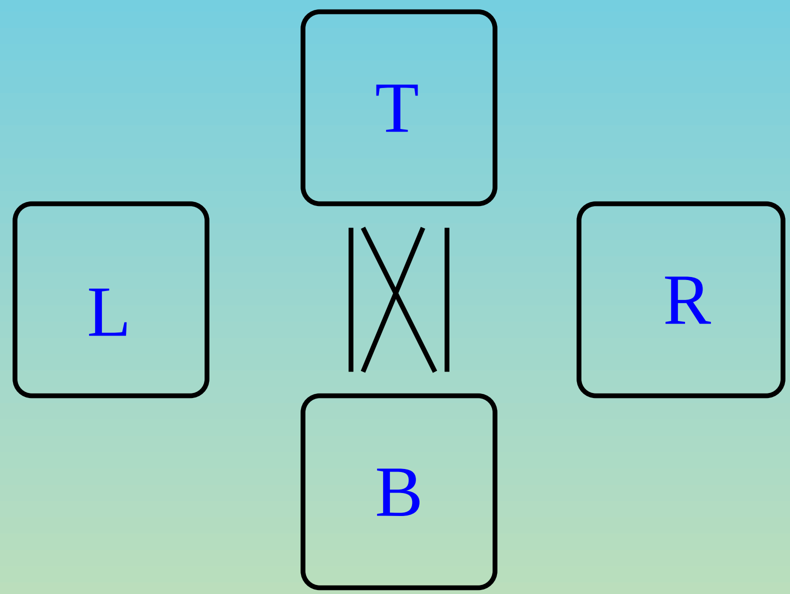


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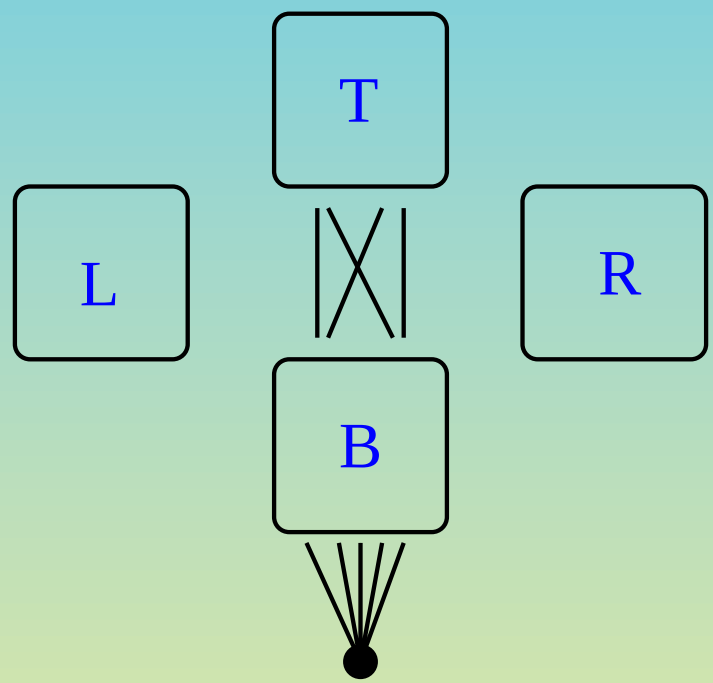

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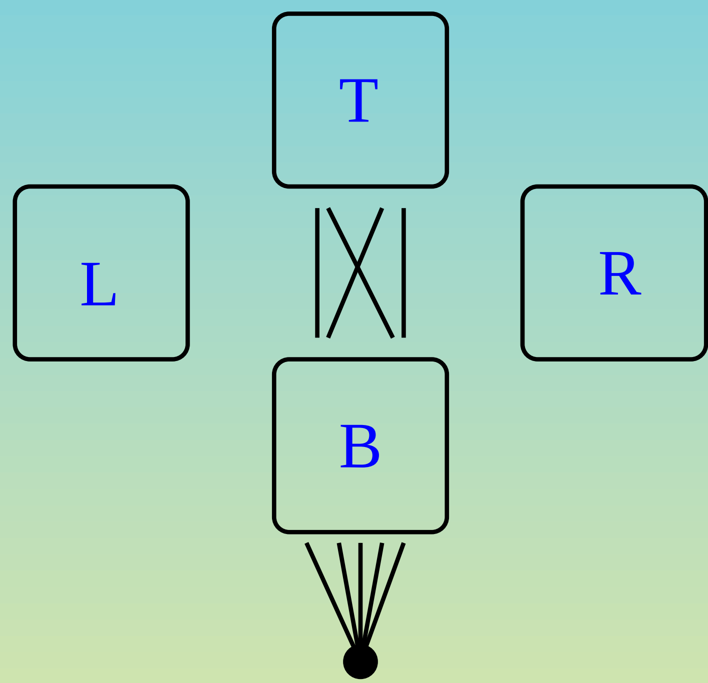


THM No minimum imperfect graph has an even skew partition.

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$(1)$ is bipartite, or

$(2)$ is a line graph of a bipartite graph, or

$(3)$ is a bicograph, or

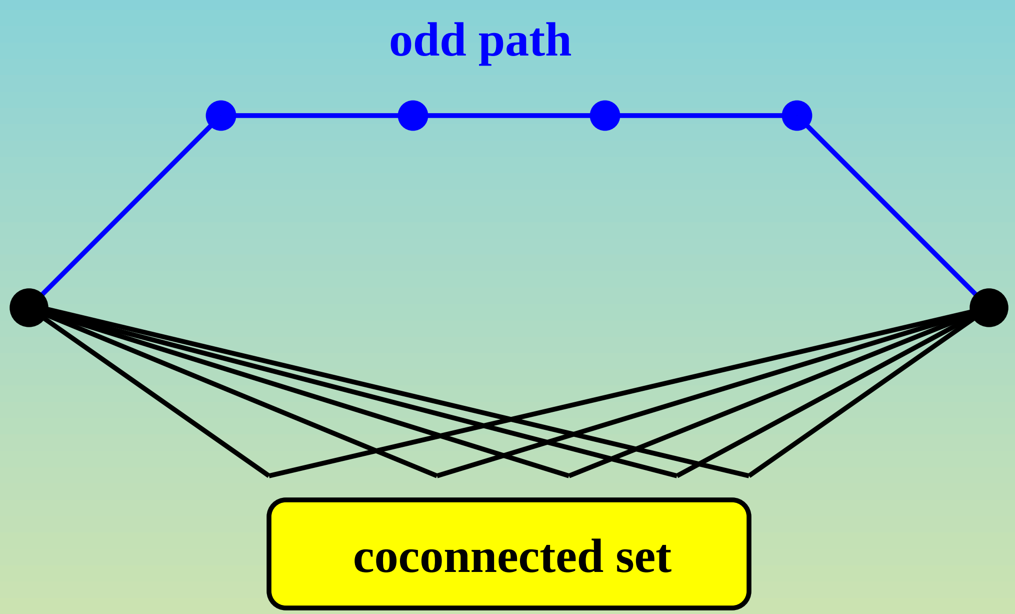
$(4)$ has an even skew partition, or

$(5)$ has a $2$-join, or

$(6)$ has an M-join.

# A LEMMA ABOUT ODD PATHS

Roussel & Rubio, RST In a Berge graph, if

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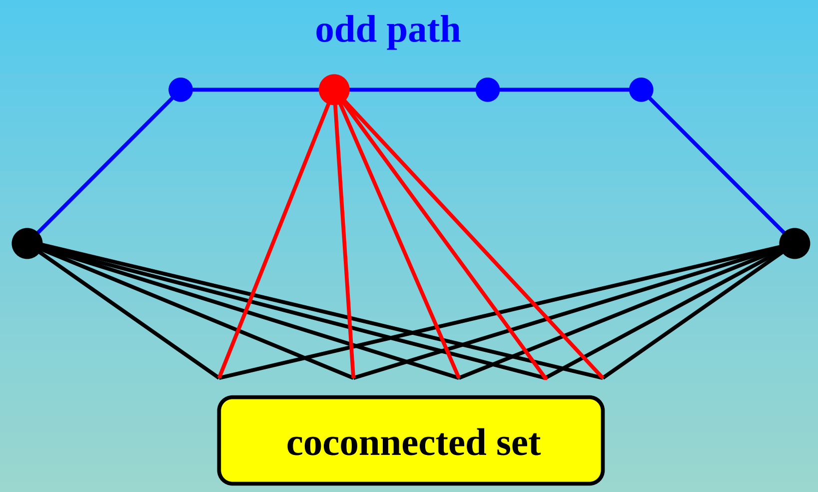
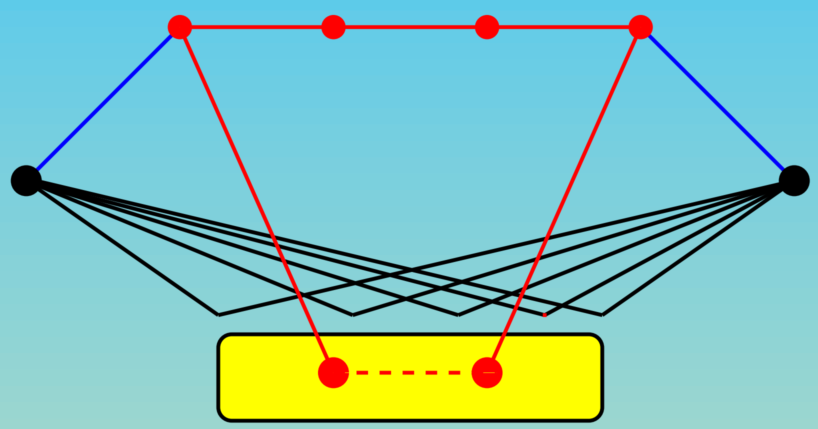
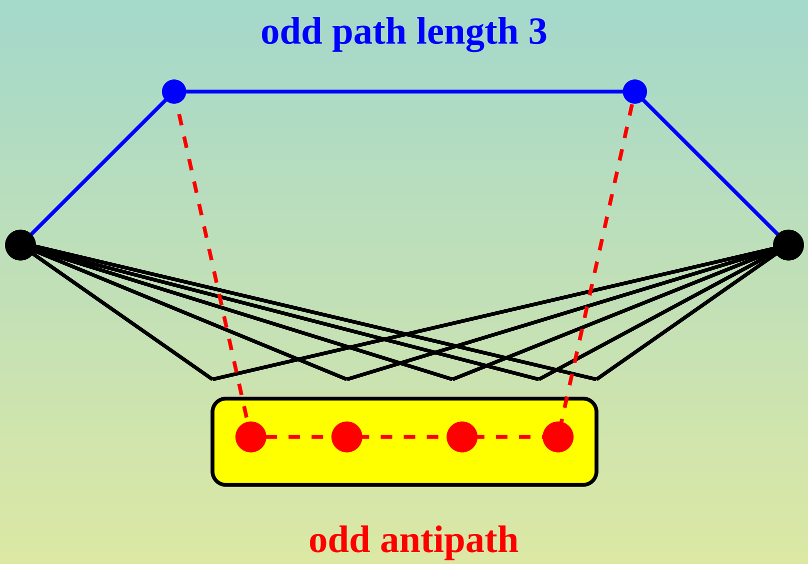


odd path
coconnected set

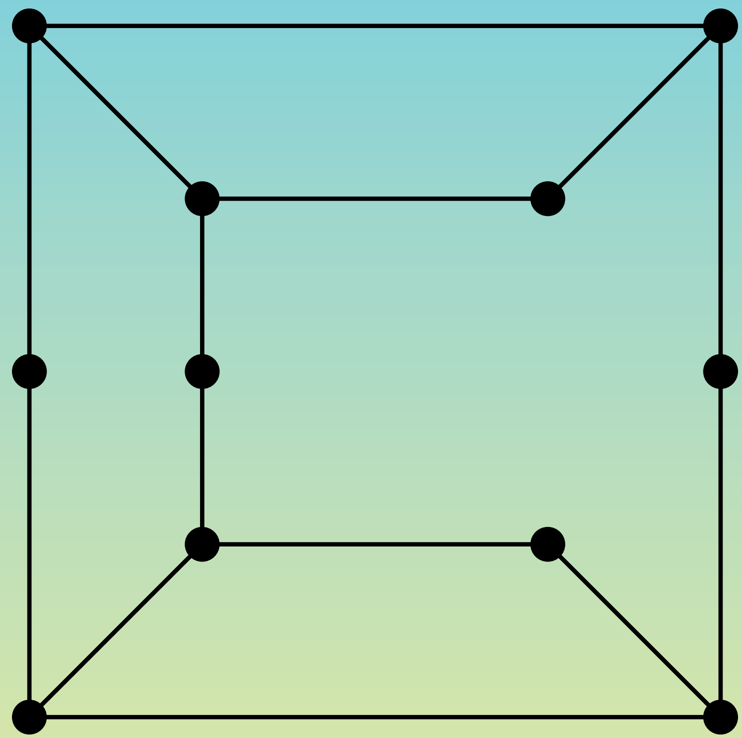


odd path length 3
odd antipath

# MAIN CASES

- $G$ contains a "large" line graph of a bipartite graph
- $G$ contains a "wheel"
- neither of the above

# LINE GRAPHS



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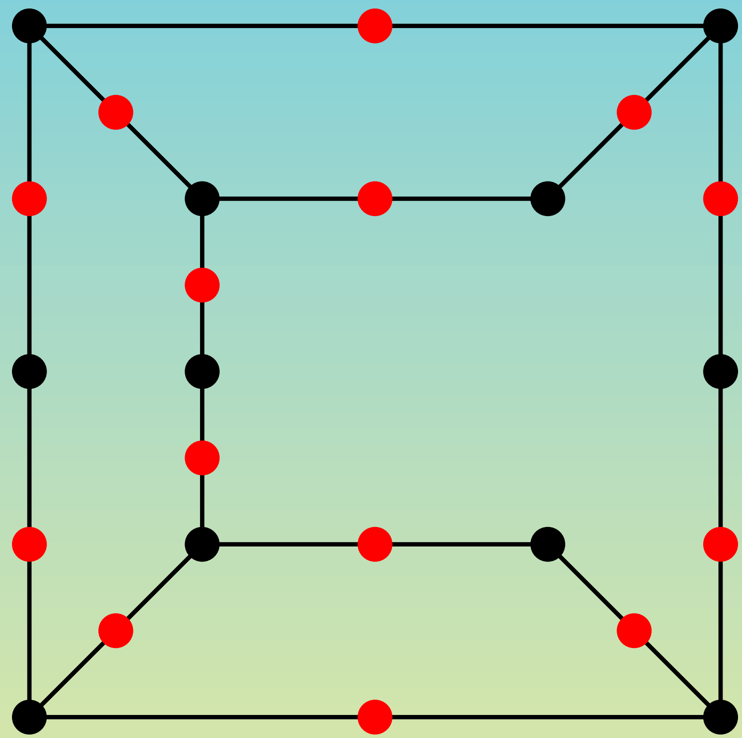

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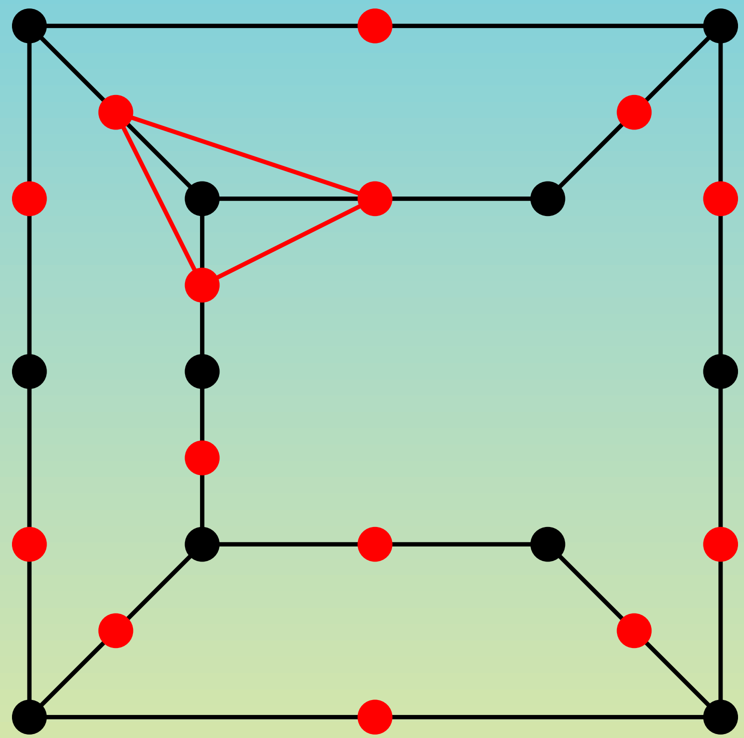

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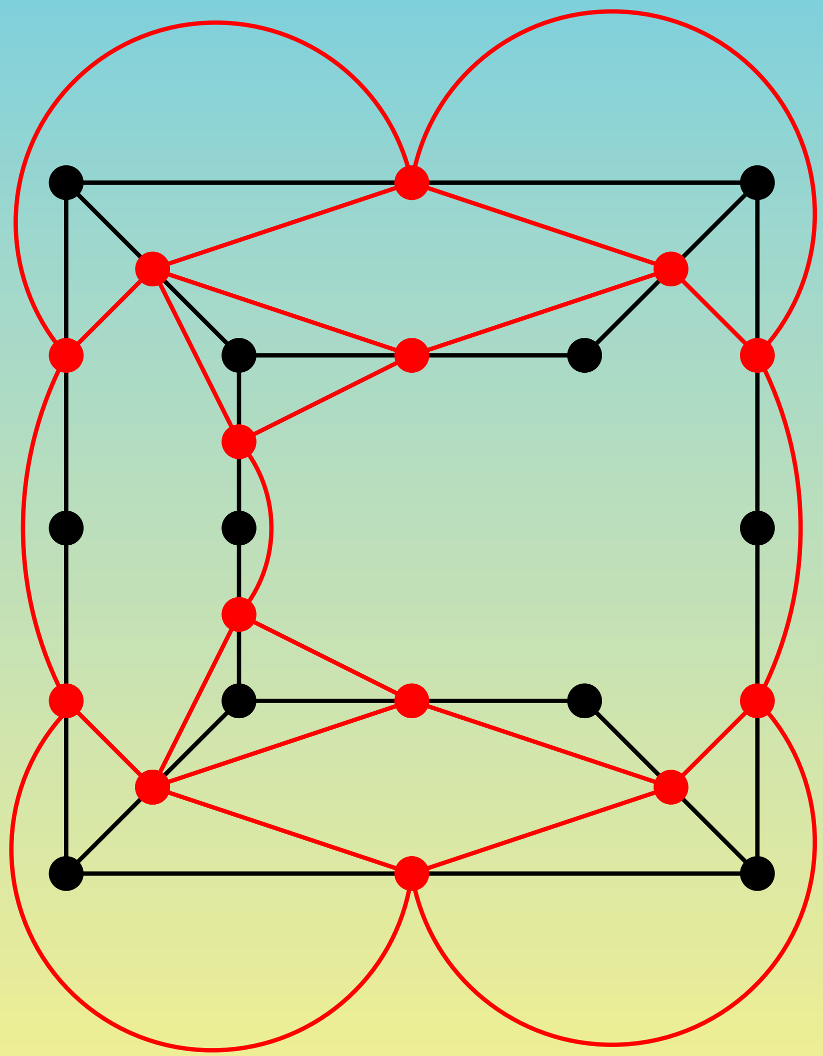

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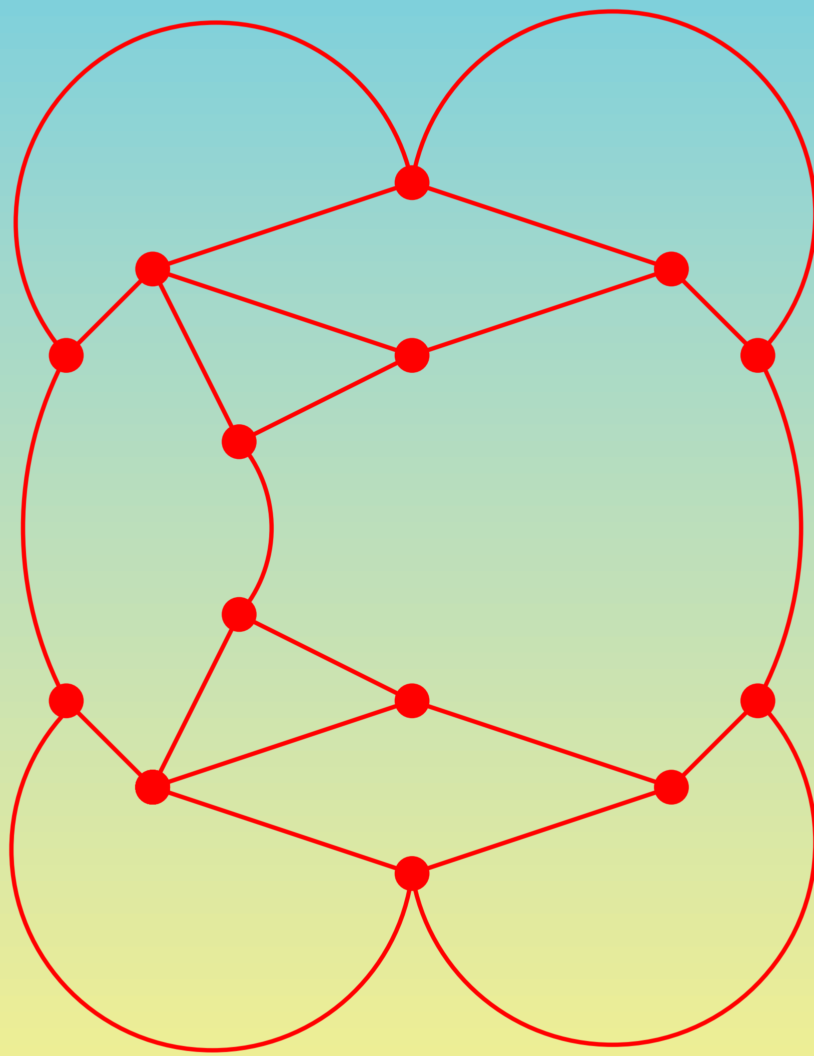

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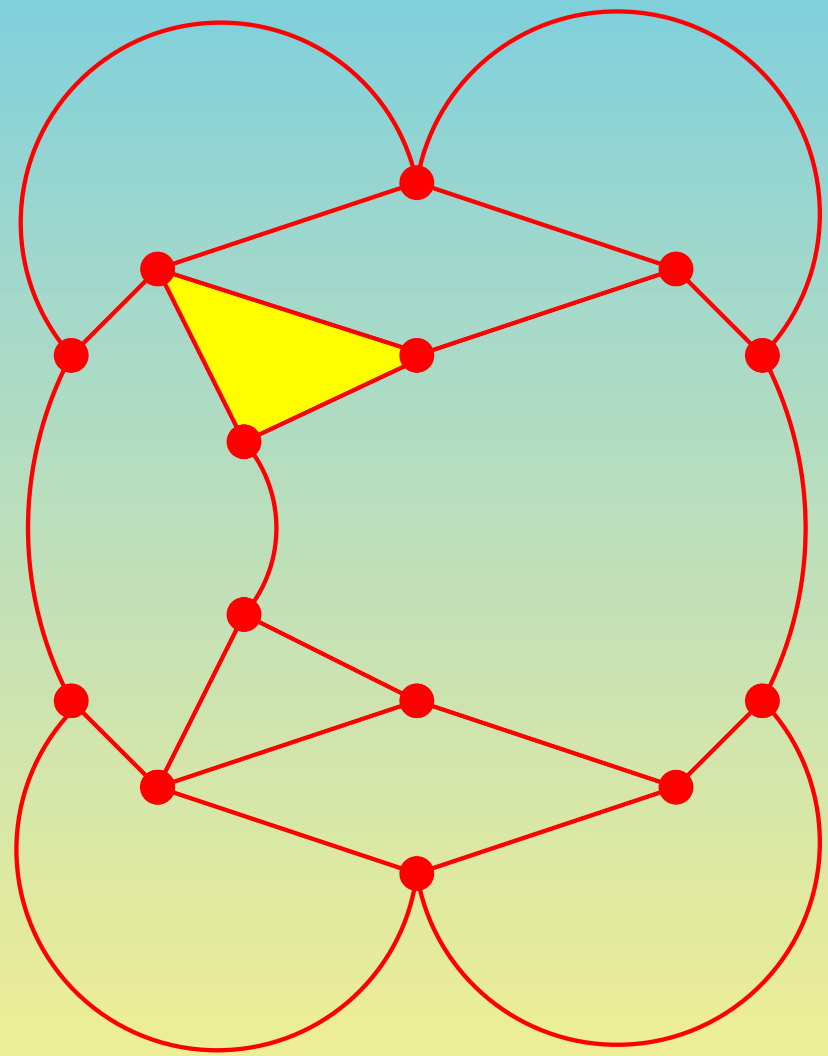

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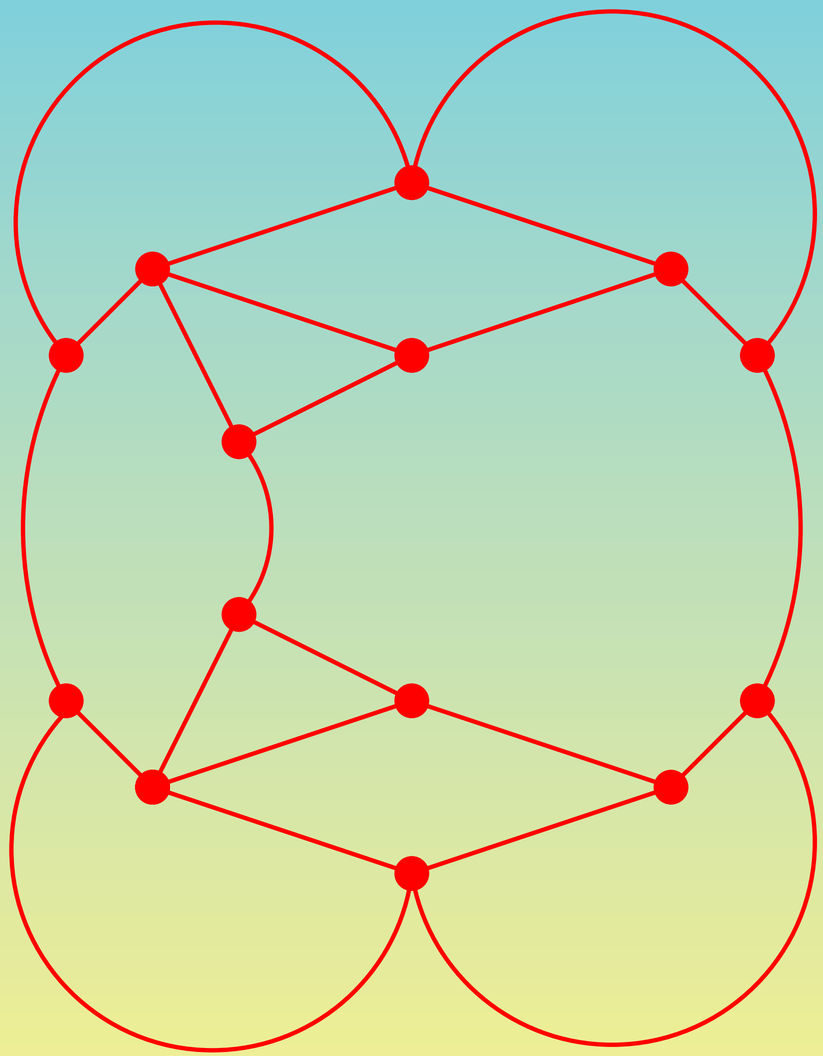
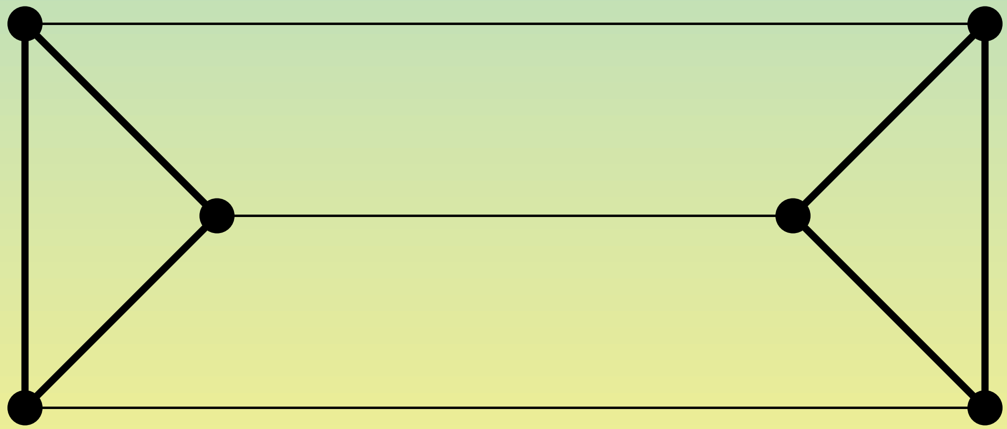


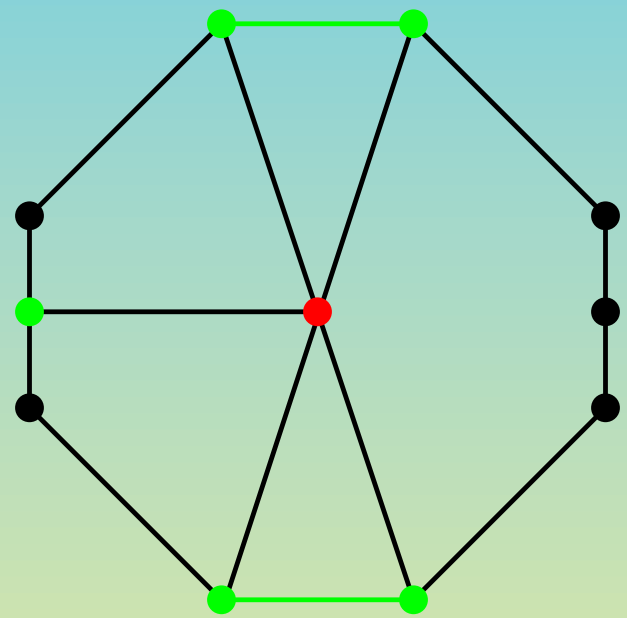
# PRISMS

THEOREM If a Berge graph has a prism, then it or its complement is a line graph of a bipartite graph, is a bicograph, has skew partition, has a $2$-join or has an M-join (and hence satisfies the conclusion of the main theorem).



# SECOND STEP: WHEELS

A wheel consists of a hole of length $\geq 6$ ("rim") and a vertex ("hub") forming $\geq 2$ triangles.



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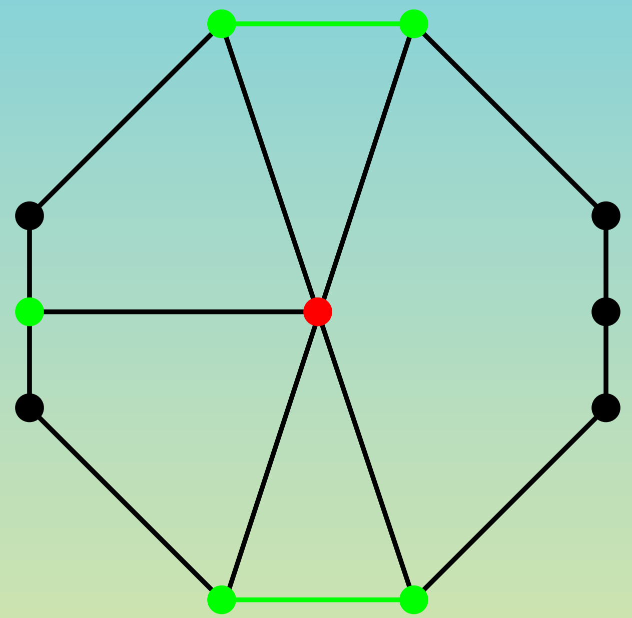
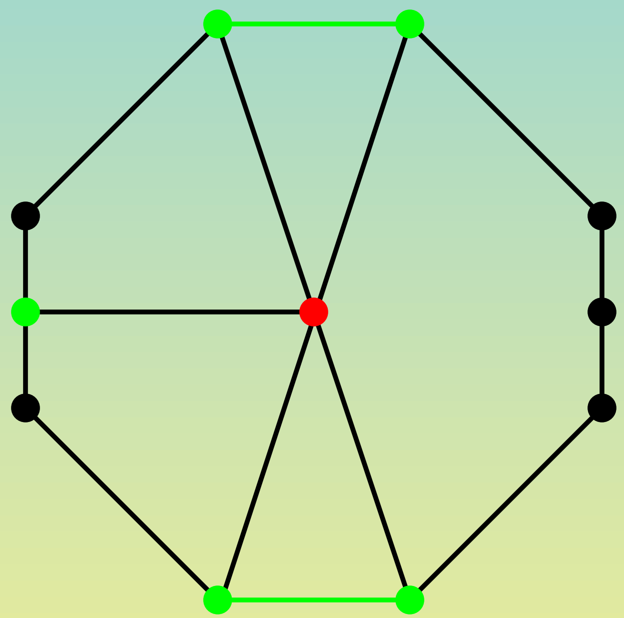


THEOREM If a Berge graph has a wheel, then it or its complement has a prism, a skew partition or a $2$-join.

# STRATEGY

Take a maximal coconnected set of hubs such that their common neighbors on the rim form $\geq 2$ edges. Take all common neighbors of hubs. This tends to be a skew cutset.



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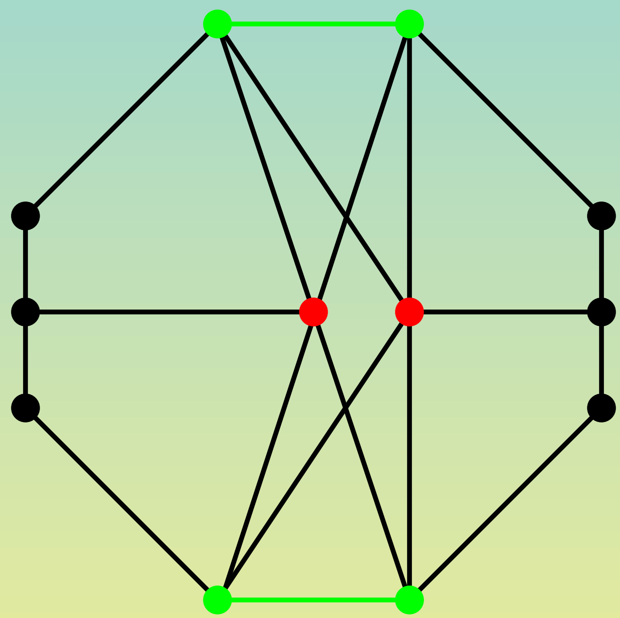

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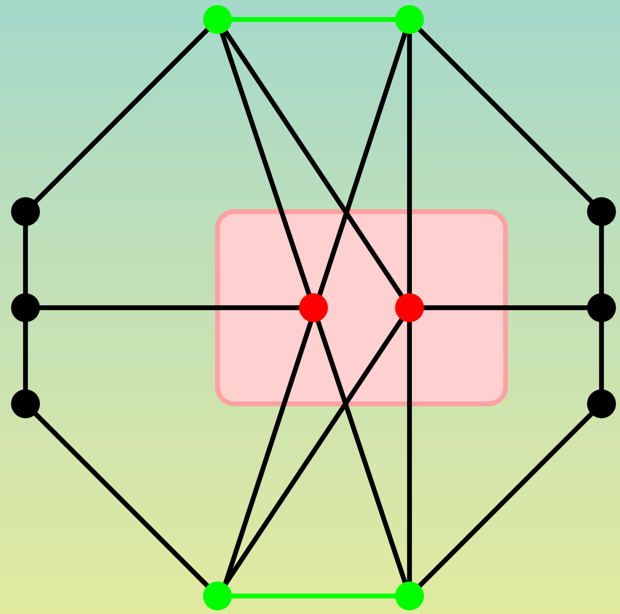

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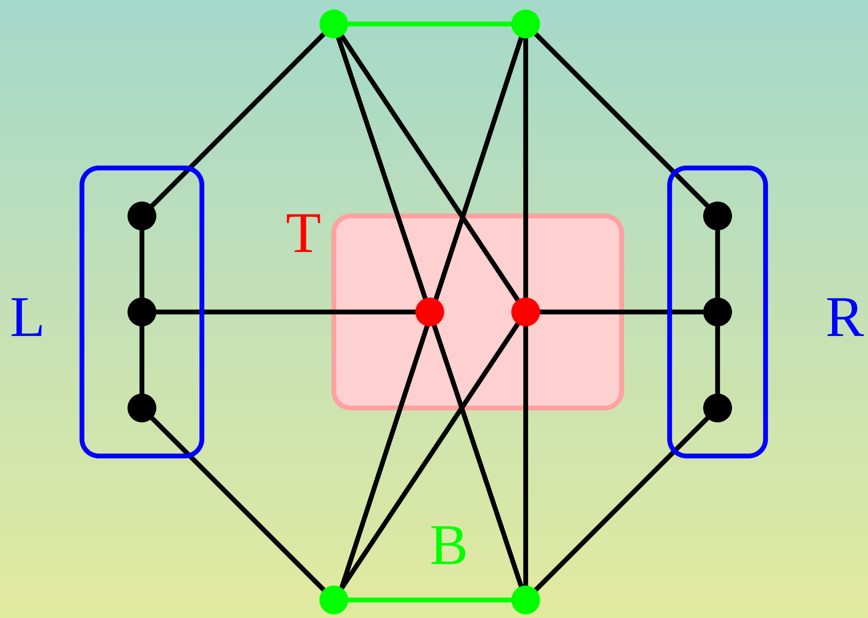
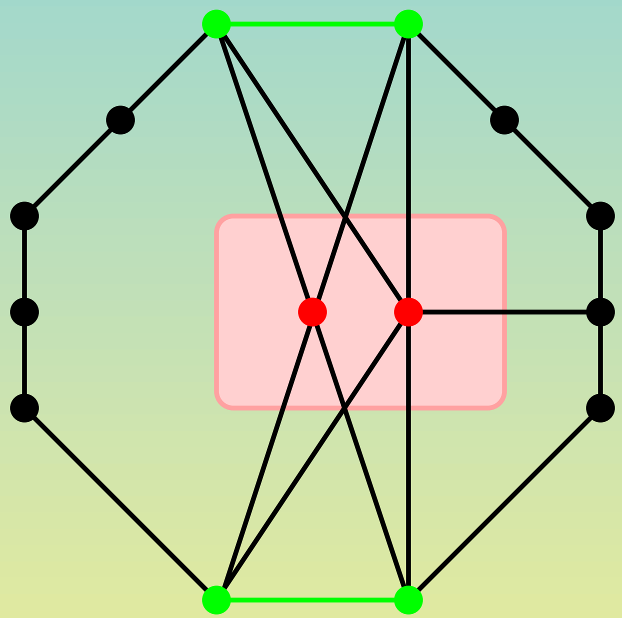


# APPLICATION OF ODD PATH LEMMA

Every subpath of the rim between common neighbors of hubs is even or an edge.



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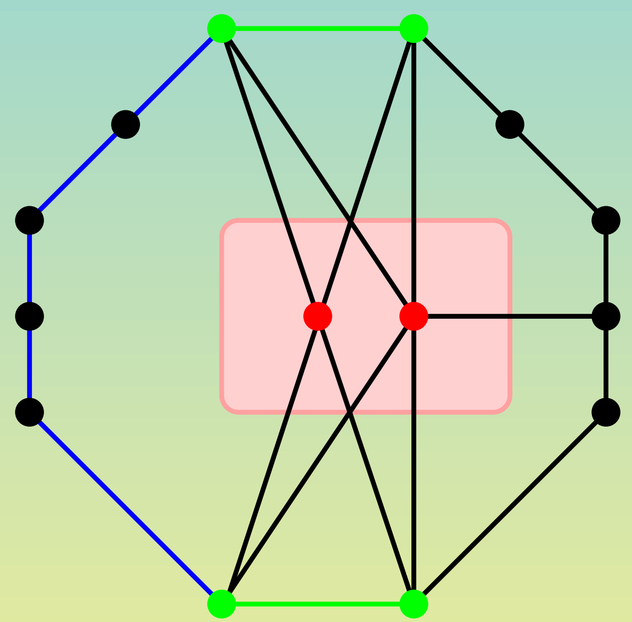

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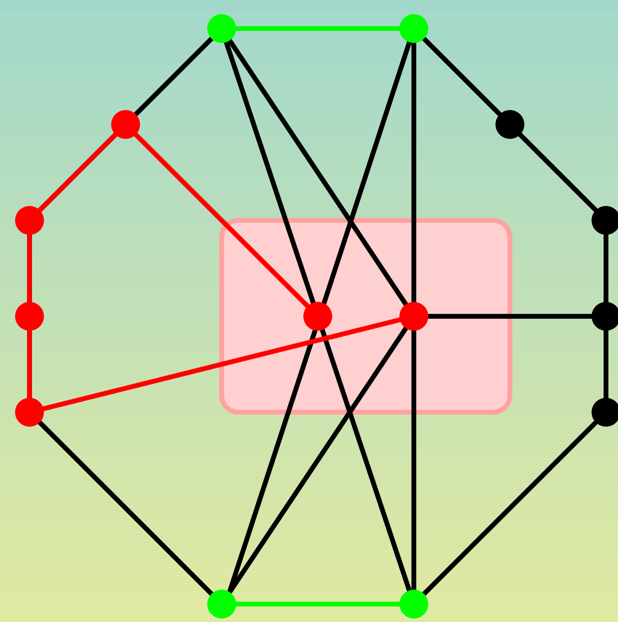

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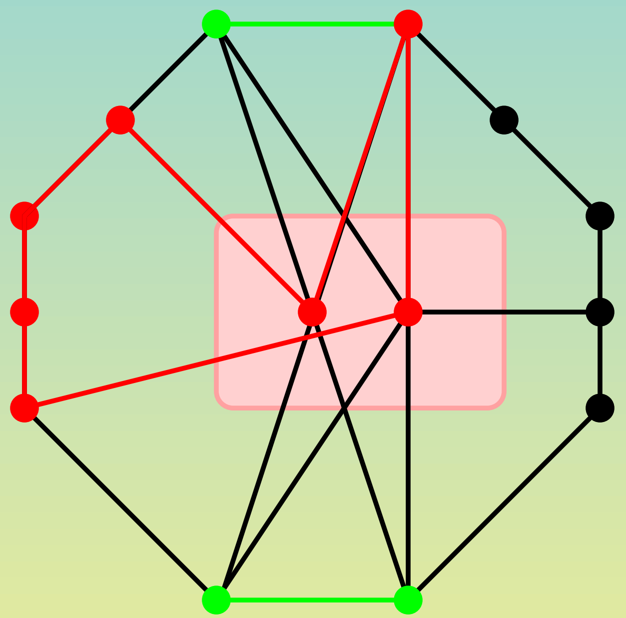

# THIRD STEP

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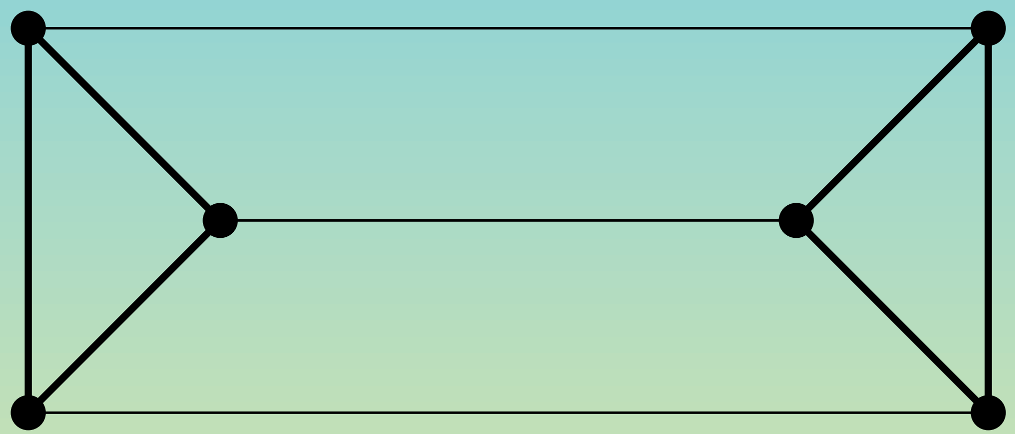

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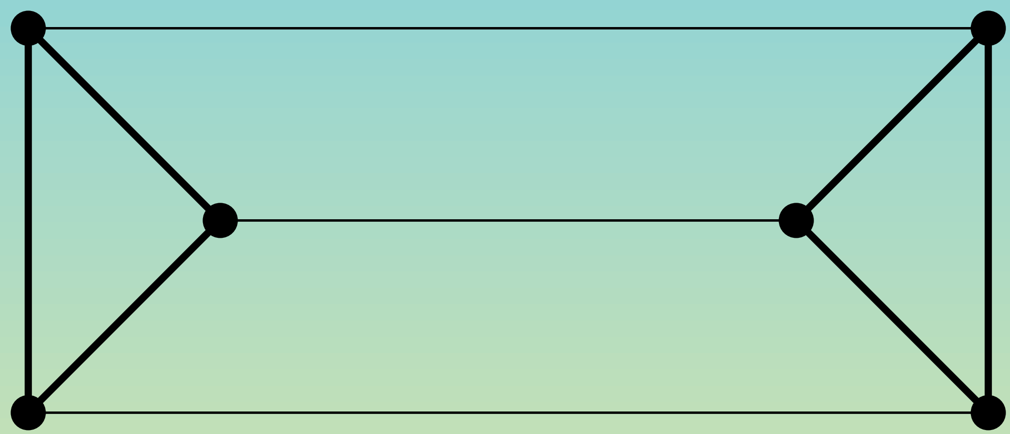

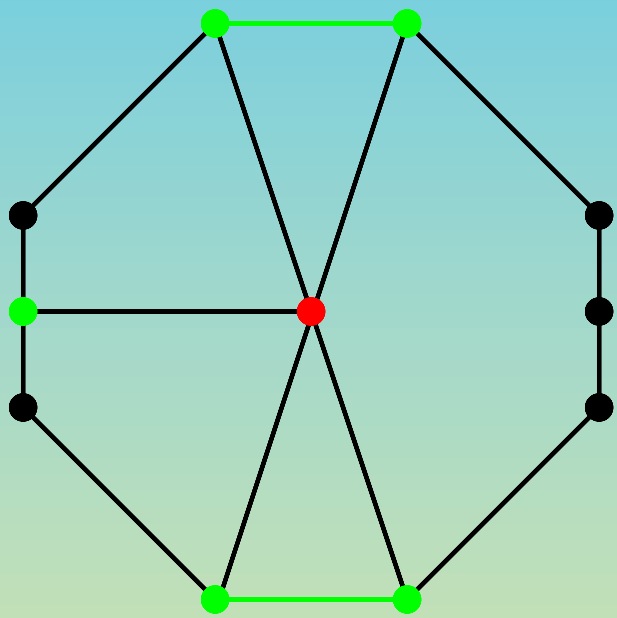

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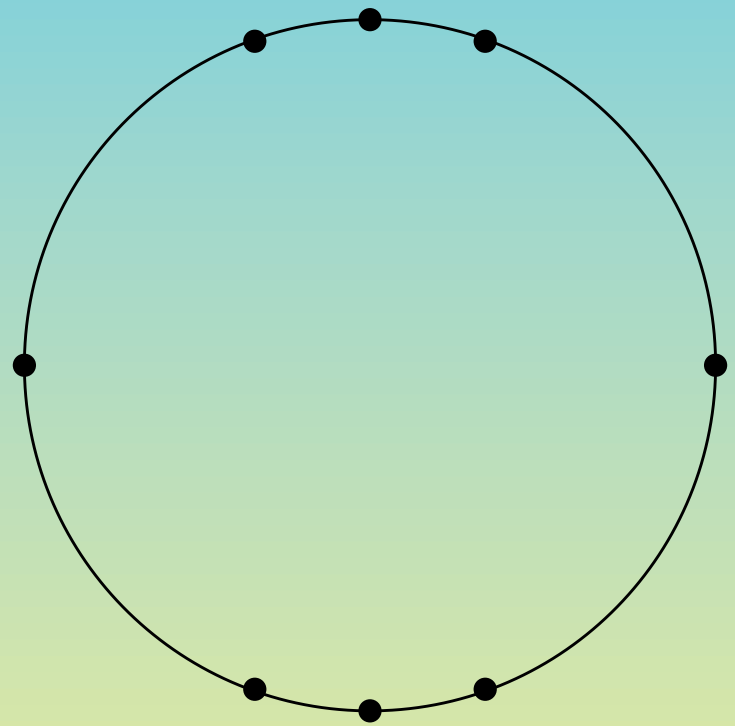

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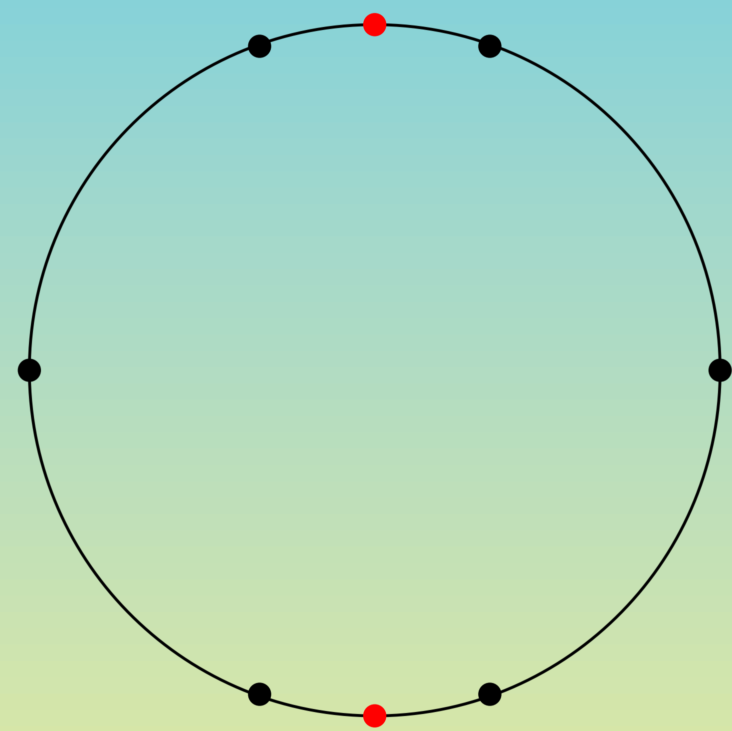

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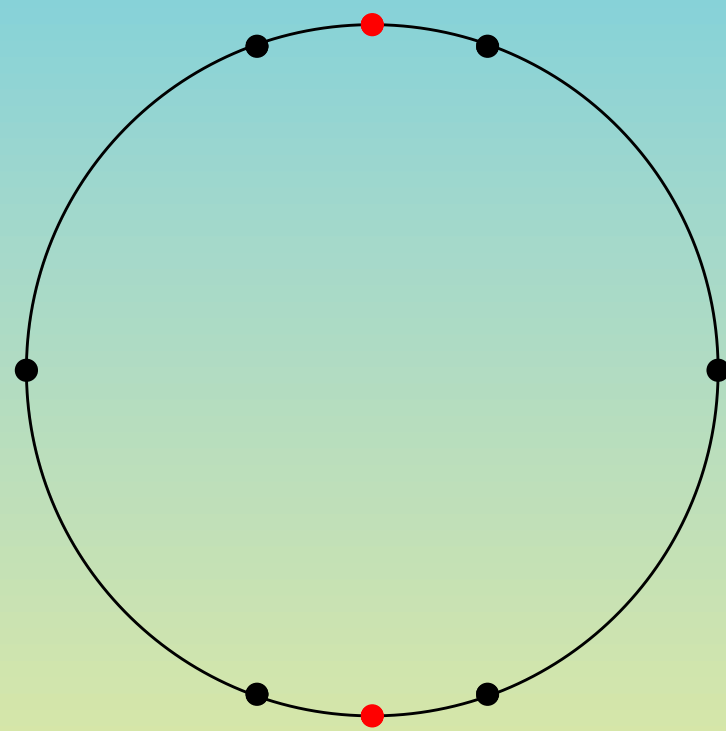

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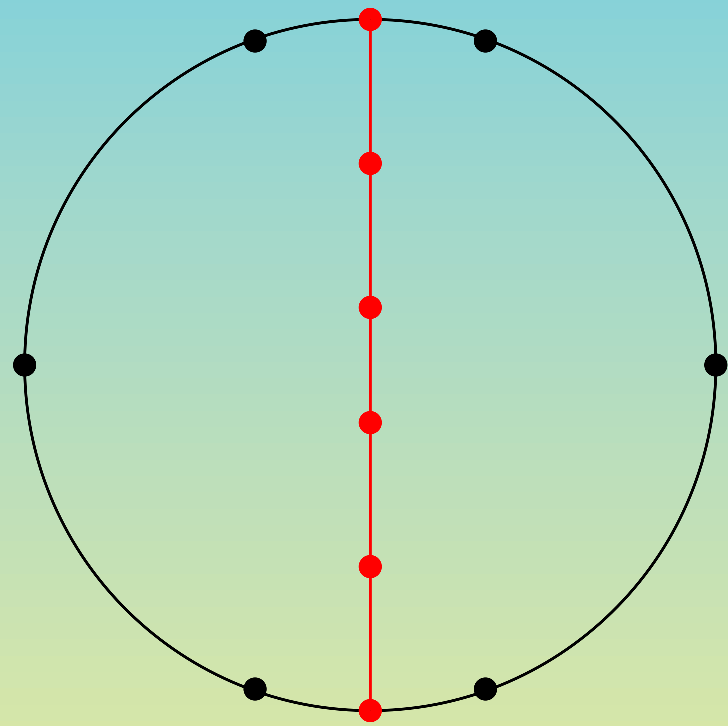

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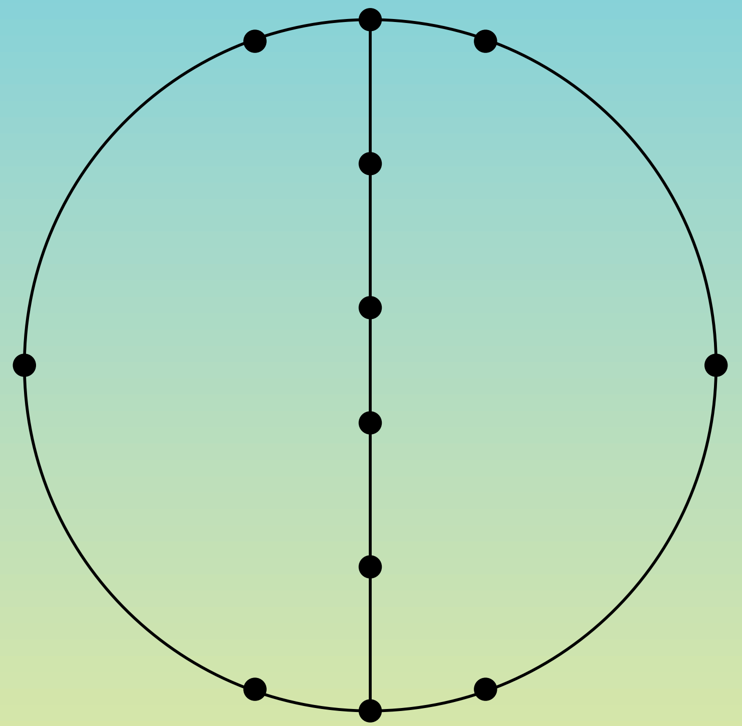

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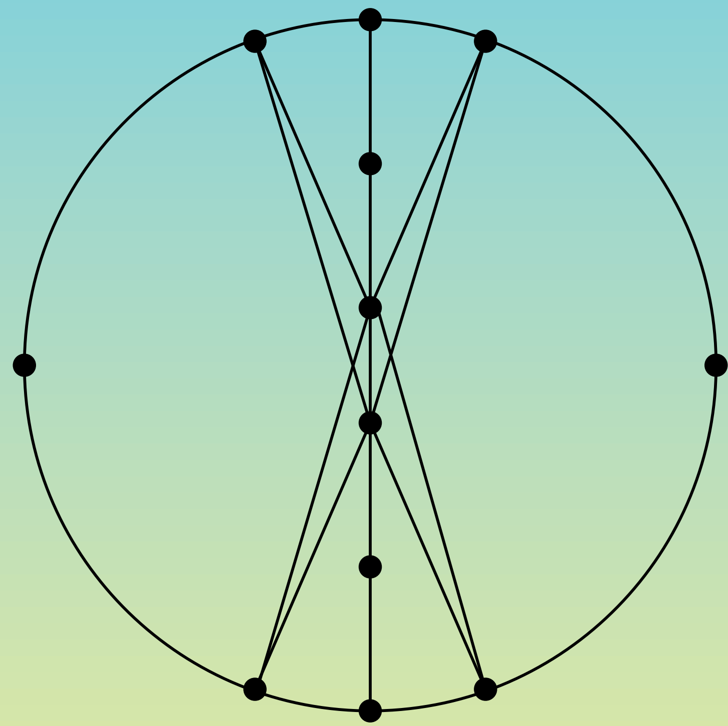

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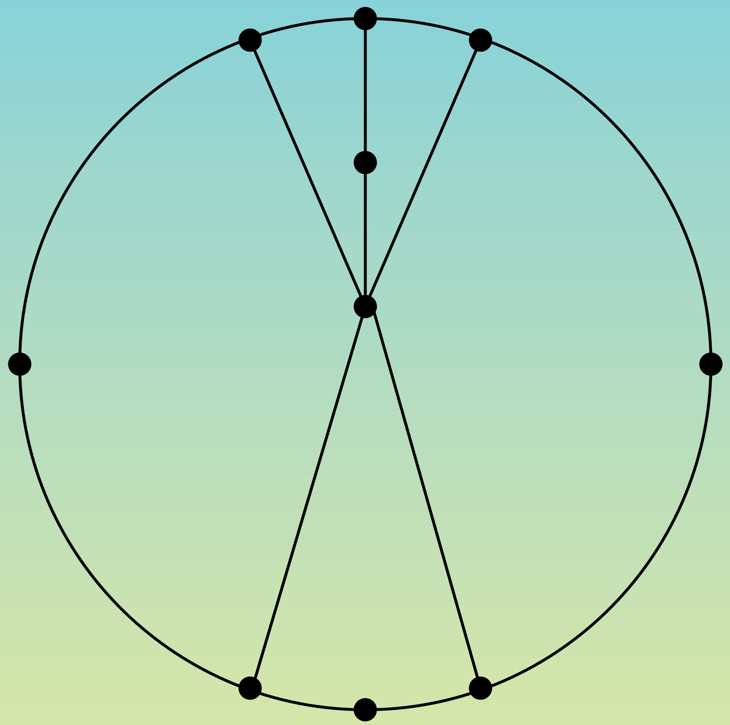

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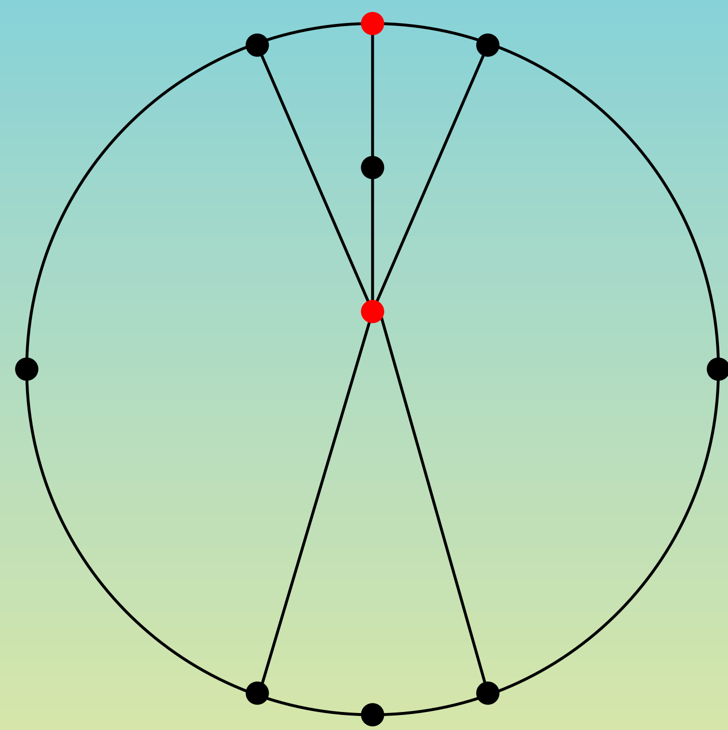

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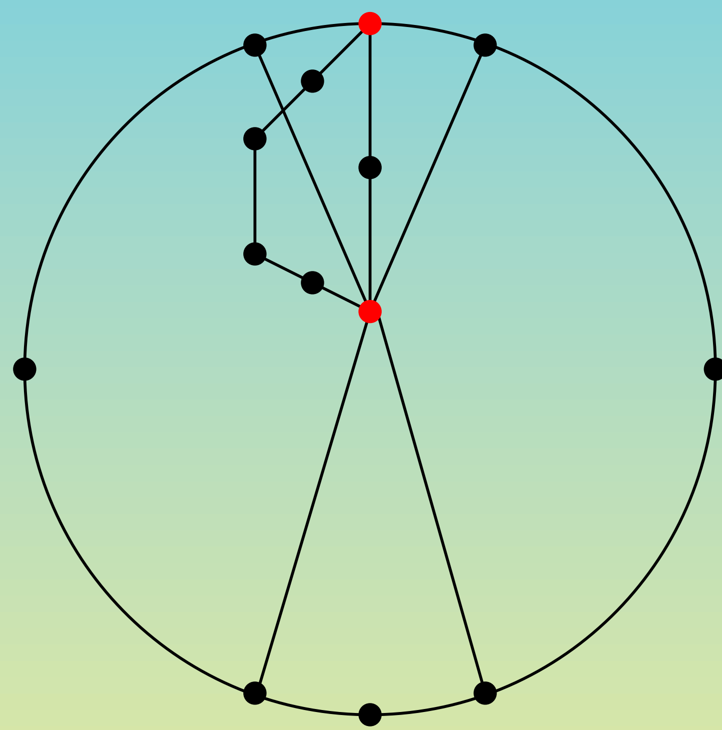

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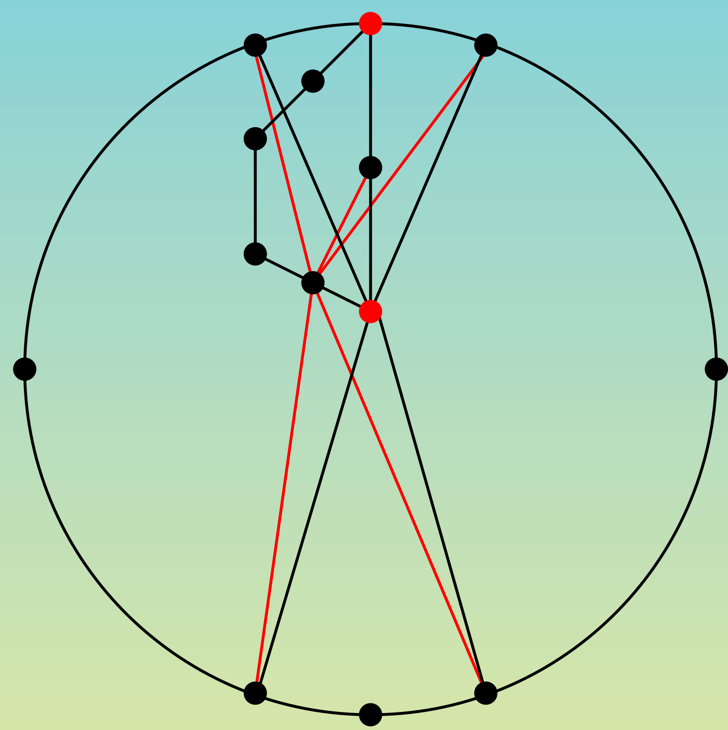

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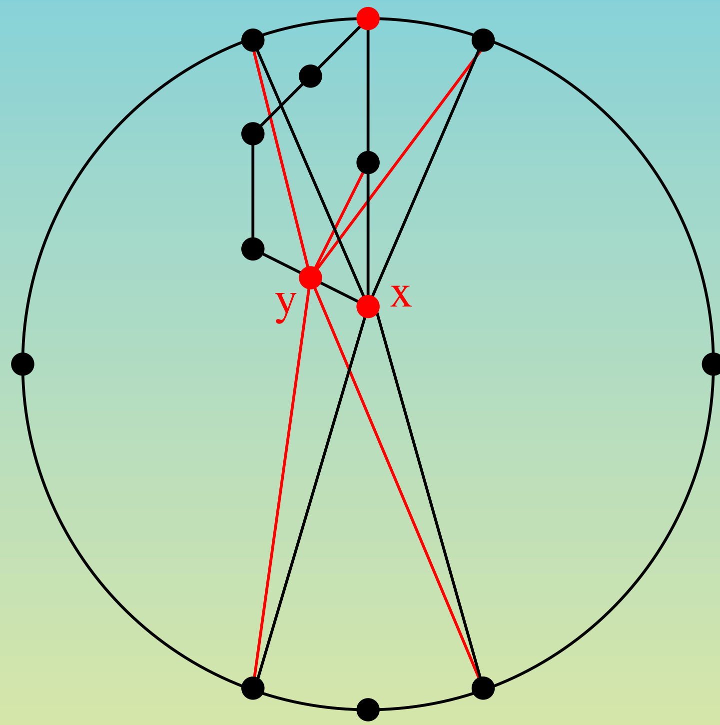

# ALGORITHMS

CONJECTURE Perfection can be tested in polynomial time.

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CONJECTURE Perfection can be tested in polynomial time.

THEOREM (Conforti, Cornuéjols, Kapoor, Vušković) There is a polynomial-time algorithm to test if an input graph has an even hole.