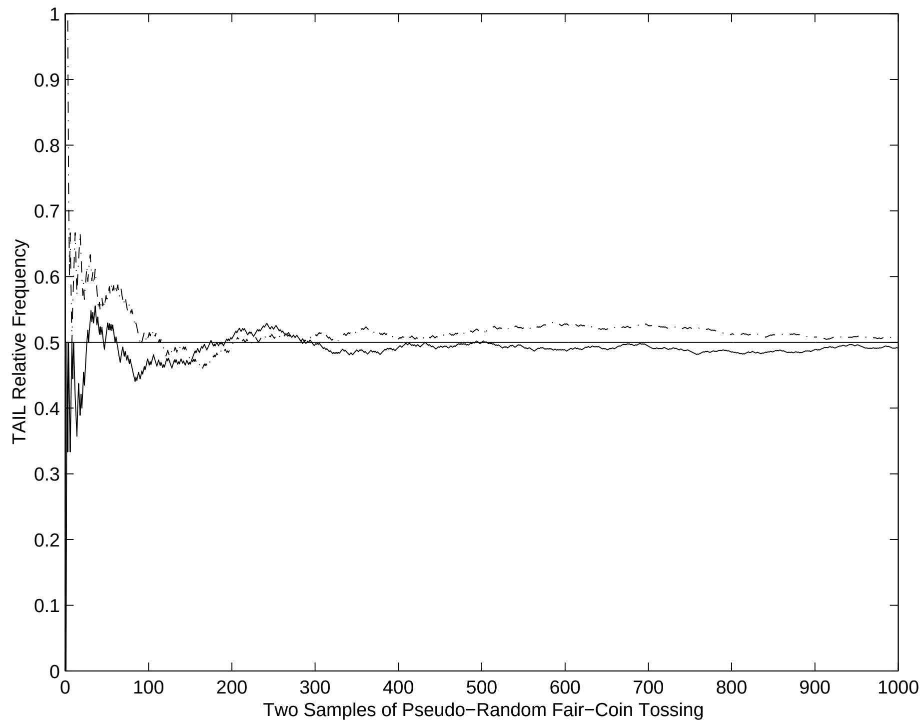
July 16, 2008 sylF08.tex

ECE 313 Probability and Random Variables, Fall 2008



Instructor: Michael Thomason
Class: 1:25-2:15MWF C 206 in the Claxton outpost.
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If you send e-mail, recognize its limitations. The volume of e-mail, including SPAM, is very large, so you have to expect delays. e-mail isn't viable for detailed technical questions and answers: use the office hours or make an appointment on a timely basis.

Example: One of the world's favorite experiments with a random outcome—flip a fair coin.

We will be studying probability models. Here's an example. Suppose we flip a *fair coin* independently in the same way again and again, so that it comes up H(EAD) or T(AIL) nondeterministically on each flip. Let n_H denote the number of times H occurs in n flips; then n_H/n is the *relative frequency of H* in n flips and, similarly, n_T/n is the *relative frequency of T* in n flips. What could we expect a plot of n_T/n vs n to look like as n goes from 1 flip to 1000 flips?

By definition, a *fair coin* is as likely to be H as T on each independent flip. We will see that the axioms of probability require $0 \leq p \leq 1$ for every probability p. Since our "flip-a-coin" experiment allows no outcome other than H or T per flip[1], the probabilities must be $p_H = p_T = 0.5$. If we flip the coin again and again, we expect to see "about the same number of H as T in the long run" i.e., we expect both the relative frequencies n_H/n and n_T/n to be about 0.5 as n gets larger and larger; however, we should be surprised if n_H/n exactly equals 0.5 for all n. (In fact, for odd n, we can't even have n_H/n equal precisely to 0.5.) Even when n is 1000, the probability that n_H/n will be precisely 0.5 is pretty small—we will see that it's about 0.025 and the corresponding probability that n_H/n will not be exactly 0.5 is approximately 1 - 0.025 = 0.975.

There is, however, "high probability" for flips of a fair coin that n_H/n and n_T/n each will be "close" to 0.5 after 1000 flips. The figure above plots the relative frequency n_T/n vs n for two simulations of 1000 flips. Since I don't have a certified fair coin (and wouldn't flip it 1000 times even if I did), this was run in MATLAB using a pseudo-random number generator. In neither plot does the relative frequency settle down to exactly 0.5 after 1000 flips. In fact, there are $2^{1000} \approx$ 1.1e+301 different H-T sequences of length 1000 and, in this fair-coin model, each individual sequence has probability

$$0.5^{1000} \approx \frac{1}{1.1 \times 10^{301}} \approx 9.33 \times 10^{-302}$$

of occurring. Plotted in the figure is n_T/n for just two of these individual sequences. That's a probability model in action, folks.

Course Description

Required Text: *Probability and Stochastic Processes: A Friendly Introduction for Electrical and Computer Engineers*, 2nd Edition, Wiley, 2005, by R.D. Yates and D.J. Goodman. We will cover large parts of chapters {1,2,3}, much of chapter {4}, and parts of chapters {5,6,7}. The objectives are to introduce sets and probability as an axiomatic mathematical system, develop some important properties, look at examples of probability distributions and computations, and show some of the connections with statistics.

Topics (expect some real-time tuning and adjustments):

Chap. 1: Elementary set theory; Venn diagrams; operations on sets

Chap. 1: Probability as an axiomatic mathematical system; the axioms and properties arising from them; conditional probability; independent events; simple combinatorics

Chap. 2: Discrete probabilty distributions; random variables; functions of random variables; expected value (expectation, mean value) and variance

Chap. 3: Continuous probabilty distributions; random variables; functions of random variables; expected value (expectation, mean value) and variance

Chaps. 4 and 5: Pairs of random variables; joint distributions; random vectors

Chaps. 6, 7, 8, and 9 (as time permits): Random samples and empirical distributions; large number laws and the Central Limit Theorem

There will be illustrations of applications as time permits. You are responsible for all assignments in the text and all handouts in class.

[1]Think of flipping the coin as an *experiment*. Each flip produces one outcome of the experiment. An experiment has a set of elementary outcomes called its *sample space*. In our experiment, the sample space is {H,T}; thus, in this model, the coin cannot balance on its edge, cannot evaporate during the flip, etc.: these are not outcomes in the sample space of this experiment.

Prereq and Grading: Prereq is M231. ECE 313 is an introduction to probability, not a course intended for people who already have a background in the topic.

There will be three in-class (50 minute, closed book and no calculator) exams for 100 points each. Exams will be spaced about evenly through the semester. There will also be 100 points total of graded homework spread over three or more problems. Homework to be turned in for grading will be handed out in class with a specific due-date and must get grade 0 if late. The course letter grade will be based on the percentage (rounded to uint8) of points earned out of the maximum points possible: 90 to 100% A, 80 to 89% B, 70 to 79% C, 60 to 69% D, $<$ 60% F. There *may* be a curve downward (like 79% for B-) depending on the class distribution. The breakpoints for B+/B- and C+/C- will depend on the class distribution.

Students who have a disability that requires accomodation should make an appointment with the Office of Disability Services to discuss specific needs.