Problem Set 12:

Computational Complexity

Due: Thursday, April 24, 2014, at the beginning of class

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0. Complete the course SAIS evaluation for this course, and turn in your confirmation sheet. I value your constructive feedback!

1. Consider again the 0-1 Knapsack problem: A thief robbing a store finds n items. The ith item is worth v_i dollars and weighs w_i pounds, where v_i and w_i are integers. The thief wants to take as valuable a load as possible, but s/he can carry at most W pounds in her/his knapsack, for some integer W. There is dynamic programming solution to this problem that runs in $O(n\ W)$ time. [An aside: a useful study exercise for the final is to develop this dynamic programming solution. But, you don't have to show it for this problem set.]

   For this homework, answer this question: Is this dynamic programming solution (which runs in $O(n\ W)$ time) a polynomial-time algorithm? Explain your answer.

2. Show that an otherwise polynomial-time algorithm that makes at most a constant number of calls to polynomial-time subroutines runs in polynomial time, but that a polynomial number of calls to polynomial-time subroutines may result in an exponential-time algorithm.

3. Show that if HAM-CYCLE $\in$ P, then the problem of printing (in order) the vertices of a Hamiltonian cycle is polynomial-time solvable. In other words, give a polynomial-time algorithm that prints out (in order) the vertices of a Hamiltonian cycle in a graph using the assumed polynomial-time subroutine that decides HAM-CYCLE.

4. Let 5-CLIQUE = {<G> | G is an undirected graph having a complete subgraph with 5 nodes}. Show that 5-CLIQUE is in P.

5. We define a monotone Boolean formula as a formula with no negated variables. We further define the 2-monotone-small-SAT problem is as follows: Given a monotone Boolean formula Φ in 2-CNF form (yes, that's two-CNF) and an integer k, determine if Φ has a truth assignment with no more than k variables assigned "True" (i.e., "1"). Show that this problem is NP-complete. [Hint: Use Vertex Cover]