

Problem Set 2:
Recurrences and Divide and Conquer

Due: Thursday, January 23, 2014, at the beginning of class

1. Solve the following recurrences, showing your work. Assume $T(1) = 1$ in all cases. Give your answers in Θ notation.

a. $T(n) = 4T(n/3) + \sqrt[3]{n}$

b. $T(n) = 4T(n/3) + n\sqrt[3]{n}$

c. $T(n) = 4T(n/3) + (n^{\log_3 4}) \log_3 n$

d. $T(n) = T(\sqrt{n}) + 1$ [Hint: consider change of variable, as discussed on page 86.]

2. Use Strassen's algorithm to compute the following matrix product. Show your work.

$$\begin{pmatrix} 1 & 3 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 6 & 8 \\ 4 & 2 \end{pmatrix}$$

3. What is the largest k such that if you can multiply 3×3 matrices using k multiplications (not assuming commutativity of multiplication), then you can multiply $n \times n$ matrices in time $o(n^{\lg 7})$? What would the running time of the algorithm be? Show your work. [Hint: Make use of the master method to reason about this problem.]