Homework 4:

Map Representation and Path Planning

(UNDERGRADUATE VERSION)

Note: If you prefer, you may turn in the *graduate* version of HW #4 instead of this homework, with the same due date as the graduate students. It is your choice. The graduate version is primarily a programming assignment. This undergraduate version involves no programming.

Assigned: Thursday, October 16, 2008
Due: Thursday, October 30, 2008, at the beginning of class (no later than 5:10 PM)
(Hard copy is acceptable.)

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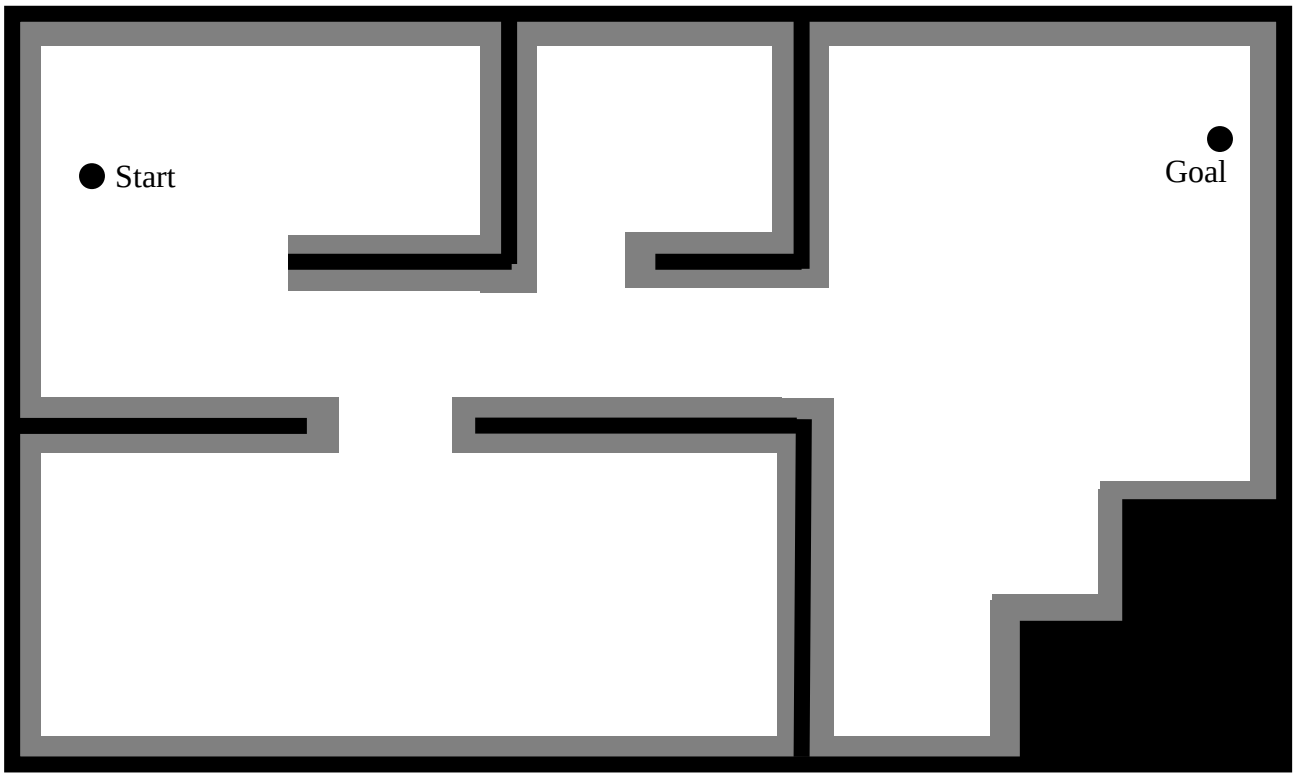
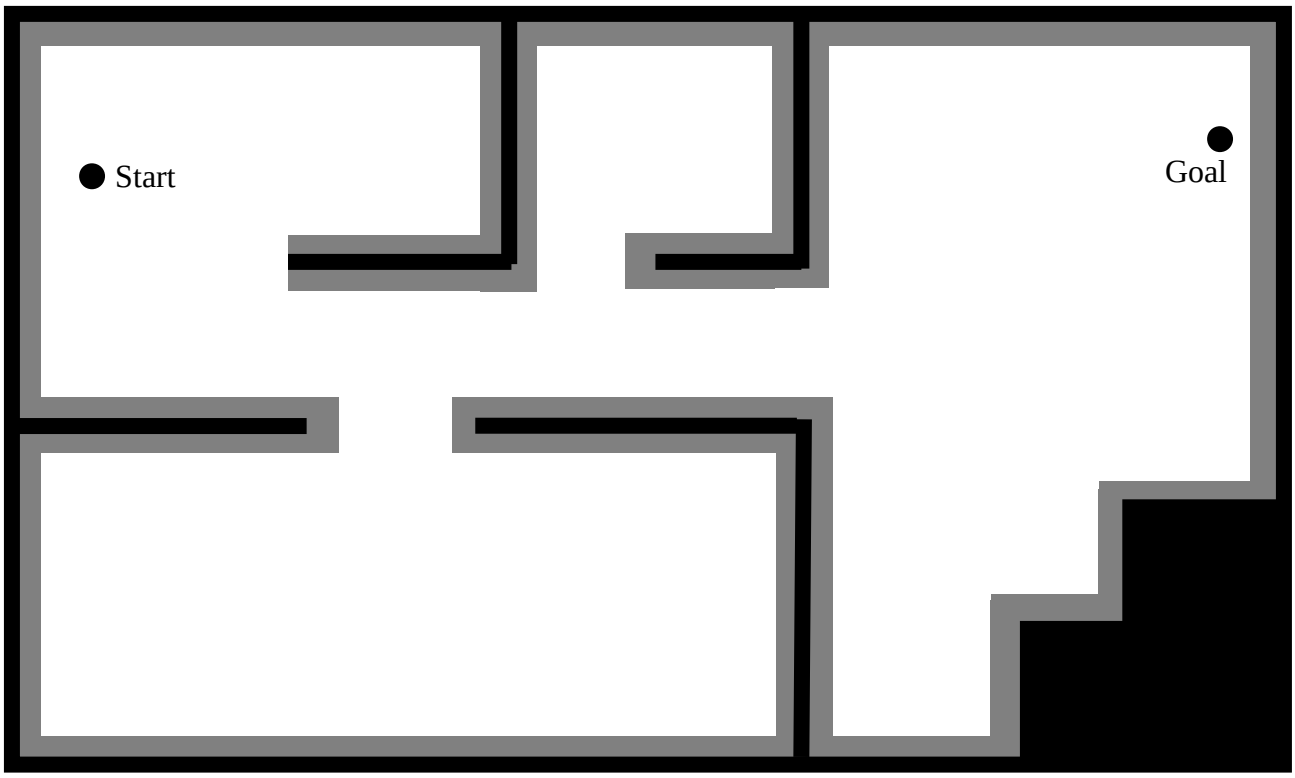
1. **Map Representations.** For the graph below (which is replicated 3 times), show the CVG, meadow map, and visibility graph representations. Here, the obstacles have already been grown.

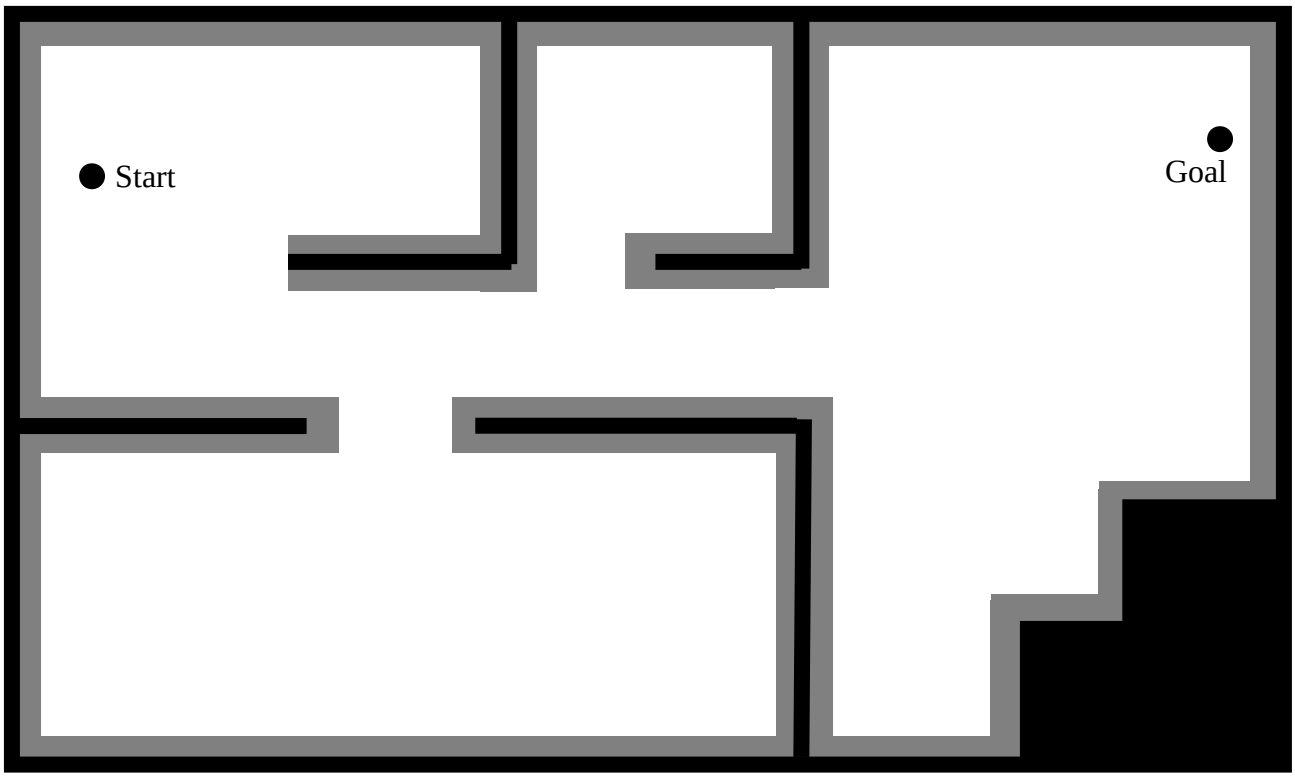
a. CVG Representation:



b. Meadow map representation:

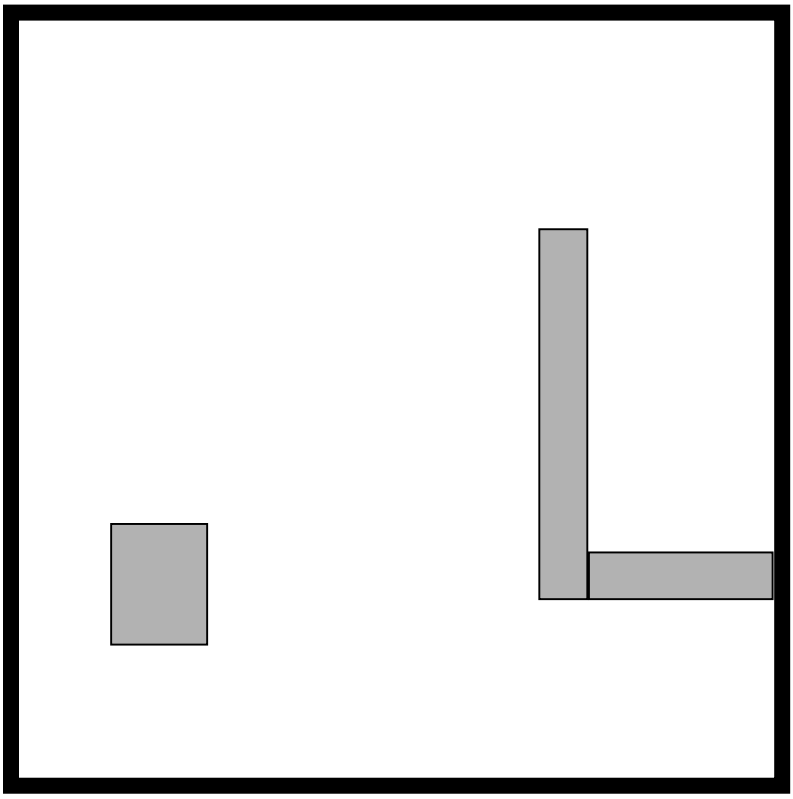


c. Visibility graph representation:



2. Quadtree representation

a. Below is a robot environment. Draw the *quadtree* grid representation of this environment, assuming the minimum grid size is the width of the top of the "L" shape on the right. (Do NOT grow object boundaries.)



b. Approximately what percentage space savings is achieved by your quadtree, compared to a regular grid that has the same lowest-level resolution?

c. What is *digitization bias*, in the context of regular grids?

3. You are developing a path planner for a planetary exploration robot, and you have decided to use the A* search algorithm. You need to come up with an estimate of the remaining cost to travel from any particular position in the environment to the goal location (i.e., $h^*(n)$). Here, cost is measured in terms of distance traveled. Which of the following are *admissible* estimates of this cost? (Circle all that are correct.)

i) 0
ii) $+\infty$
iii) -12
iv) Length of the straight-line path to the goal
v) Twice the length of the straight-line path to the goal
vi) Half of the length of the straight-line path to the goal
vii) Distance traveled so far
viii) Twice the distance traveled so far
ix) Half the distance traveled so far

4. Consider a regular grid of size 20 by 20. How many edges will a graph have if the neighbors are:

a. 4-connected?

b. 8-connected?

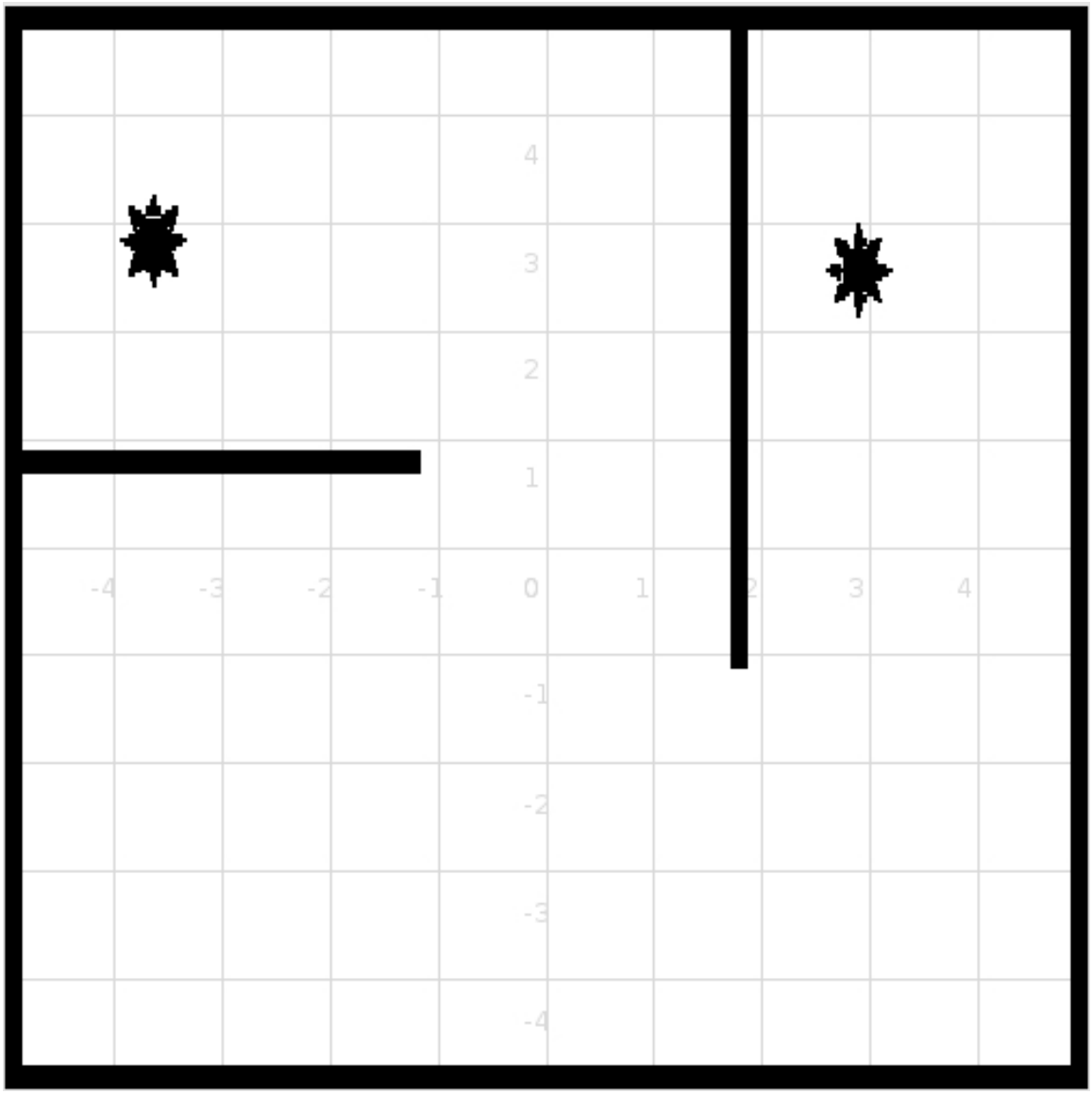
5. Convert your meadow map from part 1b into a graph using:

a. the midpoints of the open boundaries

b. the midpoints plus the 2 endpoints.

Show your graphs. Draw a path from Start to Goal on both graphs. Describe the differences.

6. Apply the A* algorithm by hand from the starting "star" on the left to the goal "star" on the right, in the following map, using the grid cells shown. Be sure to mark cells containing obstacles as occupied, so that they are not part of the search. (The "stars" themselves are not obstacles.) Show the resulting tree that is searched as the algorithm proceeds, marking each "node" in numerically increasing order as it is expanded, so that it is clear how your A* search is operating. Clearly mark the final path generated.



7. Apply the wavefront path planning algorithm by hand from the starting "star" on the left to the goal "star" on the right, in the following map, using the grid cells shown. Be sure to mark cells containing obstacles as occupied, so that they are not part of the search. (The "stars" themselves are not obstacles.) You should mark each cell in increasing numeric order that it is visited, according to the wavefront path planning algorithm. Clearly mark your resulting path.