Adversarial Search

Chapter 6

Outline

- Perfect play
- Resource limits
- α–β pruning
- Games of chance
- Games of imperfect information

Games vs. search problems

"Unpredictable" opponent $\Rightarrow$ solution is a strategy
specifying a move for every possible opponent reply

Time limits $\Rightarrow$ unlikely to find goal, must approximate

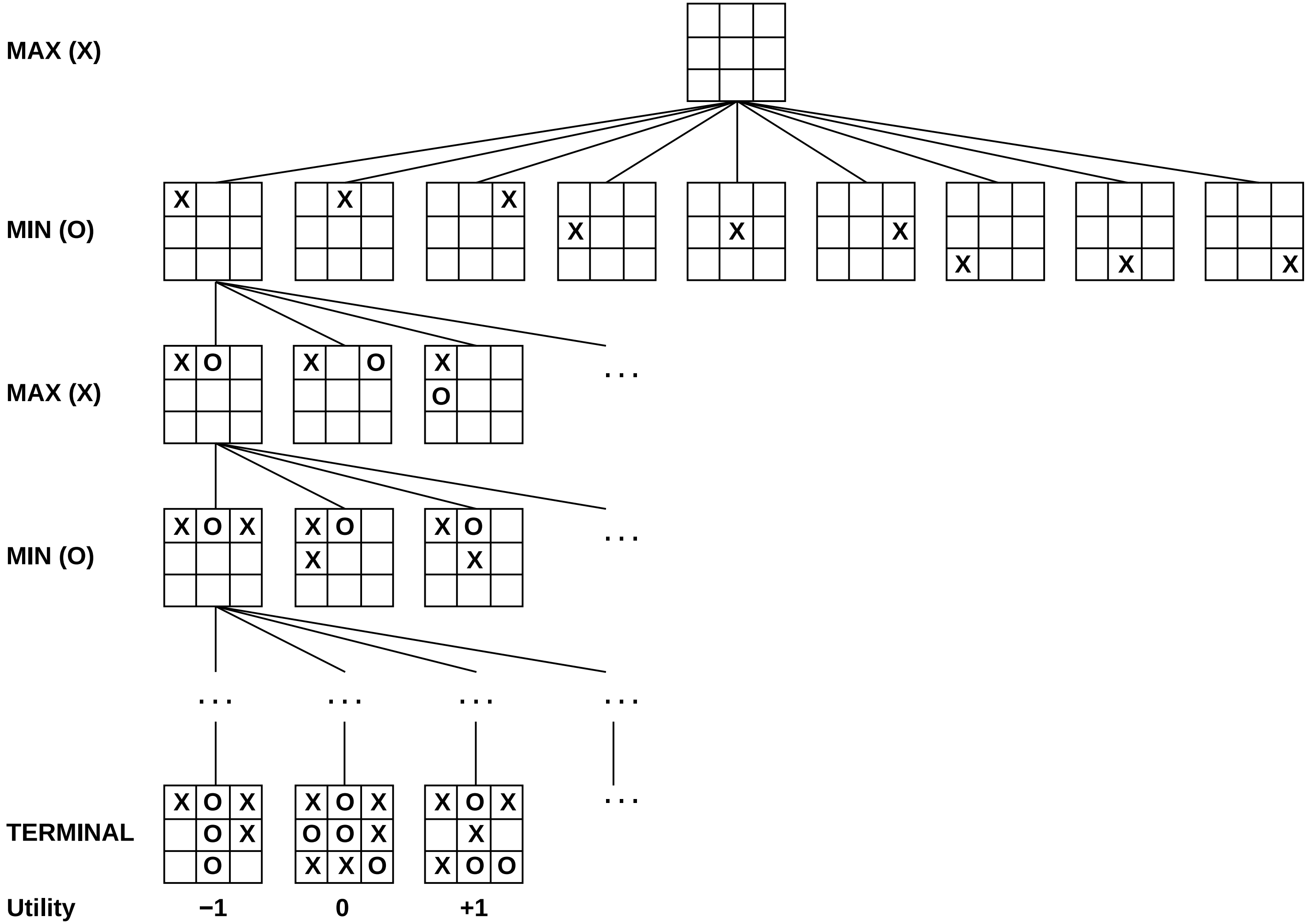
Plan of attack:

- Computer considers possible lines of play (Babbage, 1846)
- Algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
- Finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
- First chess program (Turing, 1951)
- Machine learning to improve evaluation accuracy (Samuel, 1952–57)
- Pruning to allow deeper search (McCarthy, 1956)

Types of games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information		bridge, poker, scrabble nuclear war

Game tree (2-player, deterministic, turns)

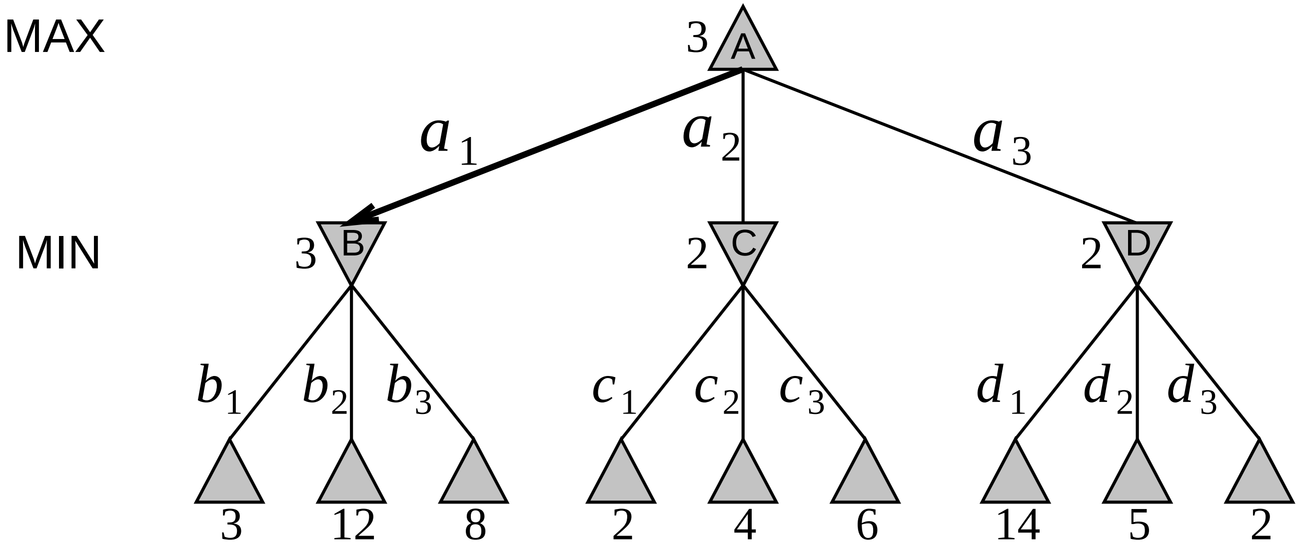


Minimax

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest *minimax value*
= best achievable payoff against best play

E.g., 2-ply game:



Minimax algorithm

```
function MINIMAX-DECISION(state, game) returns an action
    action, state ← the a, s in SUCCESSORS(state)
            such that MINIMAX-VALUE(s, game) is maximized
    return action
```

```
function MINIMAX-VALUE(state, game) returns a utility value
    if TERMINAL-TEST(state) then
        return UTILITY(state)
    else if  MAX is to move in state then
        return the highest MINIMAX-VALUE of SUCCESSORS(state)
    else
        return the lowest MINIMAX-VALUE of SUCCESSORS(state)
```

Properties of minimax

Complete??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this).
A finite strategy can exist even in an infinite tree!

Optimal??

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Space complexity??

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Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity?? $O(b^m)$

Space complexity?? $O(bm)$ (depth-first exploration)

For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
$\Rightarrow$ exact solution completely infeasible

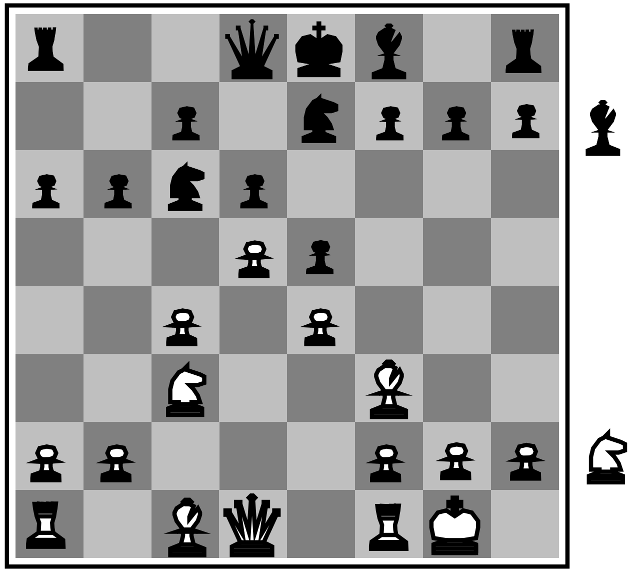
Resource limits

Suppose we have 100 seconds, explore 10^4 nodes/second
$\Rightarrow 10^6$ nodes per move

Standard approach:

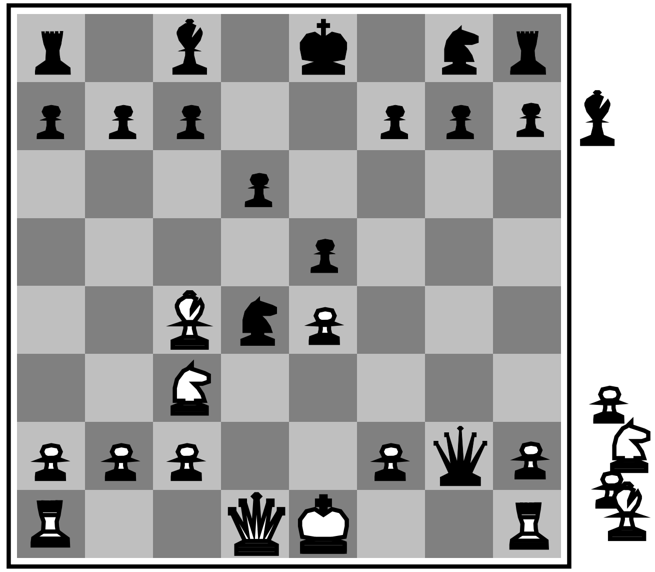
- *cutoff test*
  e.g., depth limit (perhaps add *quiescence search*)
- *evaluation function*
  $=$ estimated desirability of position

Evaluation functions



Black to move

White slightly better



White to move

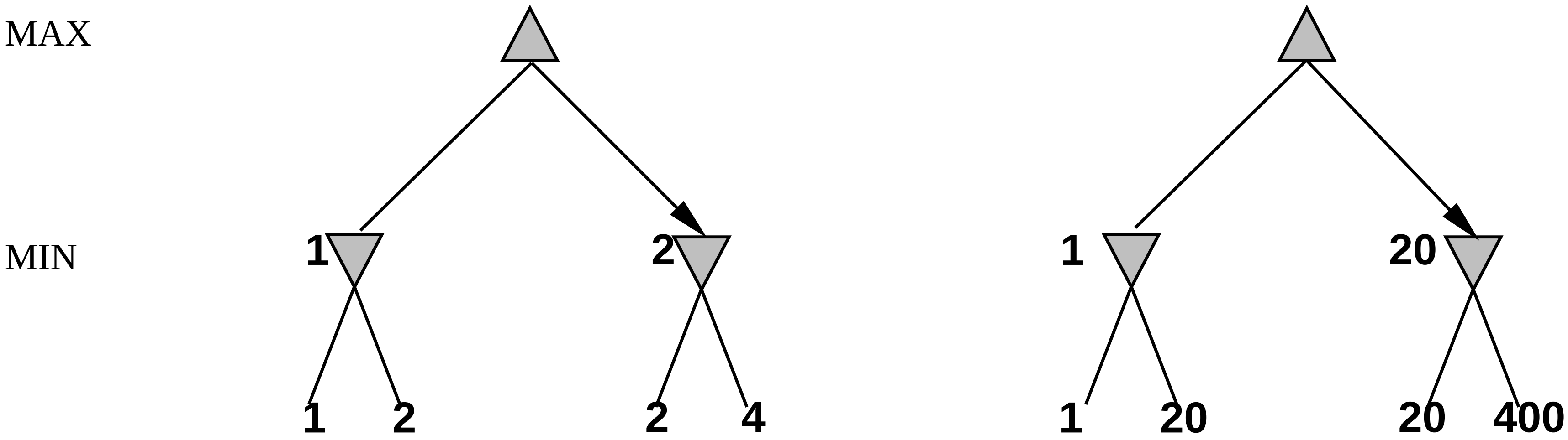
Black winning

For chess, typically *linear* weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $w_1 = 9$ with
$f_1(s) =$ (number of white queens) – (number of black queens), etc.

Digression: Exact values don't matter



Behavior is preserved under any *monotonic* transformation of EVAL

Only the order matters:

payoff in deterministic games acts as an *ordinal utility* function
(i.e., *ranking* of states, rather than meaningful numeric values)

Cutting off search

MinimaxCutoff is identical to MinimaxValue except

1. Terminal? is replaced by Cutoff?
2. Utility is replaced by Eval

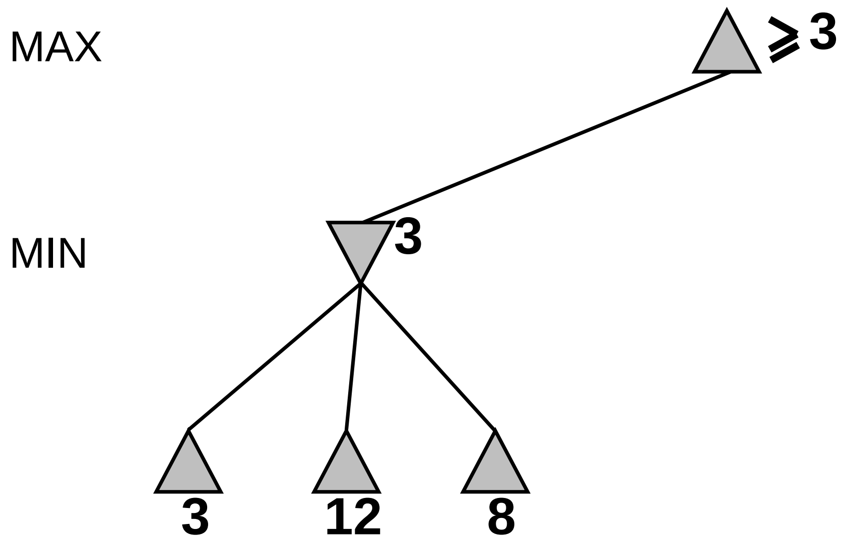
Does it work in practice?

$$b^m = 10^6, \quad b = 35 \quad \Rightarrow \quad m = 4$$

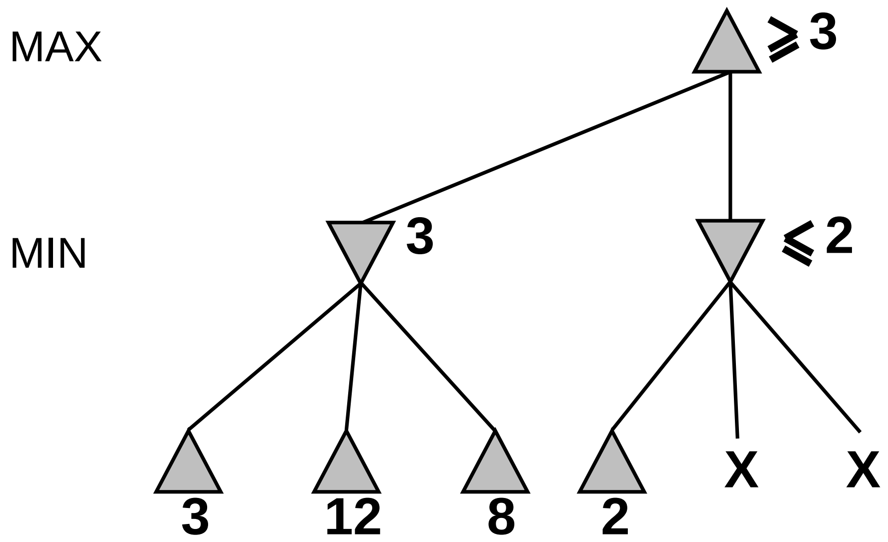
4-ply lookahead is a hopeless chess player!

4-ply $\approx$ human novice
8-ply $\approx$ typical PC, human master
12-ply $\approx$ Deep Blue, Kasparov

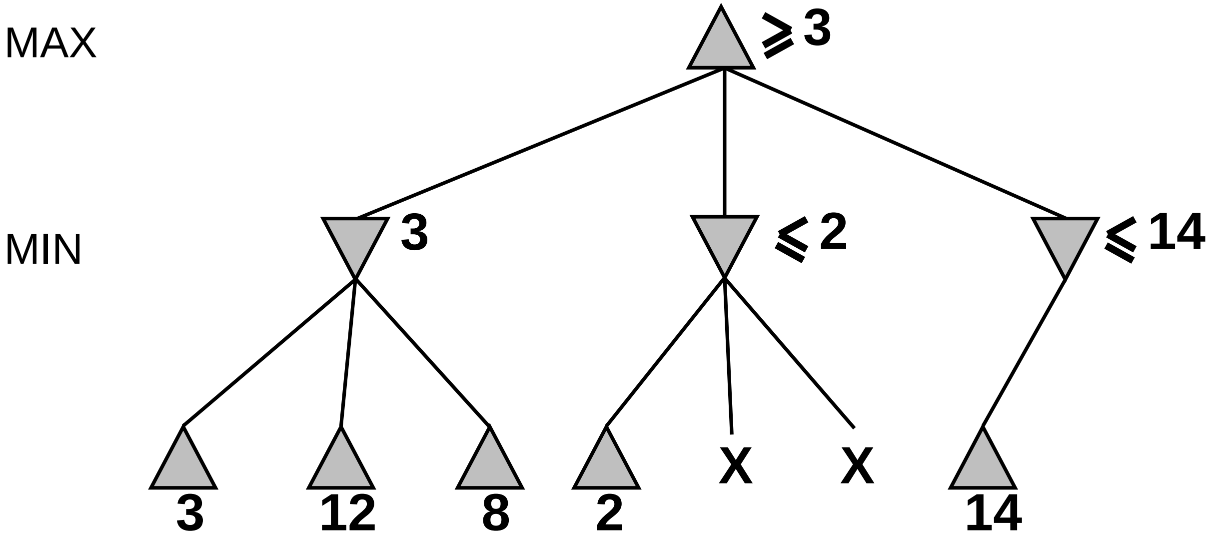
α–β pruning example



α–β pruning example



α–β pruning example



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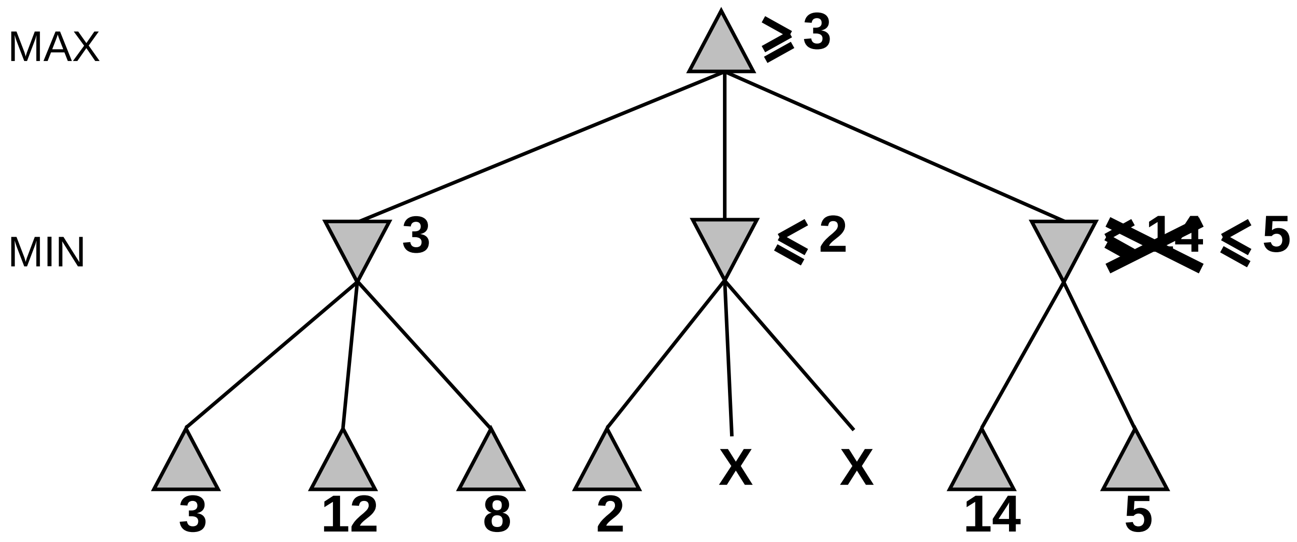
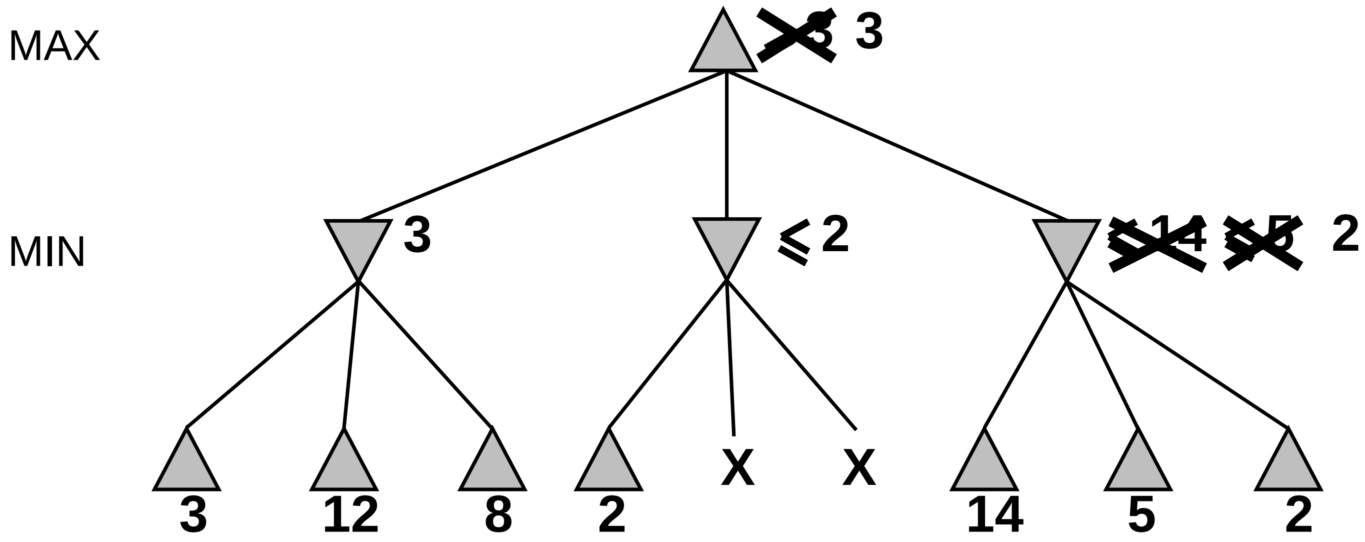


α–β pruning example



Properties of α–β

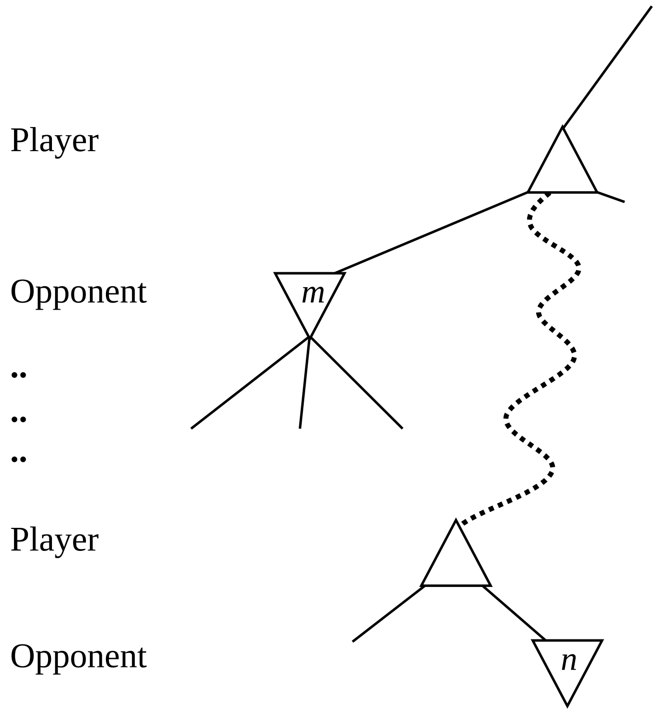
Pruning *does not* affect final result

Good move ordering improves effectiveness of pruning

With "perfect ordering," time complexity $= O(b^{m/2})$
$\Rightarrow$ *doubles* depth of search
$\Rightarrow$ can easily reach depth 8 and play good chess

A simple example of the value of reasoning about which computations are relevant (a form of *metareasoning*)

Why is it called α–β?



α is the best value (to MAX) found so far off the current path

If V is worse than α, MAX will avoid it $\Rightarrow$ prune that branch

Define β similarly for MIN

The α–β algorithm

function ALPHA-BETA-SEARCH(*state*, *game*) **returns** an action
  action, *state* ← the a, s **in** SUCCESSORS[*game*](*state*)
    such that MIN-VALUE(s, *game*, $-\infty$, $+\infty$) is maximized
  return *action*

---

function MAX-VALUE(*state*, *game*, α, β) **returns** the minimax value of *state*
  if CUTOFF-TEST(*state*) **then return** EVAL(*state*)
  for each s **in** SUCCESSORS(*state*) **do**
    α ← max(α, MIN-VALUE(s, *game*, α, β))
    if $\alpha \geq \beta$ **then return** β
  return α

---

function MIN-VALUE(*state*, *game*, α, β) **returns** the minimax value of *state*
  if CUTOFF-TEST(*state*) **then return** EVAL(*state*)
  for each s **in** SUCCESSORS(*state*) **do**
    β ← min(β, MAX-VALUE(s, *game*, α, β))
    if $\beta \leq \alpha$ **then return** α
  return β

Deterministic games in practice

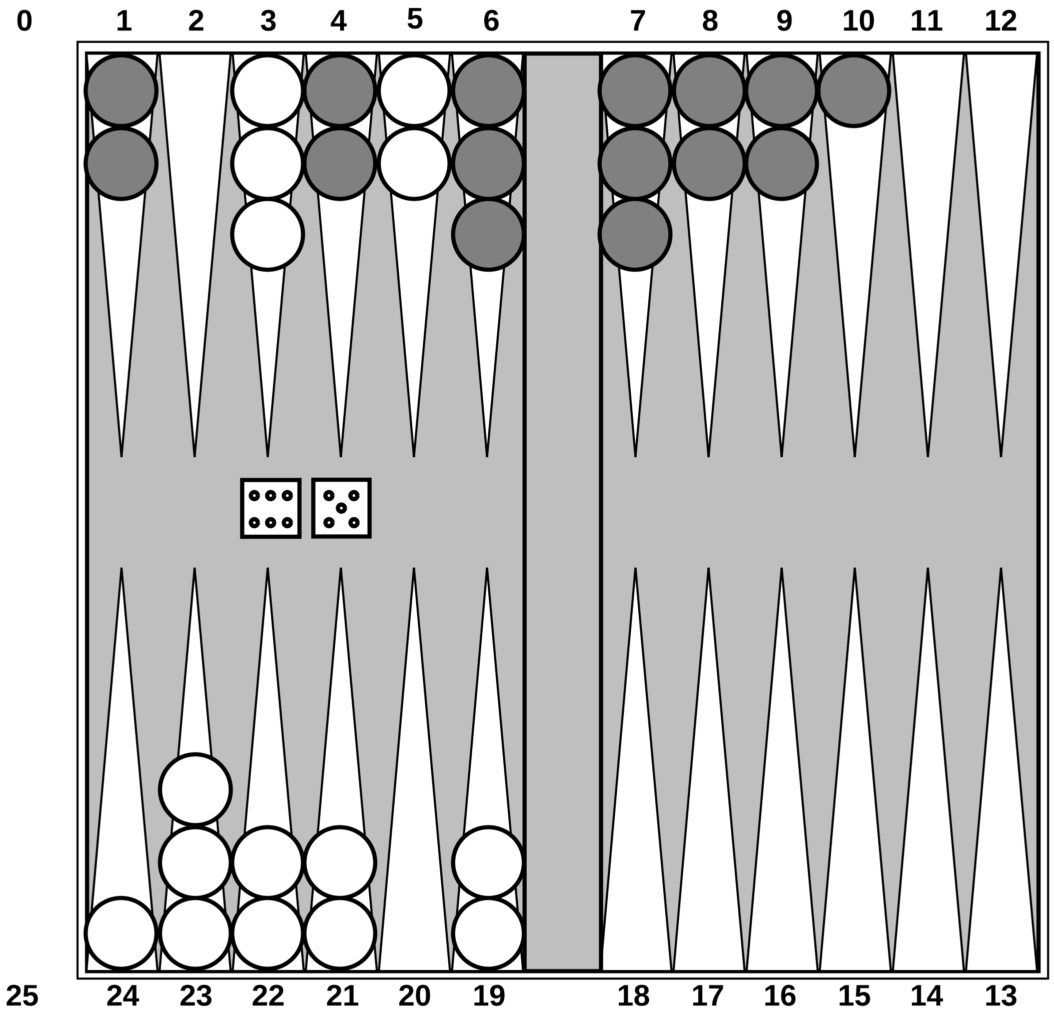
Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

Chess: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

Othello: human champions refuse to compete against computers, who are too good.

Go: human champions refuse to compete against computers, who are too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

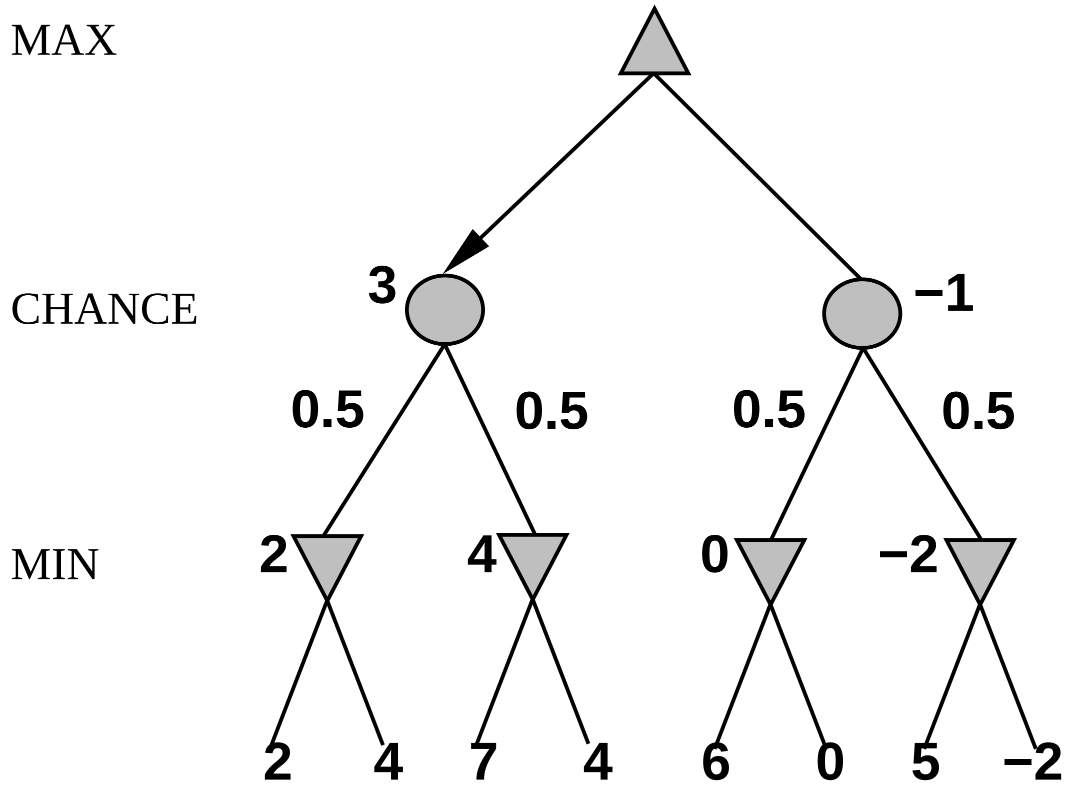
Nondeterministic games: backgammon



Nondeterministic games in general

In nondeterministic games, chance introduced by dice, card-shuffling

Simplified example with coin-flipping:



Algorithm for nondeterministic games

EXPECTIMINIMAX gives perfect play

Just like MINIMAX, except we must also handle chance nodes:

. . .
if *state* is a MAX node **then**
  return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a MIN node **then**
  return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a chance node **then**
  return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
. . .

Nondeterministic games in practice

Dice rolls increase b: 21 possible rolls with 2 dice
Backgammon $\approx$ 20 legal moves (can be 6,000 with 1-1 roll)

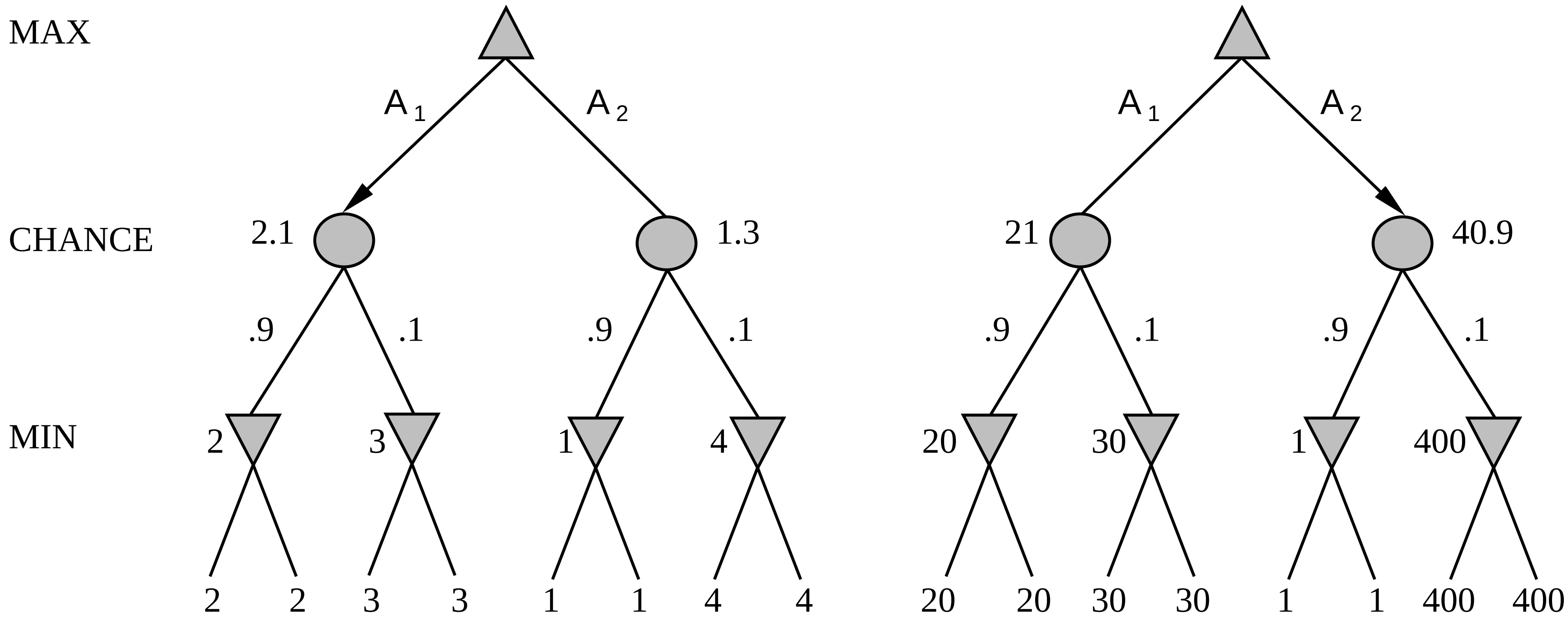
$$\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$$

As depth increases, probability of reaching a given node shrinks
$\Rightarrow$ value of lookahead is diminished

α–β pruning is much less effective

TDGAMMON uses depth-2 search + very good EVAL
$\approx$ world-champion level

Digression: Exact values DO matter



Behavior is preserved only by *positive linear* transformation of EVAL

Hence EVAL should be proportional to the expected payoff

Games of imperfect information

E.g., card games, where opponent's initial cards are unknown

Typically we can calculate a probability for each possible deal

Seems just like having one big dice roll at the beginning of the game*

Idea: compute the minimax value of each action in each deal,
then choose the action with highest expected value over all deals*

Special case: if an action is optimal for all deals, it's optimal.*

GIB, current best bridge program, approximates this idea by
1) generating 100 deals consistent with bidding information
2) picking the action that wins most tricks on average

Proper analysis

* Intuition that the value of an action is the average of its values in all actual states is WRONG

With partial observability, value of an action depends on the information state or belief state the agent is in

Can generate and search a tree of information states

Leads to rational behaviors such as
- ♢ Acting to obtain information
- ♢ Signalling to one's partner
- ♢ Acting randomly to minimize information disclosure

Summary

Games are fun to work on! (and dangerous)

They illustrate several important points about AI

- ♢ perfection is unattainable $\Rightarrow$ must approximate
- ♢ good idea to think about what to think about
- ♢ uncertainty constrains the assignment of values to states

Games are to AI as grand prix racing is to automobile design

Remember: Thought Discussion for next time

- ♢ (Read pages 947-949 of our text)
- ♢ "Weak AI: Can machines act intelligently?"
  - ♢ Specifically: Consider argument from disability
    i.e., "A machine can never do X"