# Logical agents

## Chapter 7

# Outline

◊ Knowledge-based agents

◊ Wumpus world

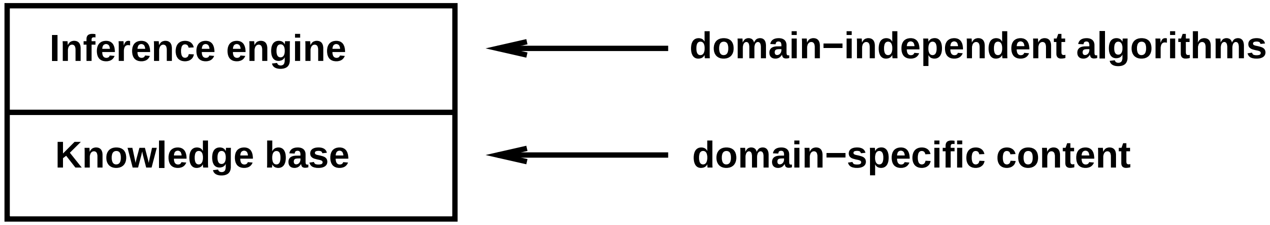
◊ Logic in general—models and entailment

◊ Propositional (Boolean) logic

◊ Equivalence, validity, satisfiability

◊ Inference rules and theorem proving
  – forward chaining
  – backward chaining
  – resolution

# Knowledge bases




Knowledge base = set of sentences in a formal language

Declarative approach to building an agent (or other system):

TELL it what it needs to know

Then it can ASK itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level

i.e., what they know, regardless of how implemented

Or at the implementation level

i.e., data structures in KB and algorithms that manipulate them

# A simple knowledge-based agent

```
function KB-Agent( percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time

    Tell(KB, Make-Percept-Sentence( percept, t))
    action ← Ask(KB, Make-Action-Query(t))
    Tell(KB, Make-Action-Sentence(action, t))
    t ← t + 1
    return action
```

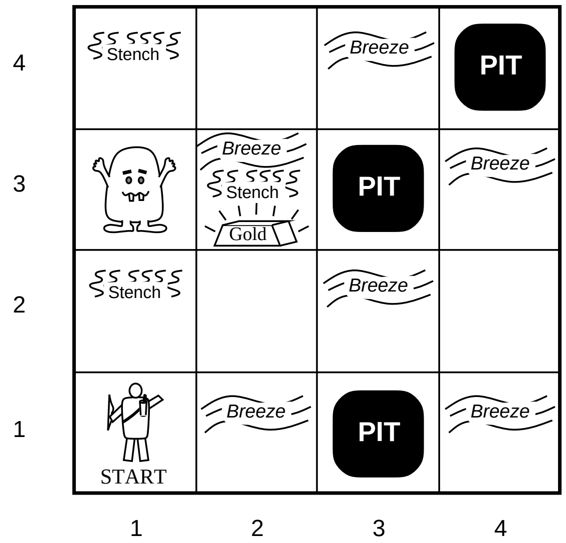
The agent must be able to:

- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world
- Deduce hidden properties of the world
- Deduce appropriate actions

# Wumpus World PEAS description

**Performance measure**
gold +1000, death -1000
-1 per step, -10 for using the arrow

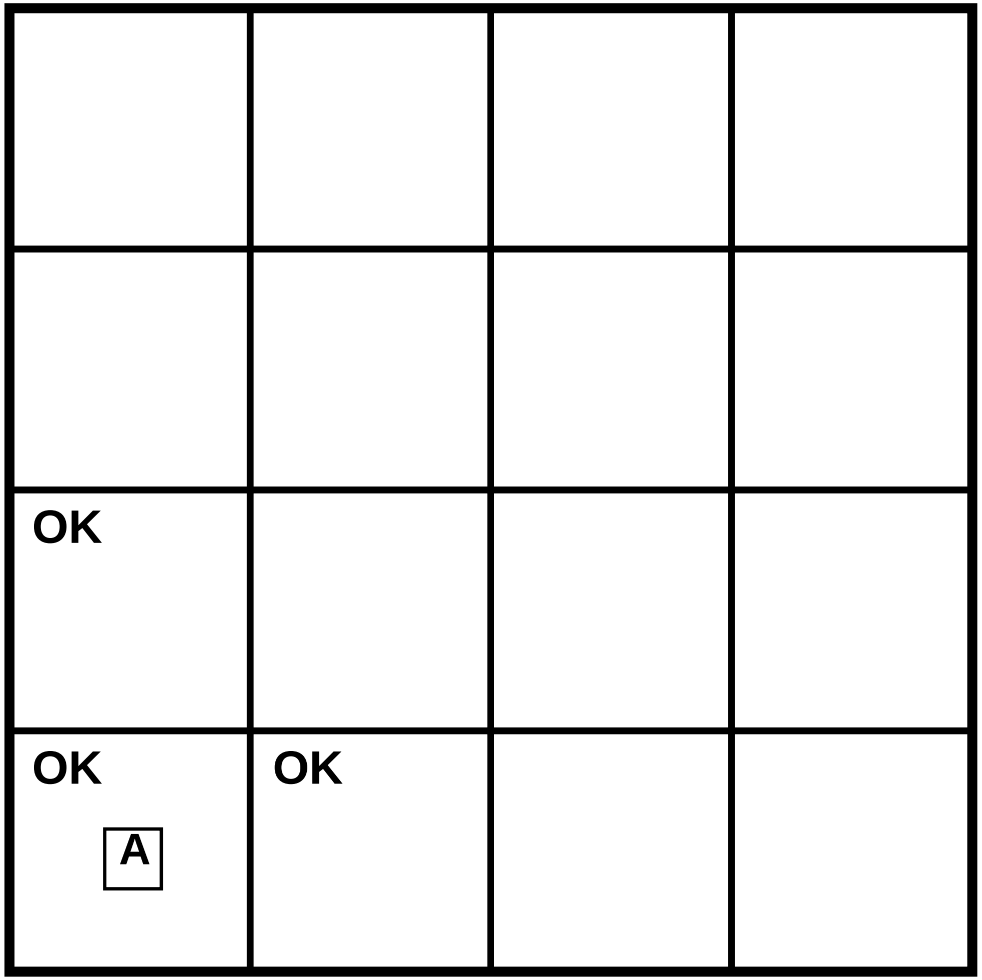
**Environment**
Squares adjacent to wumpus are smelly
Squares adjacent to pit are breezy
Glitter iff gold is in the same square
Shooting kills wumpus if you are facing it
Shooting uses up the only arrow
Grabbing picks up gold if in same square
Releasing drops the gold in same square
Feel bump when walking into a wall
Hear scream when wumpus dies




**Sensors** Smell, Breeze, Glitter, Bump, Scream

**Actuators** Left turn, Right turn, Forward, Grab, Release, Shoot

# Exploring a wumpus world



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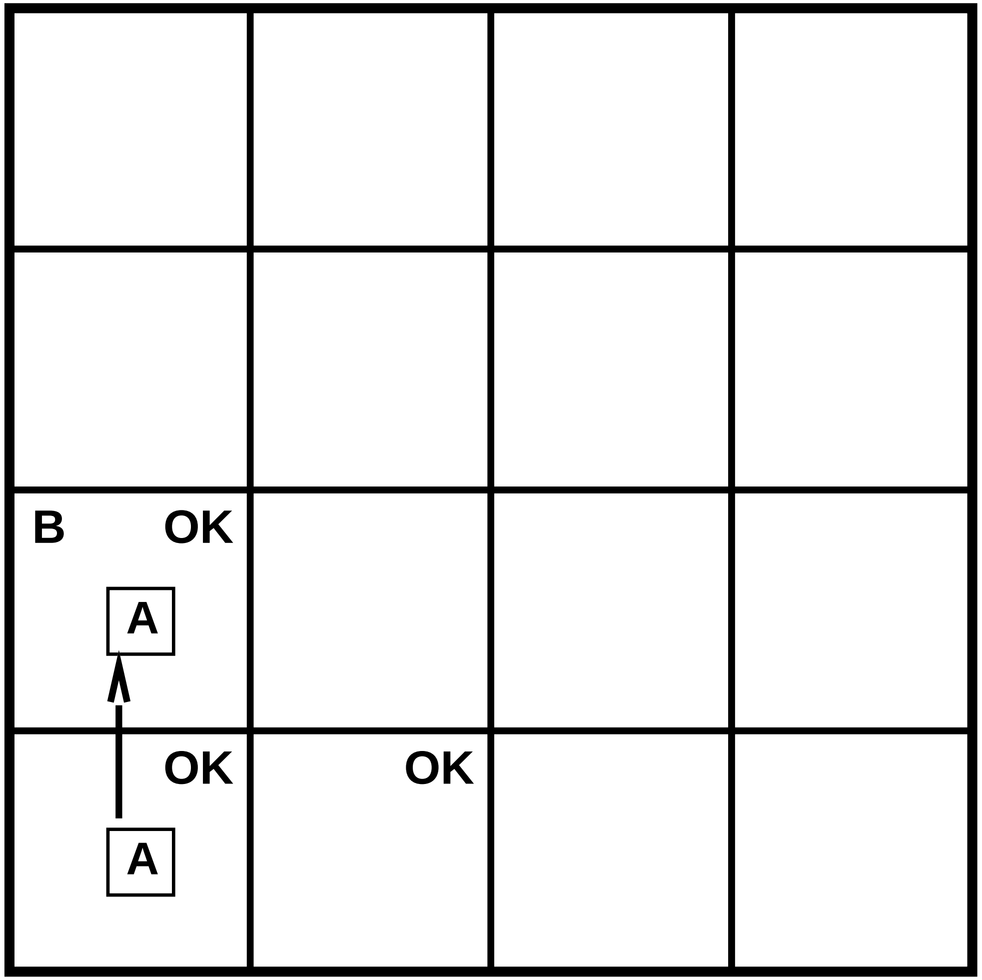

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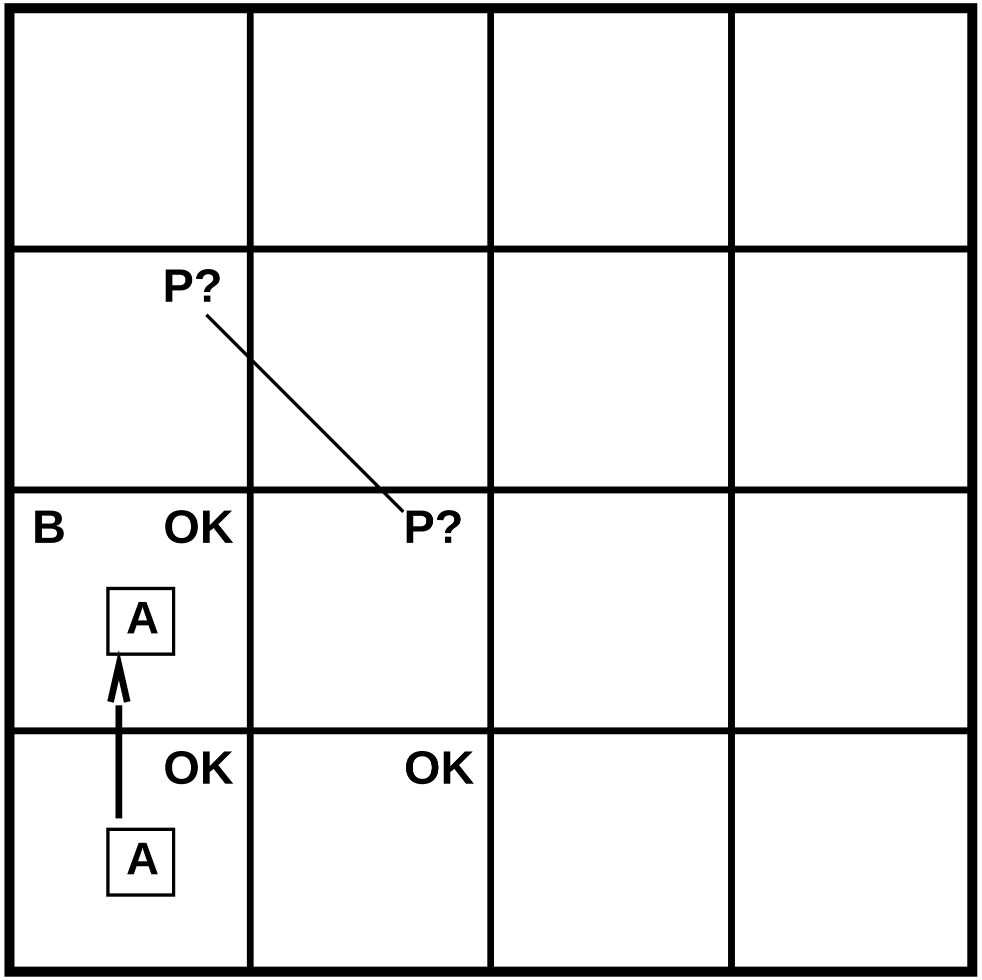

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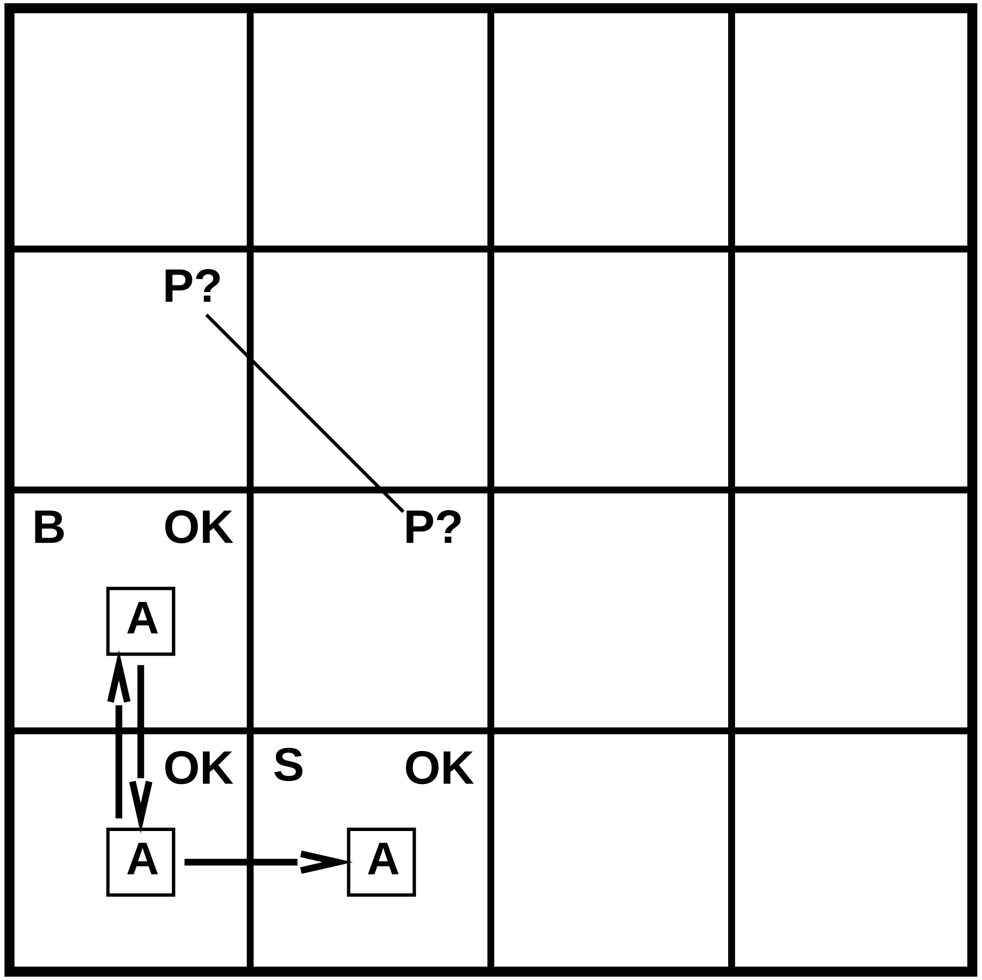

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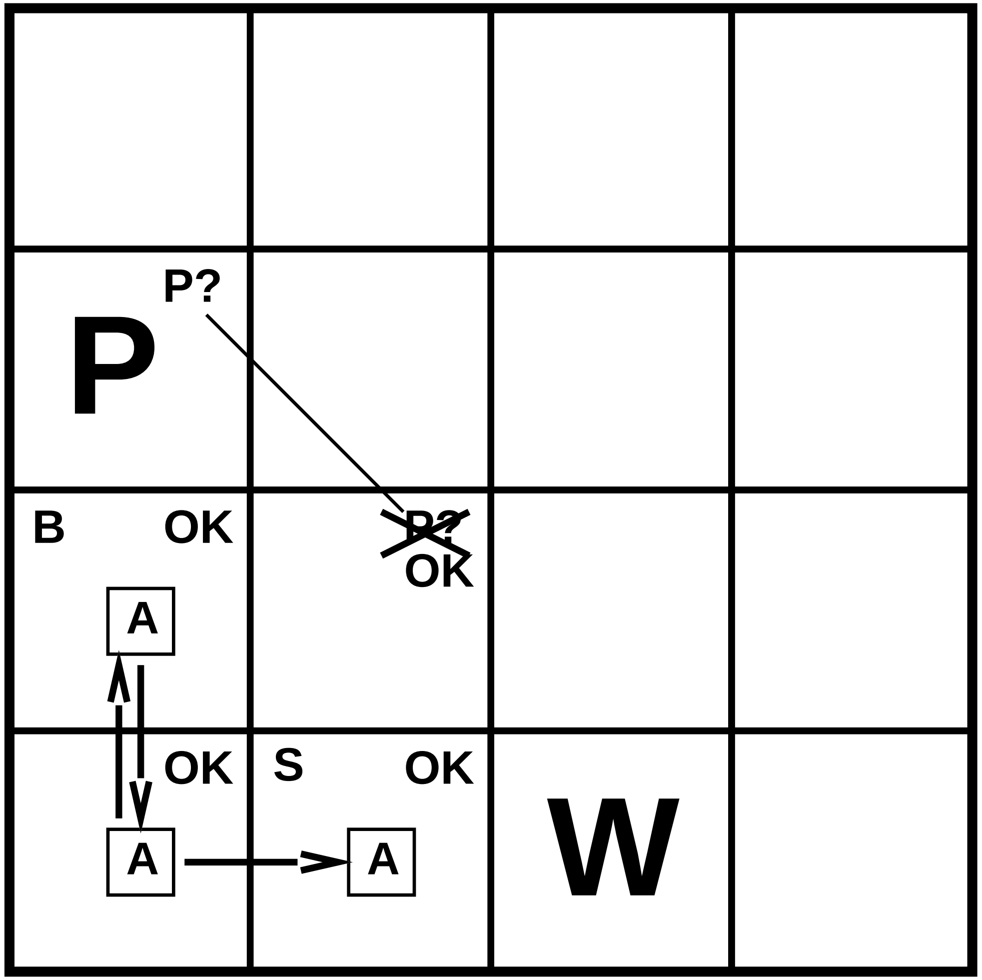

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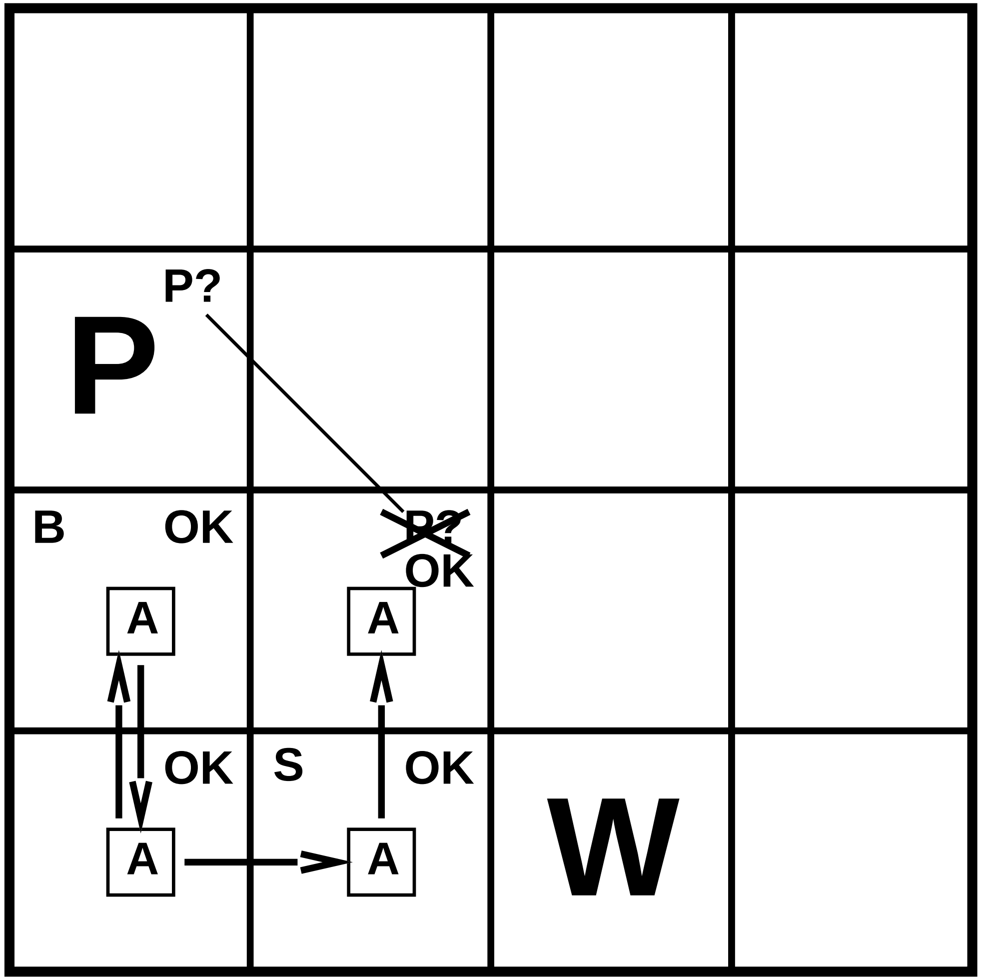

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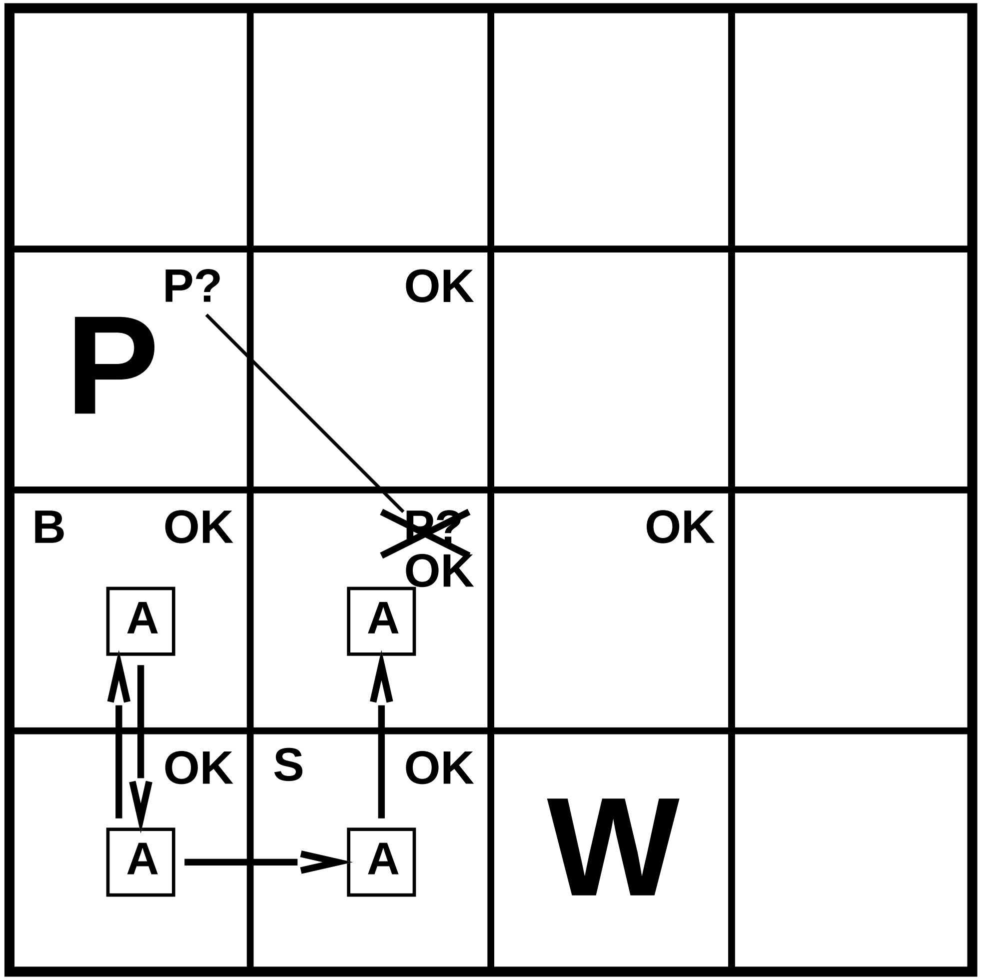

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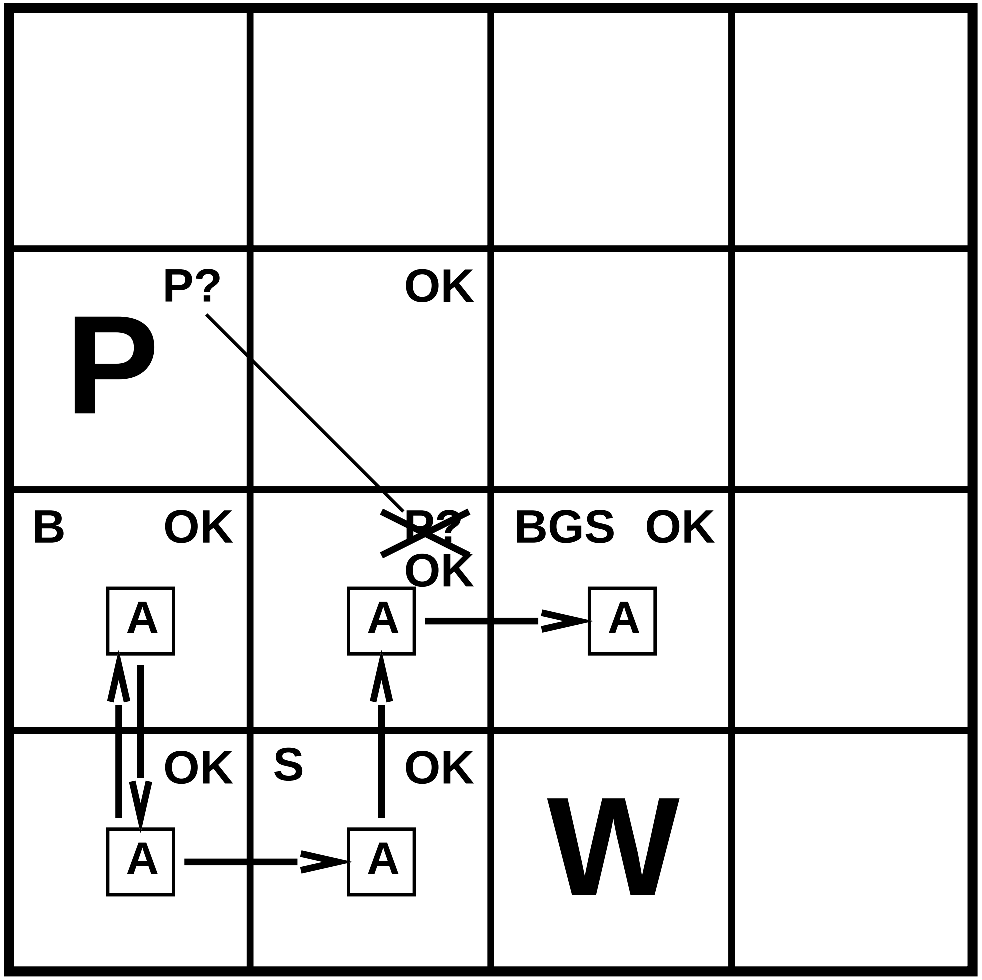
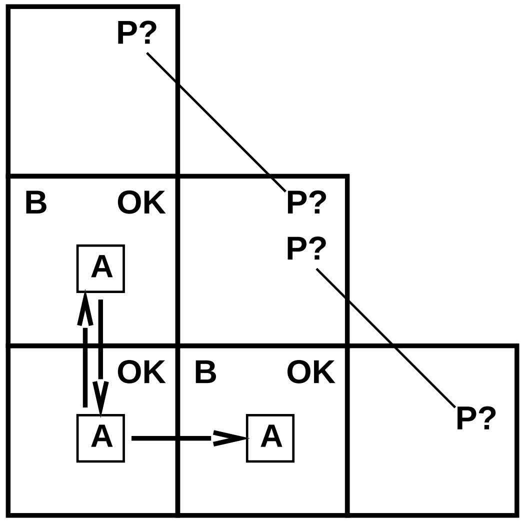
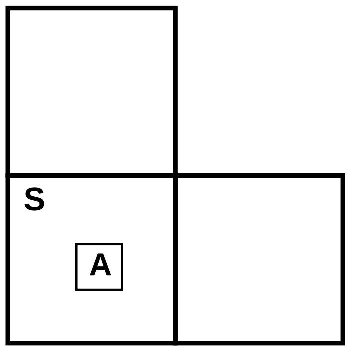


# Other tight spots




Breeze in (1,2) and (2,1)
⇒ no safe actions




Smell in (1,1)
⇒ cannot move
Can use a strategy of coercion:
shoot straight ahead
wumpus was there ⇒ dead ⇒ safe
wumpus wasn't there ⇒ safe

# Logic in general

Logics are formal languages for representing information
such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the "meaning" of sentences;
i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number $y$

$x + 2 \geq y$ is true in a world where $x = 7,\ y = 1$
$x + 2 \geq y$ is false in a world where $x = 0,\ y = 6$

# Entailment

Entailment means that one thing *follows from* another:

$KB \models \alpha$

Knowledge base $KB$ entails sentence $\alpha$
if and only if
$\alpha$ is true in all worlds where $KB$ is true

E.g., the KB containing "the Vols won" and "the Tide won"
entails "Either the Vols won or the Tide won"

E.g., $x + y = 4$ entails $4 = x + y$

Entailment is a relationship between sentences (i.e., *syntax*)
that is based on *semantics*

Note: brains process *syntax* (of some sort)

# Models

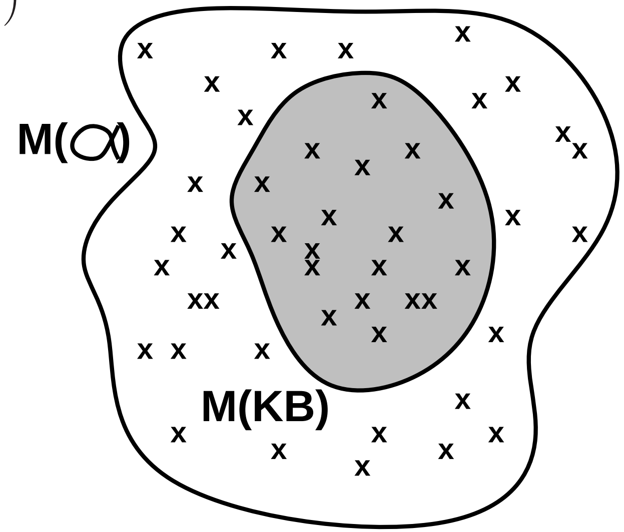
Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated

We say $m$ is a model of a sentence $\alpha$ if $\alpha$ is true in $m$

$M(\alpha)$ is the set of all models of $\alpha$

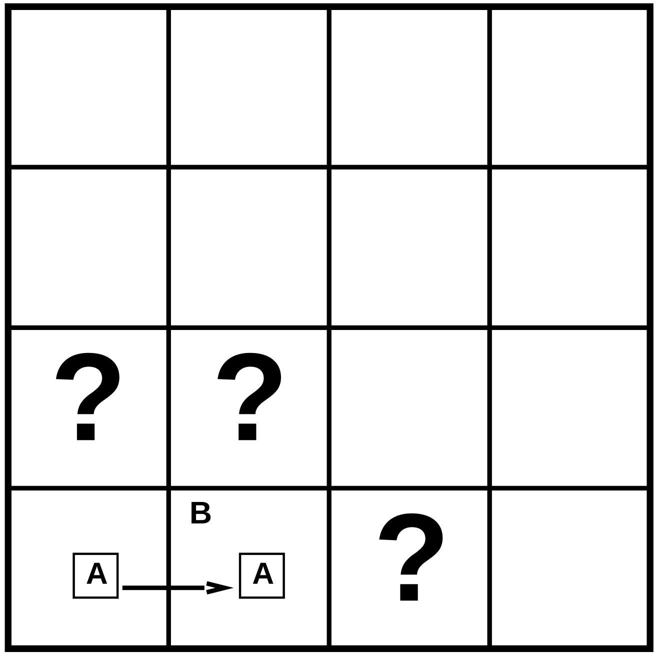
Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

E.g. $KB =$ Vols won and Tide won
$\alpha =$ Vols won



# Entailment in the wumpus world

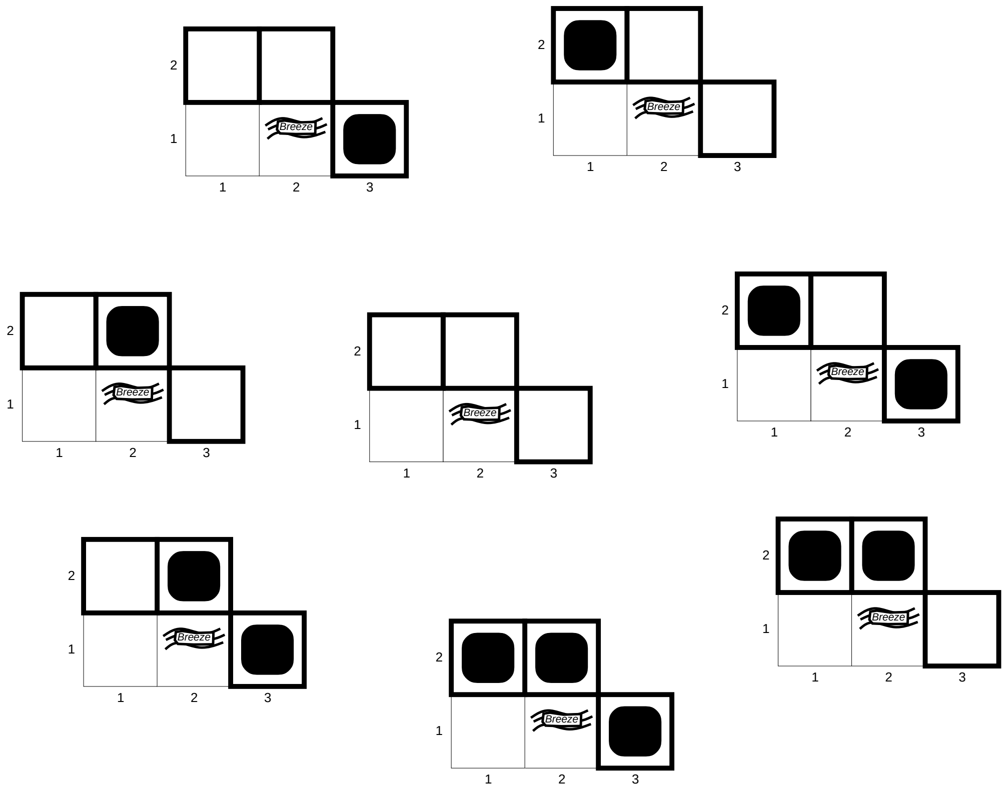
Situation after detecting nothing in [1,1], moving right, breeze in [2,1]



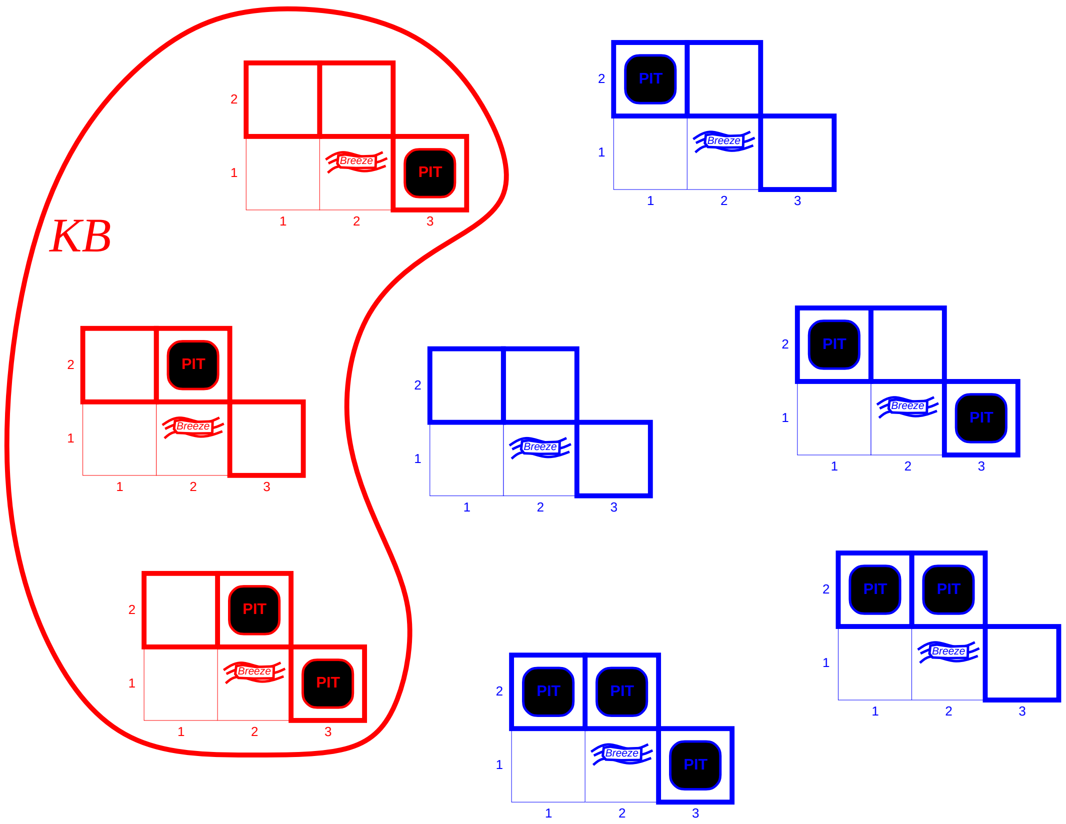

Consider possible models for ?s assuming only pits

3 Boolean choices $\Rightarrow$ 8 possible models

# Wumpus models

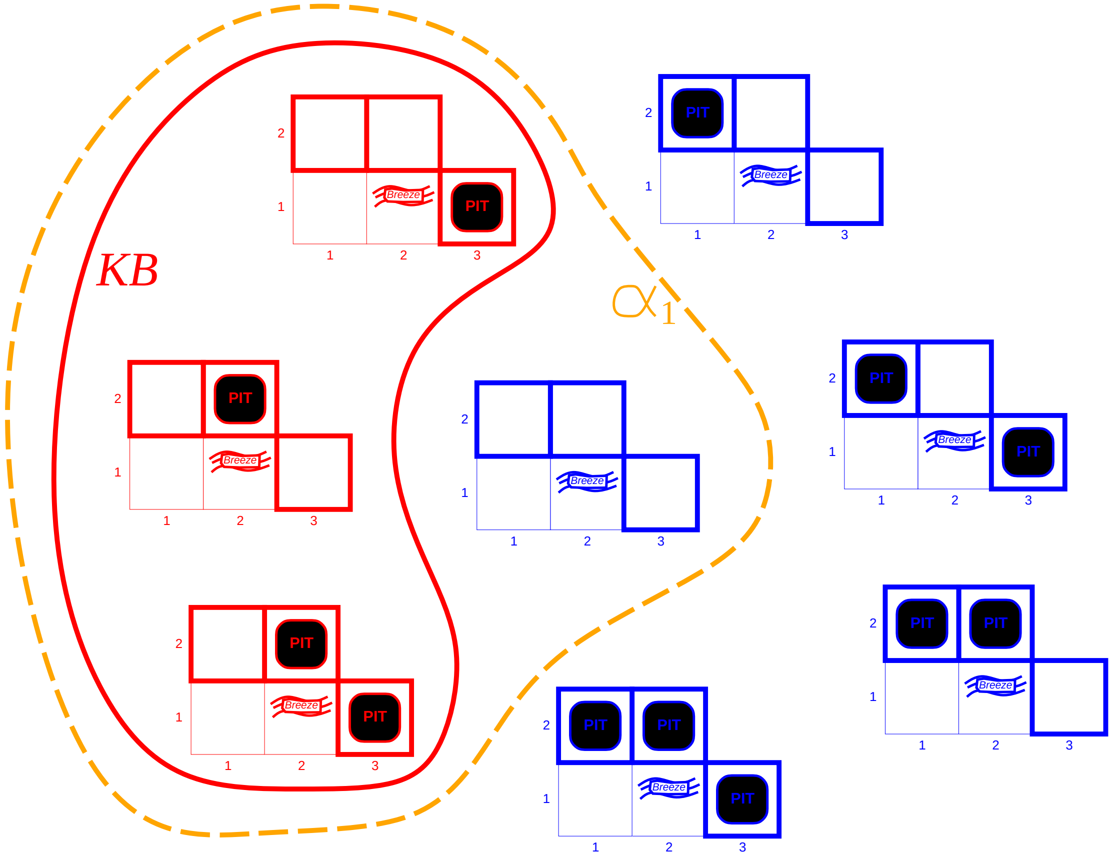


# Wumpus models




$KB$ = wumpus-world rules + observations

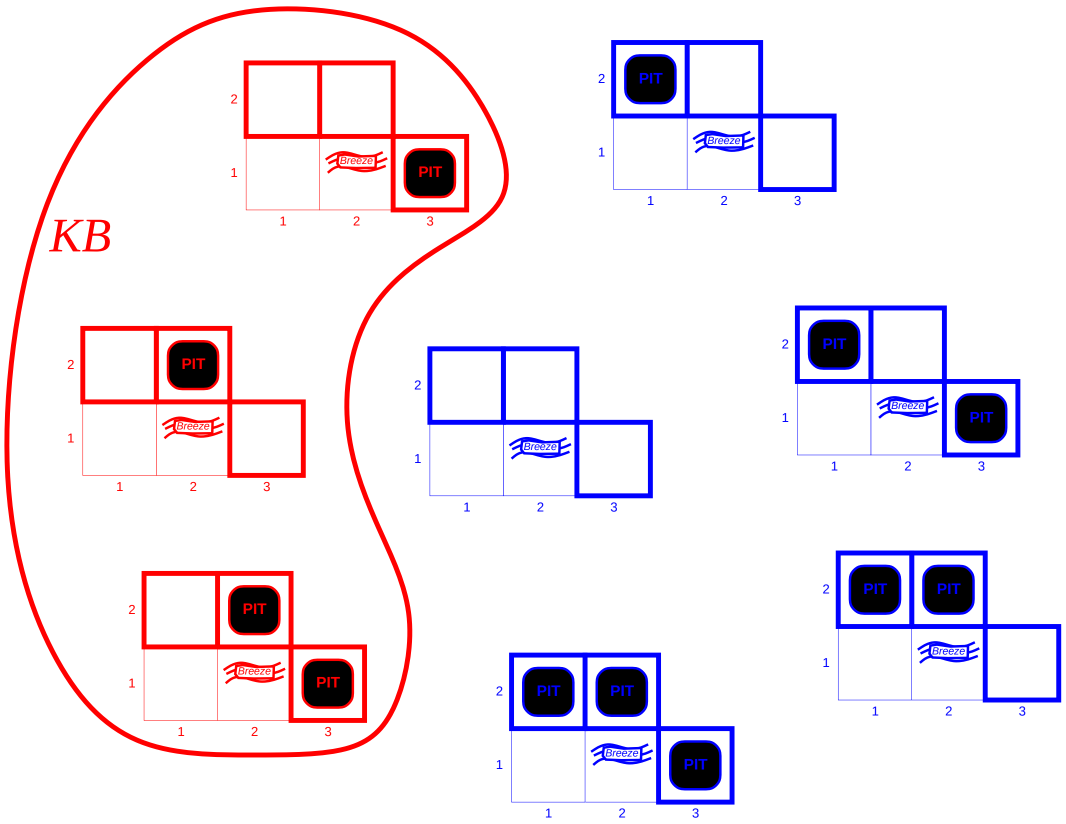
# Wumpus models




$KB$ = wumpus-world rules + observations

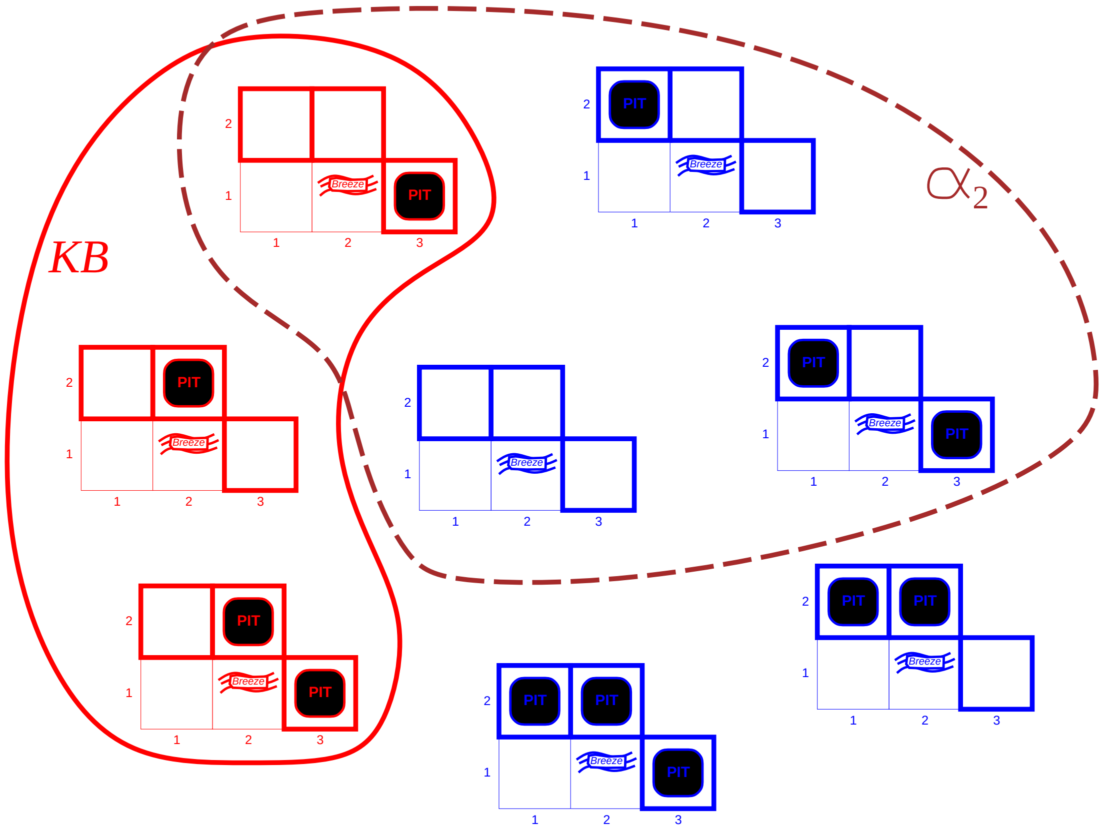
$\alpha_1$ = "[1,2] is safe", $KB \models \alpha_1$, proved by model checking

# Wumpus models




$KB$ = wumpus-world rules + observations

# Wumpus models




$KB$ = wumpus-world rules + observations

$\alpha_2$ = "[2,2] is safe", $KB \not\models \alpha_2$

# Inference

$KB \vdash_i \alpha$ = sentence $\alpha$ can be derived from $KB$ by procedure $i$

Consequences of $KB$ are a haystack; $\alpha$ is a needle.
Entailment = needle in haystack; inference = finding it

Soundness: $i$ is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: $i$ is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the $KB$.

# Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols $P_1$, $P_2$ etc are sentences

If $S$ is a sentence, $\neg S$ is a sentence (negation)

If $S_1$ and $S_2$ are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If $S_1$ and $S_2$ are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If $S_1$ and $S_2$ are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If $S_1$ and $S_2$ are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

# Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. $\begin{array}{ccc} P_{1,2} & P_{2,2} & P_{3,1} \\ false & false & true \end{array}$

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model $m$:

| | | | | | |
|---|---|---|---|---|---|
| $\neg S$ | is true iff | $S$ | is false | | |
| $S_1 \wedge S_2$ | is true iff | $S_1$ | is true and | $S_2$ | is true |
| $S_1 \vee S_2$ | is true iff | $S_1$ | is true or | $S_2$ | is true |
| $S_1 \Rightarrow S_2$ | is true iff | $S_1$ | is false or | $S_2$ | is true |
| i.e., | is false iff | $S_1$ | is true and | $S_2$ | is false |
| $S_1 \Leftrightarrow S_2$ | is true iff | $S_1 \Rightarrow S_2$ | is true and | $S_2 \Rightarrow S_1$ | is true |

Simple recursive process evaluates an arbitrary sentence, e.g.,
$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = true \wedge (false \vee true) = true \wedge true = true$

# Truth tables for connectives

| $P$ | $Q$ | $\neg P$ | $P \wedge Q$ | $P \vee Q$ | $P \Rightarrow Q$ | $P \Leftrightarrow Q$ |
|---|---|---|---|---|---|---|
| $false$ | $false$ | $true$ | $false$ | $false$ | $true$ | $true$ |
| $false$ | $true$ | $true$ | $false$ | $true$ | $true$ | $false$ |
| $true$ | $false$ | $false$ | $false$ | $true$ | $false$ | $false$ |
| $true$ | $true$ | $false$ | $true$ | $true$ | $true$ | $true$ |

# Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i,j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.

$$\neg P_{1,1}$$
$$\neg B_{1,1}$$
$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

# Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i,j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.

$$\neg P_{1,1}$$
$$\neg B_{1,1}$$
$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

$$B_{1,1} \quad \Leftrightarrow \quad (P_{1,2} \lor P_{2,1})$$
$$B_{2,1} \quad \Leftrightarrow \quad (P_{1,1} \lor P_{2,2} \lor P_{3,1})$$

"A square is breezy *if and only if* there is an adjacent pit"

# Truth tables for inference

Recall: $\alpha_1 =$ "[1,2] is safe"

Question: Does $KB \models \alpha_1$?

| $B_{1,1}$ | $B_{2,1}$ | $P_{1,1}$ | $P_{1,2}$ | $P_{2,1}$ | $P_{2,2}$ | $P_{3,1}$ | $KB$ | $\alpha_1$ |
|---|---|---|---|---|---|---|---|---|
| $false$ | $false$ | $false$ | $false$ | $false$ | $false$ | $false$ | $false$ | $true$ |
| $false$ | $false$ | $false$ | $false$ | $false$ | $false$ | $true$ | $false$ | $true$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $false$ | $true$ | $false$ | $false$ | $false$ | $false$ | $false$ | $false$ | $true$ |
| $false$ | $true$ | $false$ | $false$ | $false$ | $false$ | $true$ | $\underline{true}$ | $\underline{true}$ |
| $false$ | $true$ | $false$ | $false$ | $false$ | $true$ | $false$ | $\underline{true}$ | $\underline{true}$ |
| $false$ | $true$ | $false$ | $false$ | $false$ | $true$ | $true$ | $\underline{true}$ | $\underline{true}$ |
| $false$ | $true$ | $false$ | $false$ | $true$ | $false$ | $false$ | $false$ | $true$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $true$ | $true$ | $true$ | $true$ | $true$ | $true$ | $true$ | $false$ | $false$ |

Recall: $\alpha_2 =$ "[2,2] is safe"

Question: Does $KB \models \alpha_2$?

# Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-Entails?(KB, α) returns true or false
    symbols ← a list of the proposition symbols in KB and α
    return TT-Check-All(KB, α, symbols, [ ])
```

---

```
function TT-Check-All(KB, α, symbols, model) returns true or false
    if Empty?(symbols) then
        if PL-True?(KB, model) then return PL-True?(α, model)
        else return true
    else do
        P ← First(symbols); rest ← Rest(symbols)
        return TT-Check-All(KB, α, rest, Extend(P, true, model)) and
                   TT-Check-All(KB, α, rest, Extend(P, false, model))
```

$O(2^n)$ for $n$ symbols; problem is co-NP-complete

# Logical equivalence

Two sentences are logically equivalent iff true in same models:

$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) \quad \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) \quad \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) \quad \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) \quad \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \;\Rightarrow\; \beta) &\equiv (\neg\beta \;\Rightarrow\; \neg\alpha) \quad \text{contraposition} \\
(\alpha \;\Rightarrow\; \beta) &\equiv (\neg\alpha \lor \beta) \quad \text{implication elimination} \\
(\alpha \;\Leftrightarrow\; \beta) &\equiv ((\alpha \;\Rightarrow\; \beta) \land (\beta \;\Rightarrow\; \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg\alpha \lor \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) \quad \text{de Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) \quad \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) \quad \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

# Validity and satisfiability

A sentence is valid if it is true in *all* models,

e.g., $True$, $A \lor \neg A$, $A \Rightarrow A$, $(A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g., $A \lor B$, $C$

A sentence is unsatisfiable if it is true in no models

e.g., $A \land \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \land \neg\alpha)$ is unsatisfiable

i.e., prove $\alpha$ by *reductio ad absurdum*

# Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules
- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications
  Can use inference rules as operators in a standard search alg.
- Typically require translation of sentences into a normal form

Model checking
truth table enumeration (always exponential in $n$)
improved backtracking, e.g., Davis–Putnam–Logemann–Loveland
heuristic search in model space (sound but incomplete)
e.g., min-conflicts-like hill-climbing algorithms

# Resolution

Conjunctive Normal Form (CNF—universal)

$\qquad$ *conjunction* of $\underbrace{\textit{disjunctions} \text{ of } \textit{literals}}_{\textit{clauses}}$

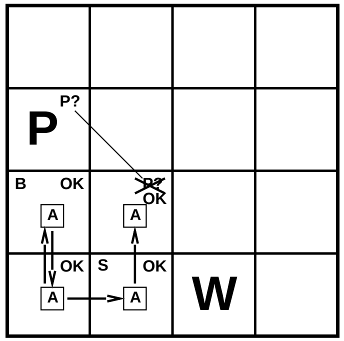
$\qquad$ E.g., $(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$

Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \lor \cdots \lor \ell_k, \qquad m_1 \lor \cdots \lor m_n}{\ell_1 \lor \cdots \lor \ell_{i-1} \lor \ell_{i+1} \lor \cdots \lor \ell_k \lor m_1 \lor \cdots \lor m_{j-1} \lor m_{j+1} \lor \cdots \lor m_n}$$

where $\ell_i$ and $m_j$ are complementary literals. E.g.,

$$\frac{P_{1,3} \lor P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$




Resolution is sound and complete for propositional logic

# Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \;\Rightarrow\; \beta) \wedge (\beta \;\Rightarrow\; \alpha)$.

   $(B_{1,1} \;\Rightarrow\; (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \;\Rightarrow\; B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributivity law ($\vee$ over $\wedge$) and flatten:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

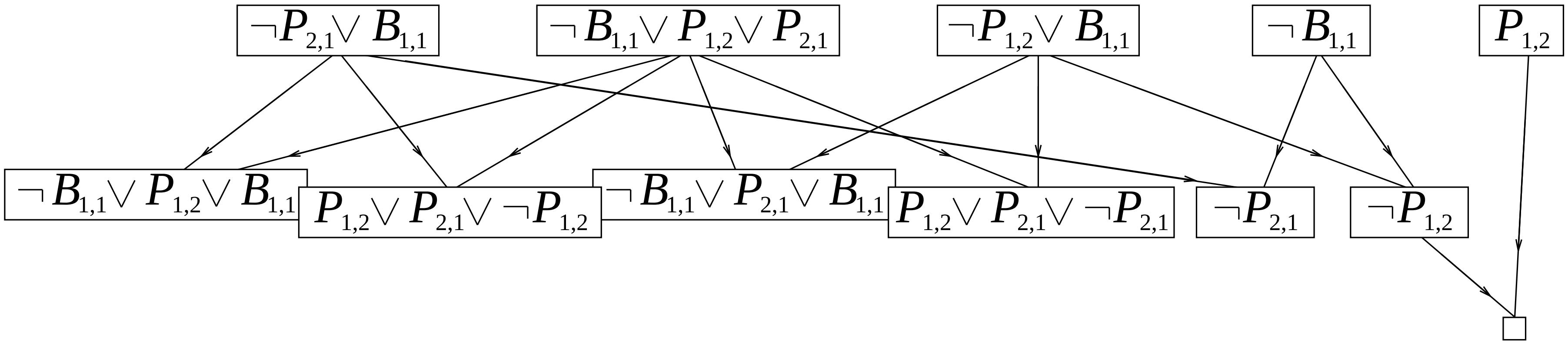
# Resolution algorithm

Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable

```
function PL-Resolution(KB, α) returns true or false
    clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
    new ← { }
    loop do
        for each Ci, Cj in clauses do
            resolvents ← PL-Resolve(Ci, Cj)
            if resolvents contains the empty clause then return true
            new ← new ∪ resolvents
        if new ⊆ clauses then return false
        clauses ← clauses ∪ new
```

# Resolution example

$KB = (B_{1,1} \;\Leftrightarrow\; (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$ $\alpha = \neg P_{1,2}$



# Forward and backward chaining

Horn Form (restricted)

KB = *conjunction* of *Horn clauses*

Horn clause =

- ♢ proposition symbol; or
- ♢ (conjunction of symbols) $\Rightarrow$ symbol

E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

Modus Ponens (for Horn Form): complete for Horn KBs

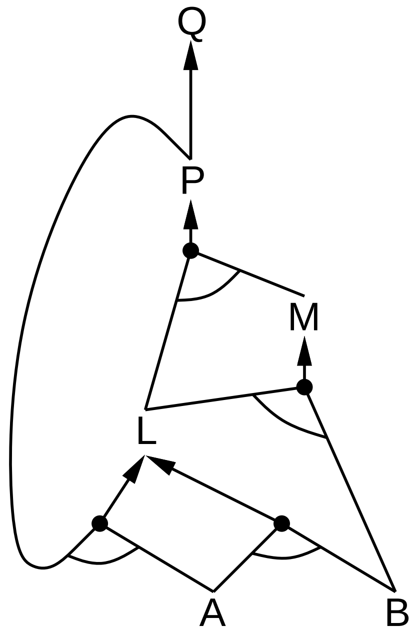
$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining.
These algorithms are very natural and run in *linear* time

# Forward chaining

Idea: fire any rule whose premises are satisfied in the $KB$,
add its conclusion to the $KB$, until query is found

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

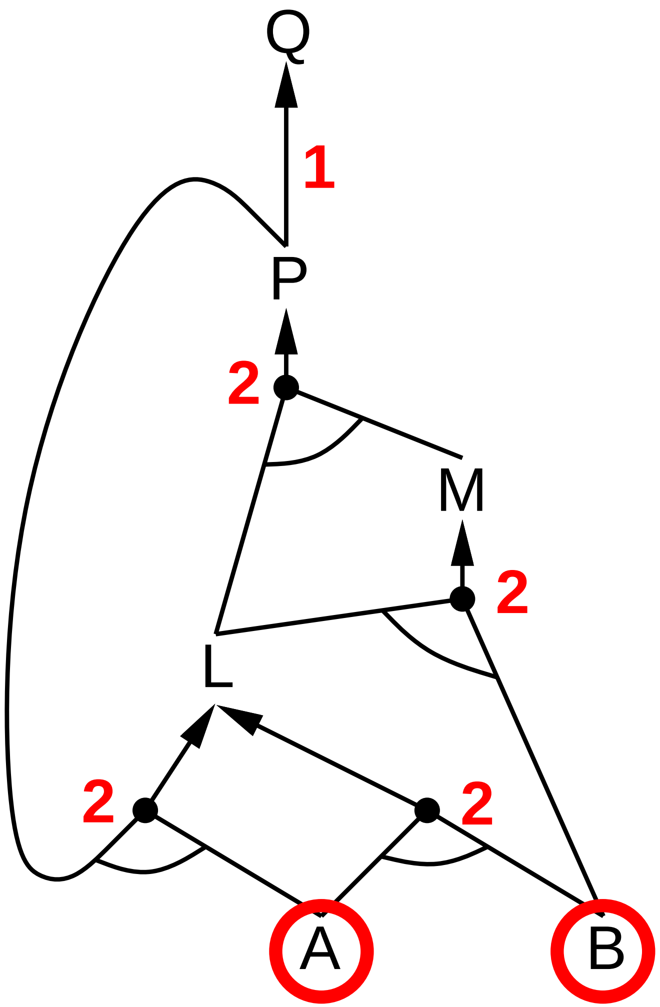


# Forward chaining algorithm

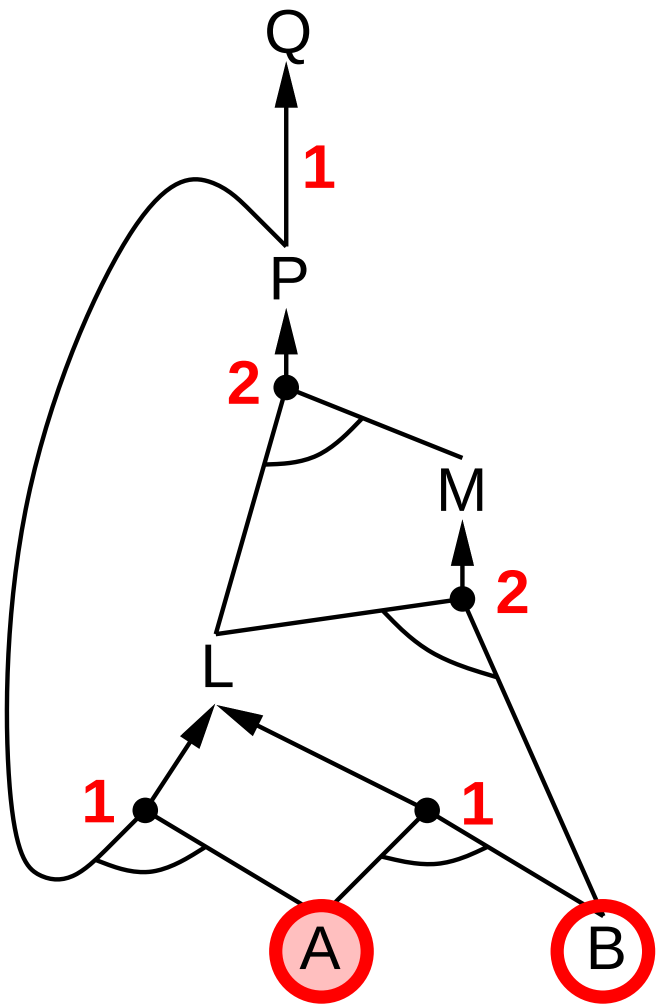
```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known to be true

  while agenda is not empty do
      p ← POP(agenda)
      unless inferred[p] do
          inferred[p] ← true
          for each Horn clause c in whose premise p appears do
              decrement count[c]
              if count[c] = 0 then do
                  if HEAD[c] = q then return true
                  PUSH(HEAD[c], agenda)
  return false
```

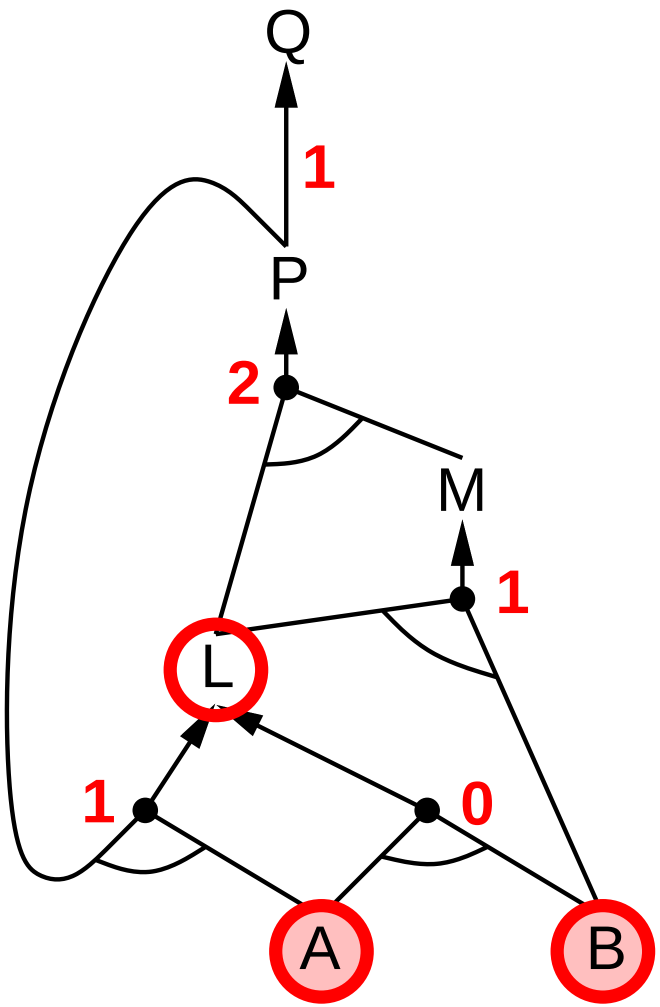
# Forward chaining example



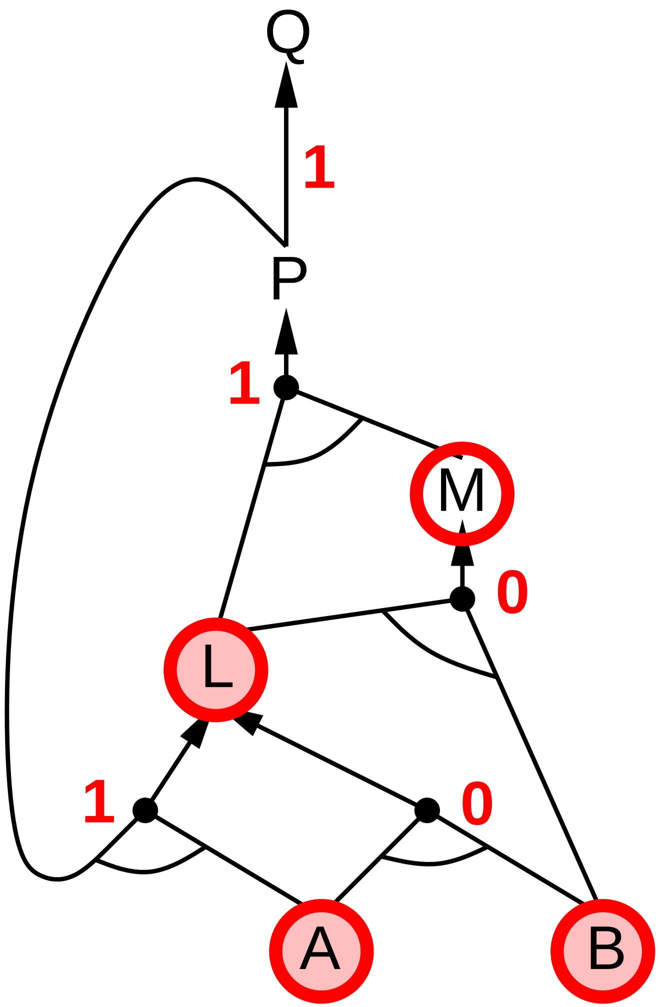
# Forward chaining example



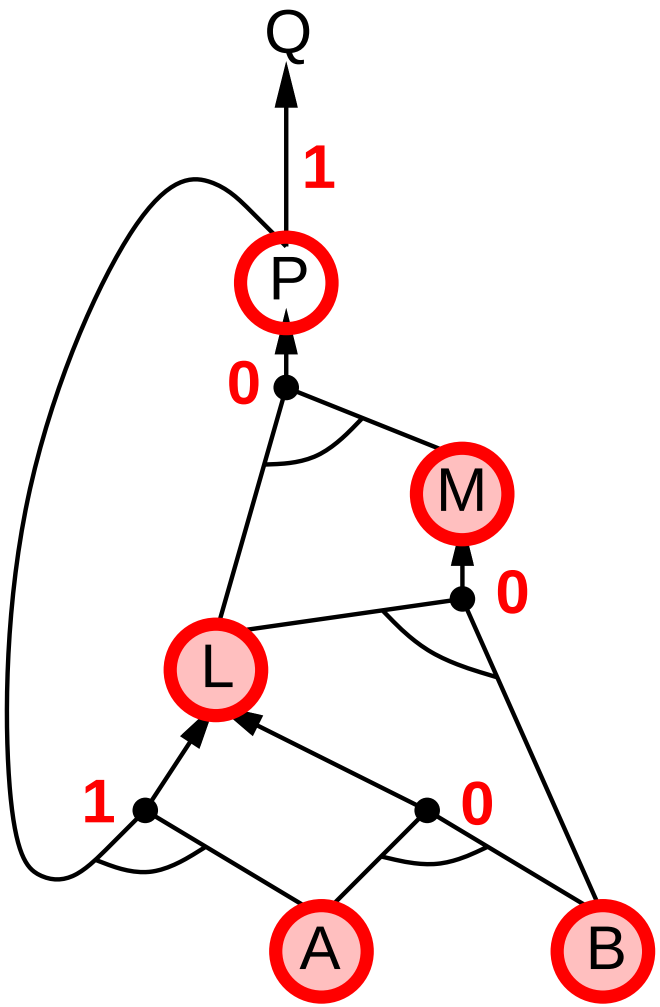
# Forward chaining example



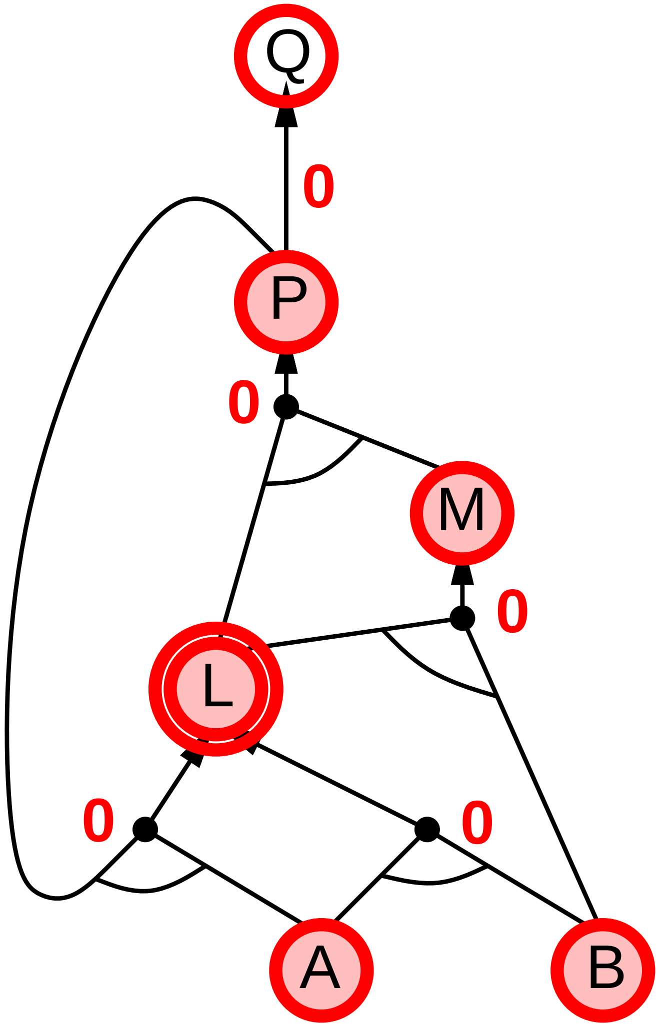
# Forward chaining example



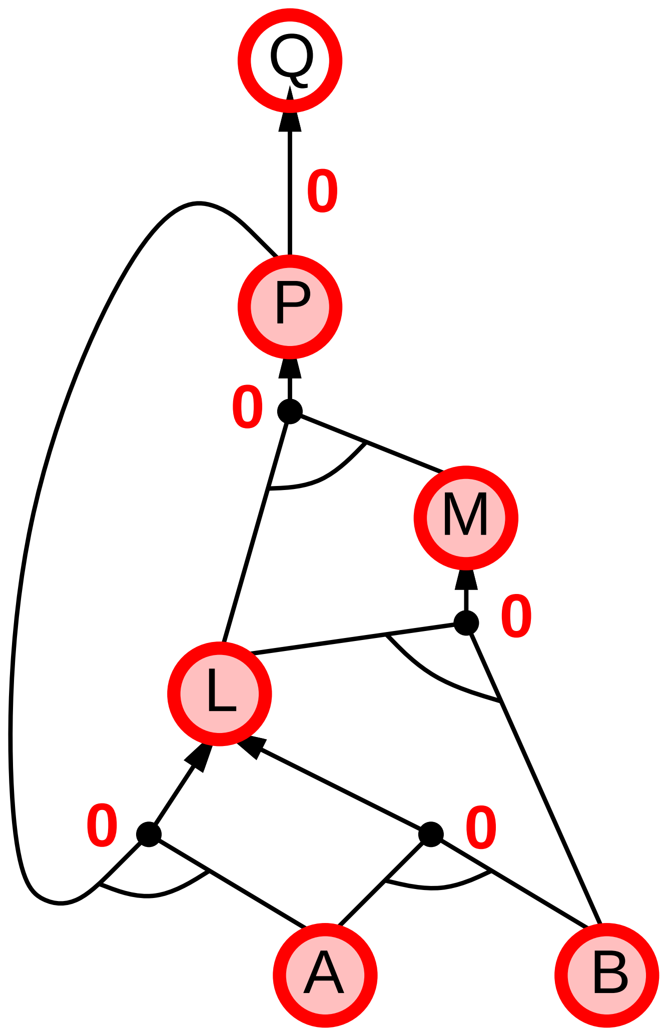
# Forward chaining example



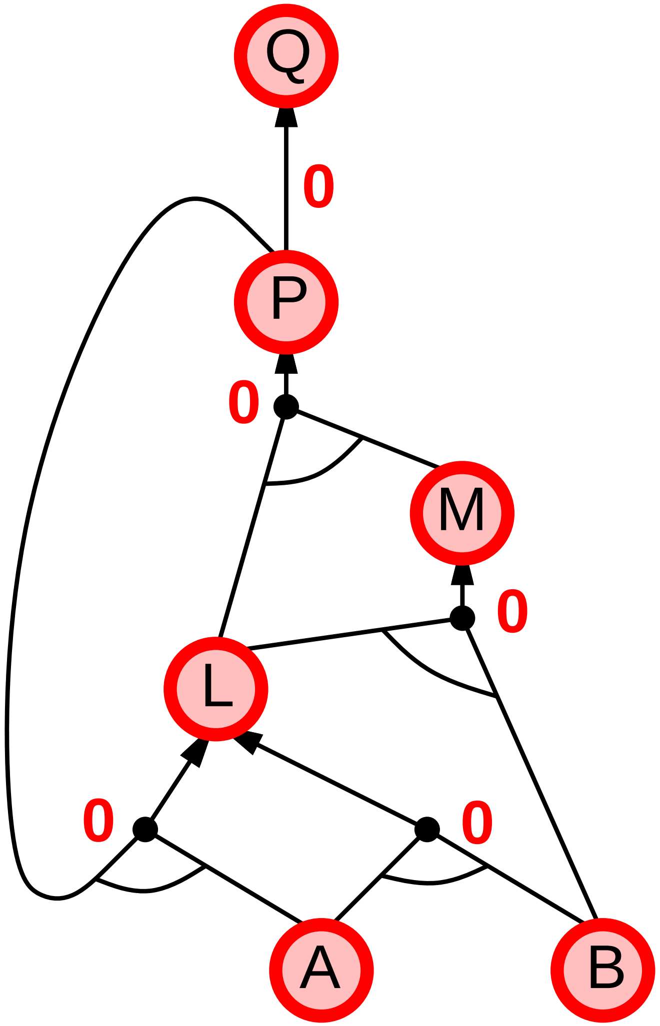
# Forward chaining example



# Forward chaining example



# Forward chaining example



# Proof of completeness

FC derives every atomic sentence that is entailed by $KB$

1. FC reaches a fixed point where no new atomic sentences are derived

2. Consider the final state as a model $m$, assigning true/false to symbols

3. Every clause in the original $KB$ is true in $m$
   *Proof*: Suppose a clause $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ is false in $m$
   Then $a_1 \wedge \ldots \wedge a_k$ is true in $m$ and $b$ is false in $m$
   Therefore the algorithm has not reached a fixed point!

4. Hence $m$ is a model of $KB$

5. If $KB \models q$, $q$ is true in *every* model of $KB$, including $m$

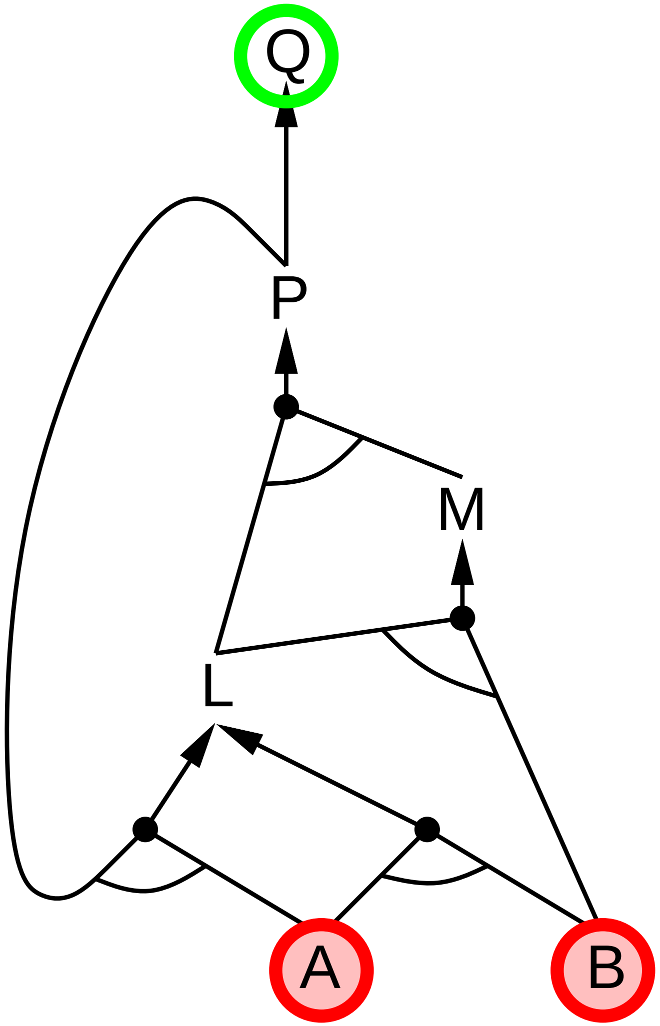
# Backward chaining

Idea: work backwards from the query $q$:
  to prove $q$ by BC,
    check if $q$ is known already, or
    prove by BC all premises of some rule concluding $q$

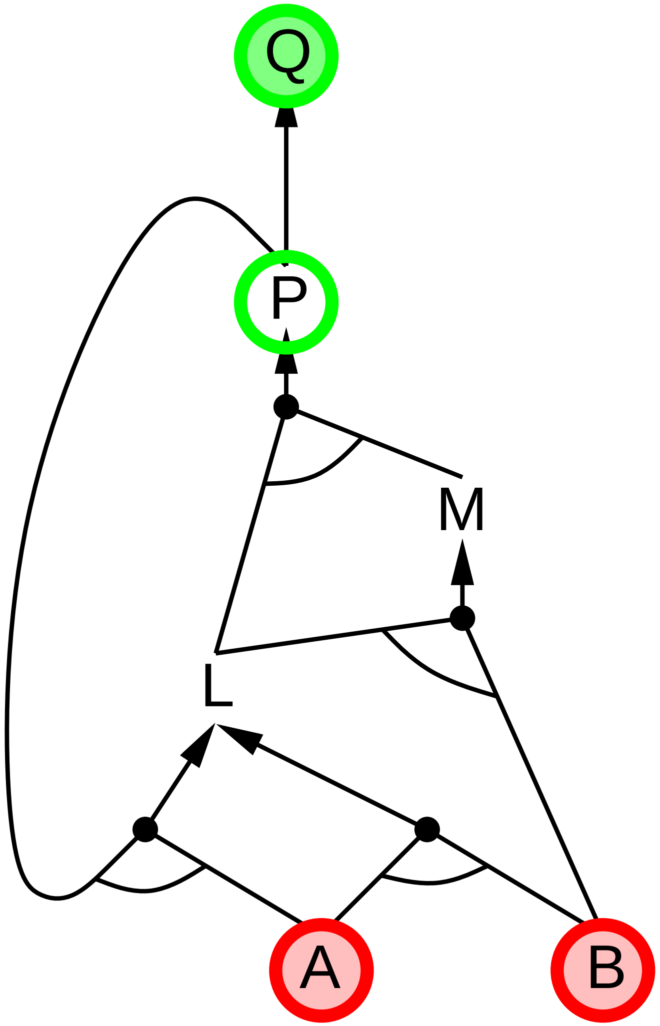
Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal
  1) has already been proved true, or
  2) has already failed

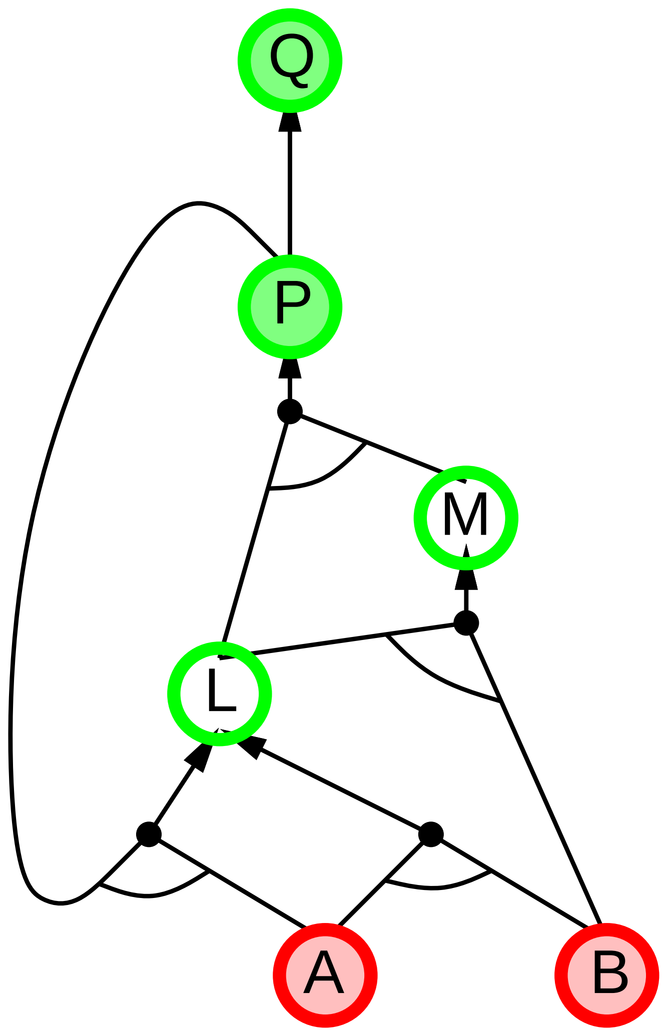
# Backward chaining example



# Backward chaining example



# Backward chaining example



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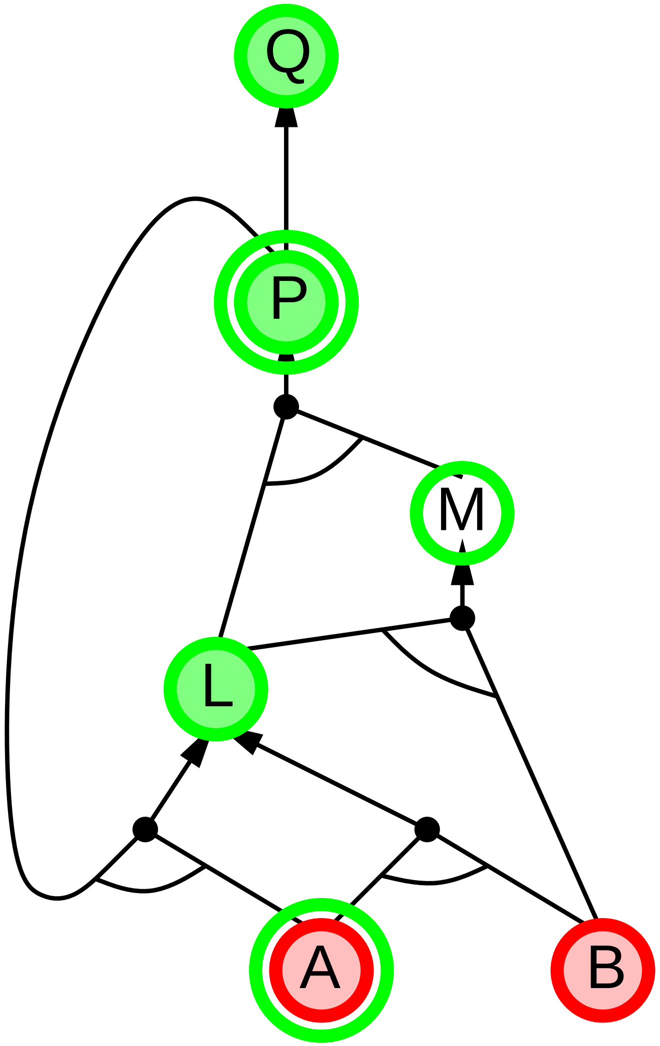

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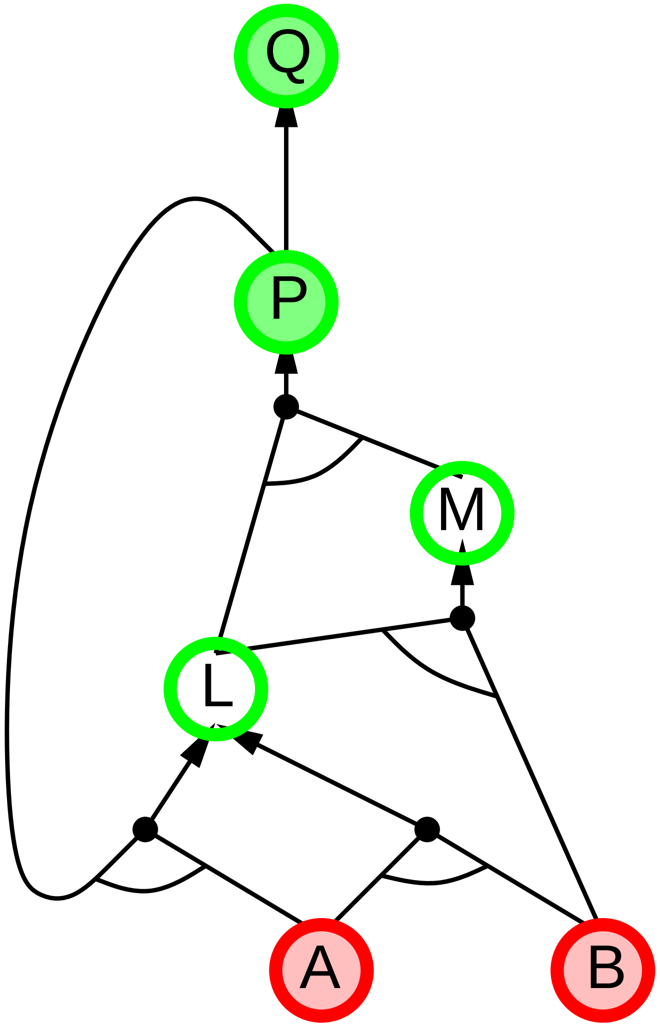

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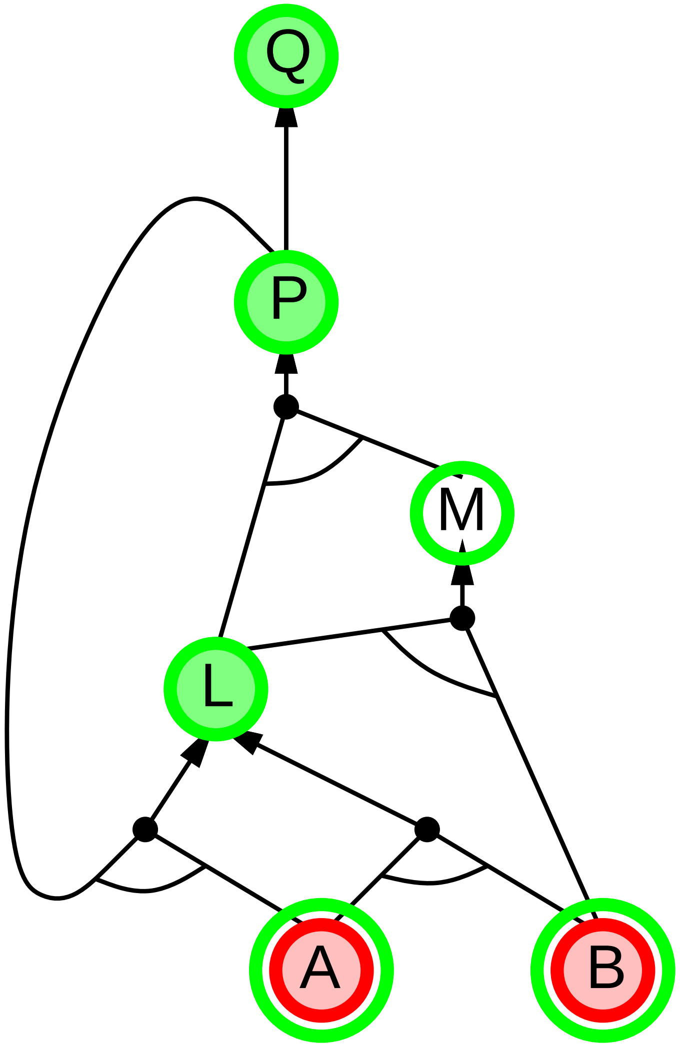

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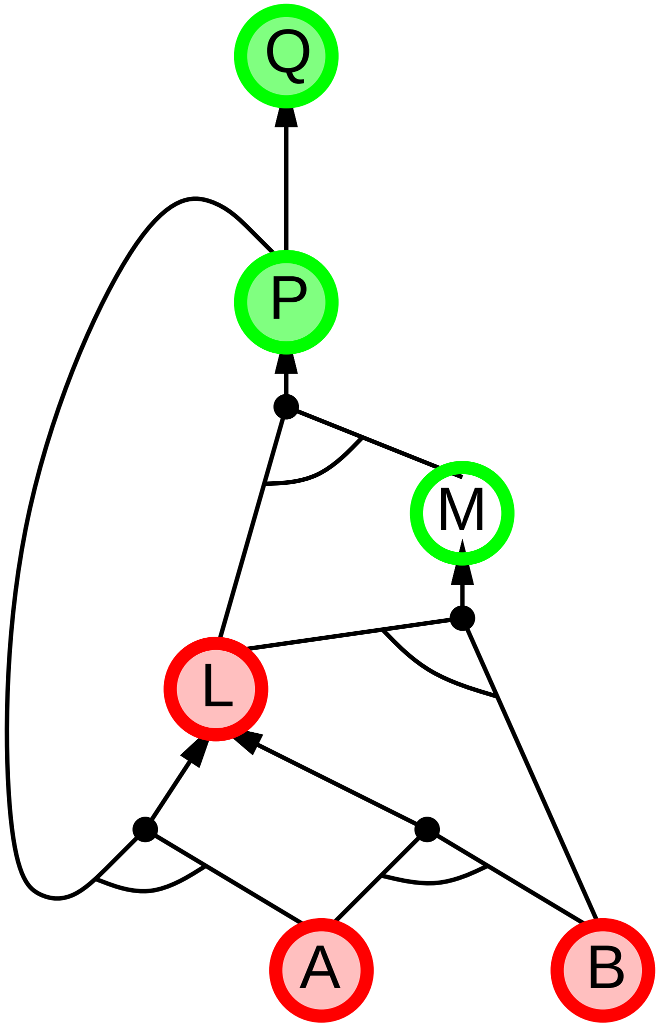

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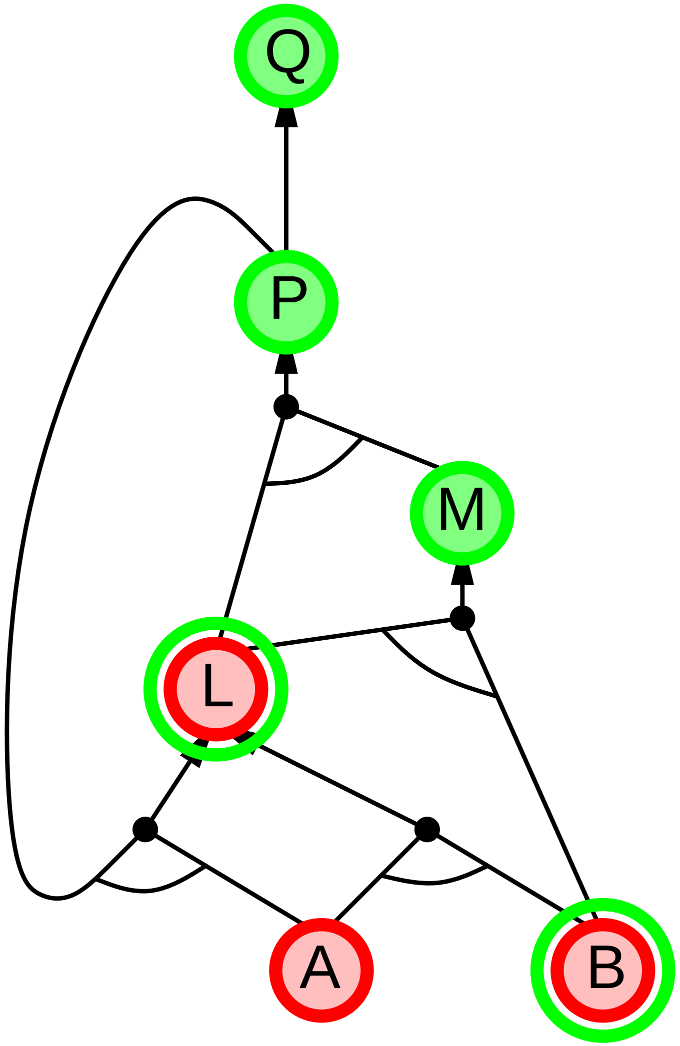

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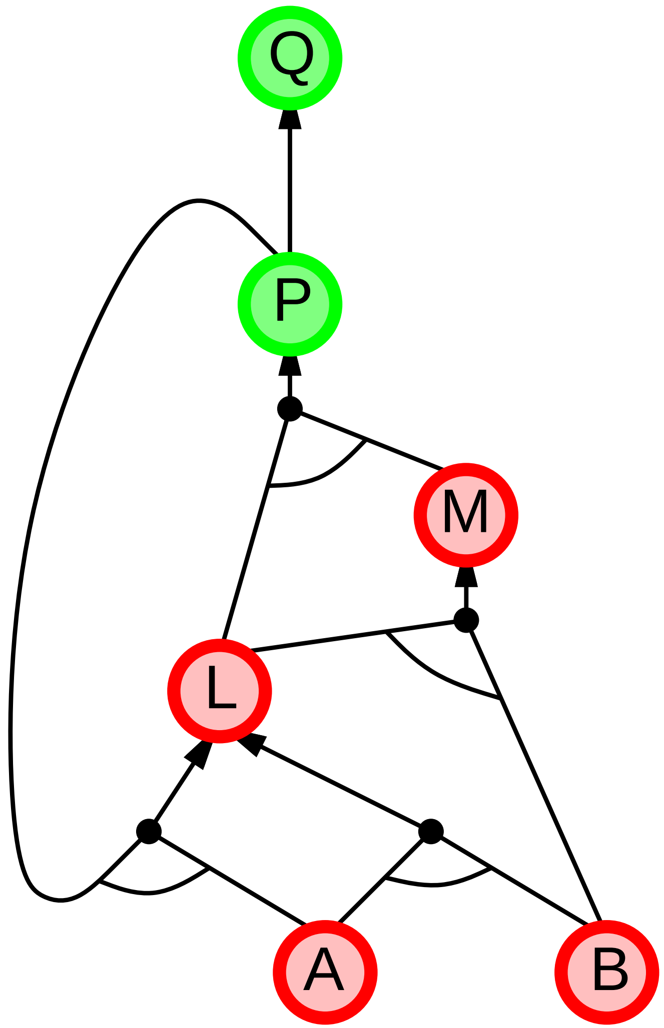

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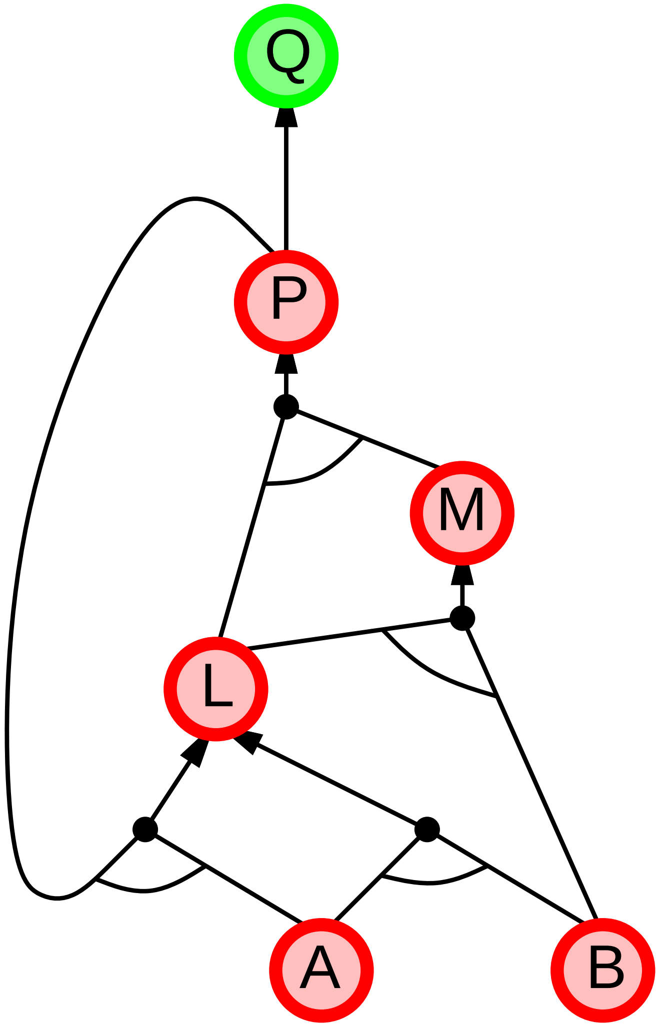

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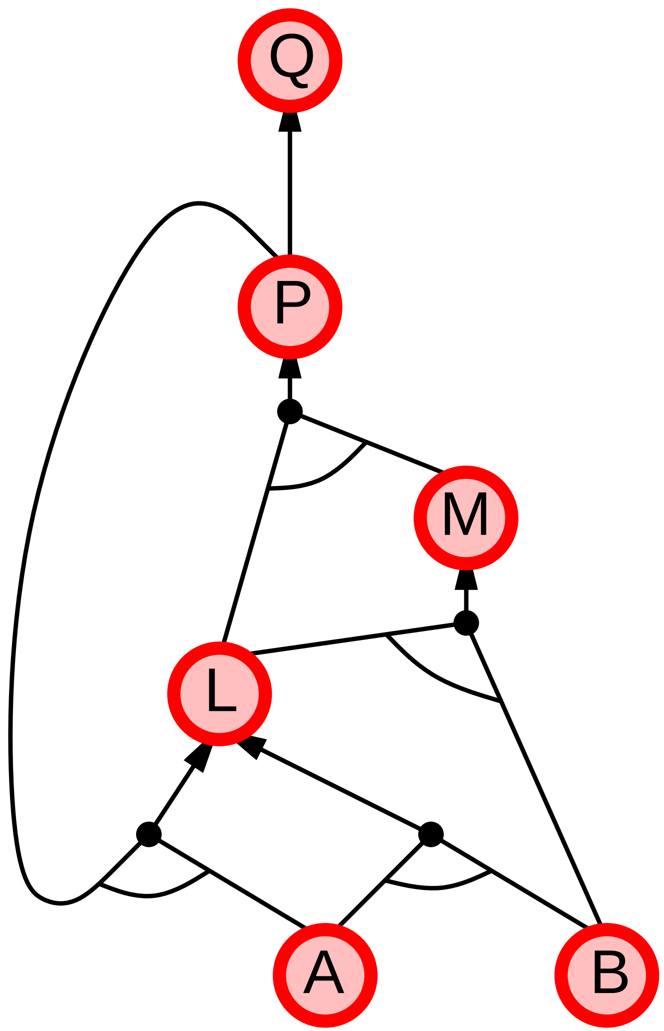

# Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing,
e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for problem-solving,
e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be *much less* than linear in size of KB

# Summary

Logical agents apply inference to a knowledge base
to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundness: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses
Resolution is complete for propositional logic

Propositional logic lacks expressive power

# Thought discussion

♢ Continuing with discussion on "Weak AI"...

♢ We'll discuss the "mathematical objection" to creating intelligent machines (read pages 949-950)