Hierarchical Data Structures, Scene Graph and Quaternion

Jian Huang

Spatial Data Structure

- Octree, Quadtree
- BSP tree
- K-D tree

Spatial Data Structures

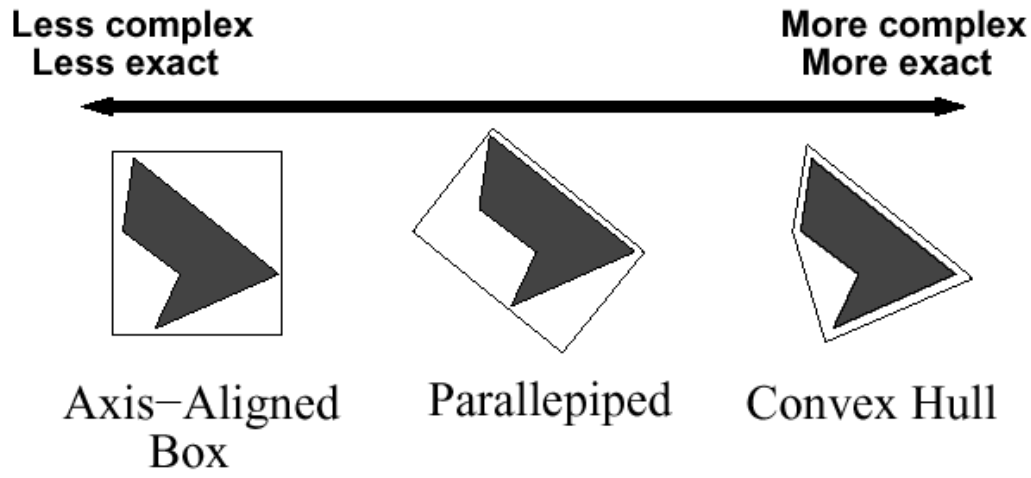
- Data structures for efficiently storing geometric information. They are useful for
  - Collision detection (will the spaceships collide?)
  - Location queries (which is the nearest post office?)
  - Chemical simulations (which protein will this drug molecule interact with?)
  - Rendering (is this aircraft carrier on-screen?), and more
- Good data structures can give speed up rendering by 10x, 100x, or more

Bounding Volume

- Simple notion: wrap things that are hard to check for ray intersection in things that are easy to check.
  - Example: wrap a complicated polygonal mesh in a box. Ray can't hit the real object unless it hits the box
- Adds some overhead, but generally pays for itself .
- Can build bounding volume hierarchies

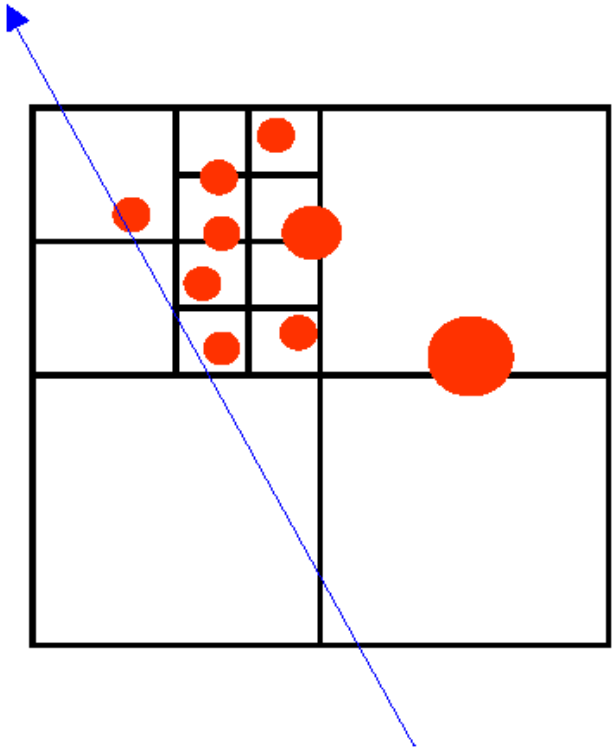
Bounding Volumes

- Choose Bounding Volume(s)
  - Spheres
  - Boxes
  - Parallelepipeds
  - Oriented boxes
  - Ellipsoids
  - Convex hulls



Quad-trees

- Quad-tree is the 2-D generalization of binary tree
  - node (cell) is a square
  - recursively split into four equal sub-squares
  - stop when leaves get “simple enough”

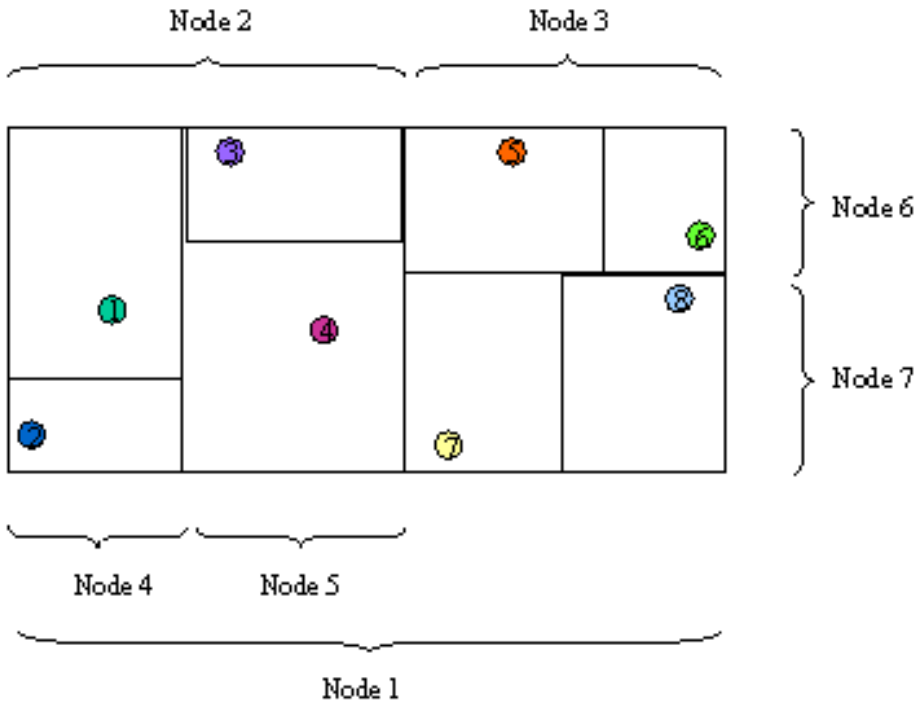


Octrees

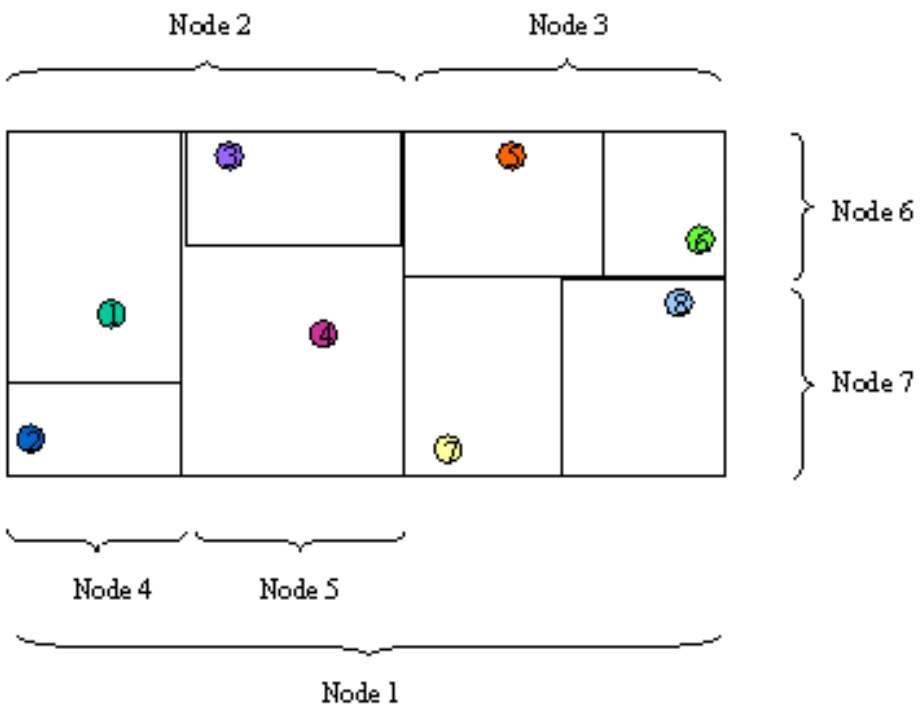
- Octree is the 3-D generalization of quad-tree
- node (cell) is a cube, recursively split into eight equal sub- cubes
  - stop splitting when the number of objects intersecting the cell gets “small enough” or the tree depth exceeds a limit
  - internal nodes store pointers to children, leaves store list of surfaces
- more expensive to traverse than a grid
- adapts to non-homogeneous, clumpy scenes better

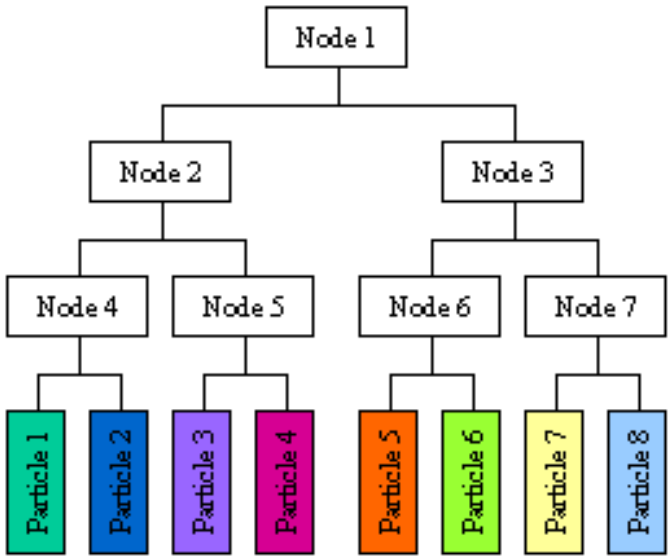
K-D tree

- The K-D approach is to make the problem space a rectangular parallelepiped whose sides are, in general, of unequal length.
- The length of the sides is the maximum spatial extent of the particles in each spatial dimension.



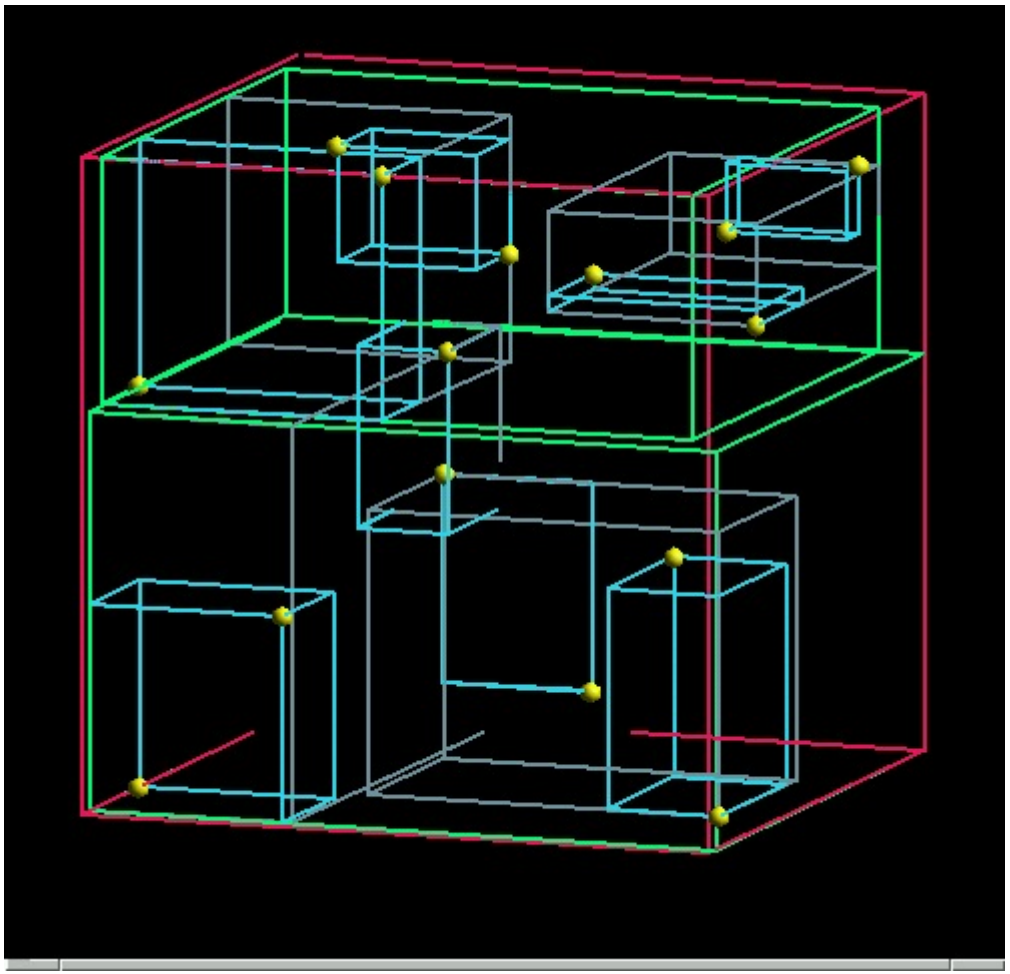
K-D tree


Node 2
Node 3
Node 6
Node 7
Node 4
Node 5
Node 1


Node 1
Node 2
Node 3
Node 4
Node 5
Node 6
Node 7
Particle 1
Particle 2
Particle 3
Particle 4
Particle 5
Particle 6
Particle 7
Particle 8

K-D Tree in 3-D

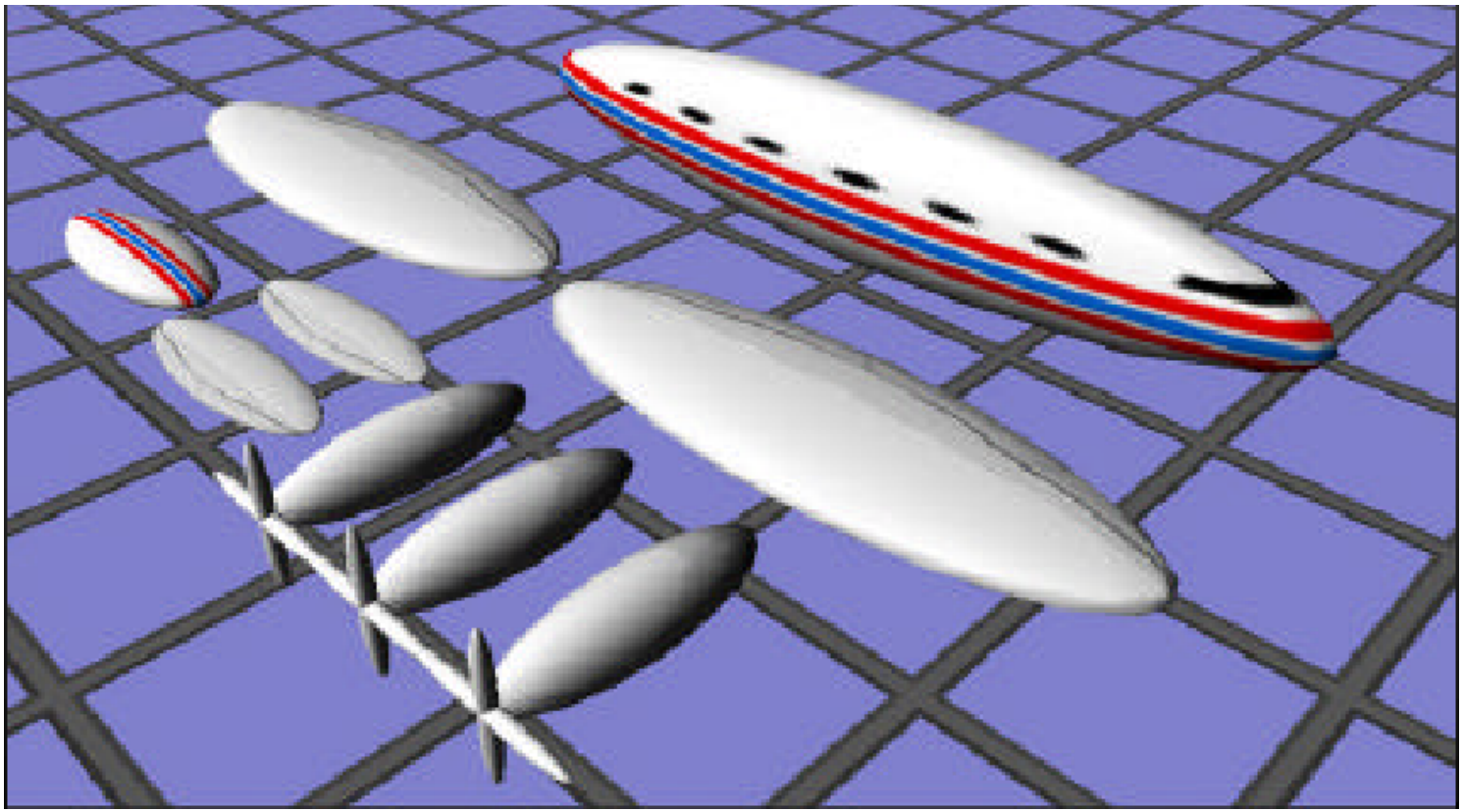
- Similarly, the problem space in three dimensions is a parallelepiped whose sides are the greatest particle separation in each of the three spatial dimensions.



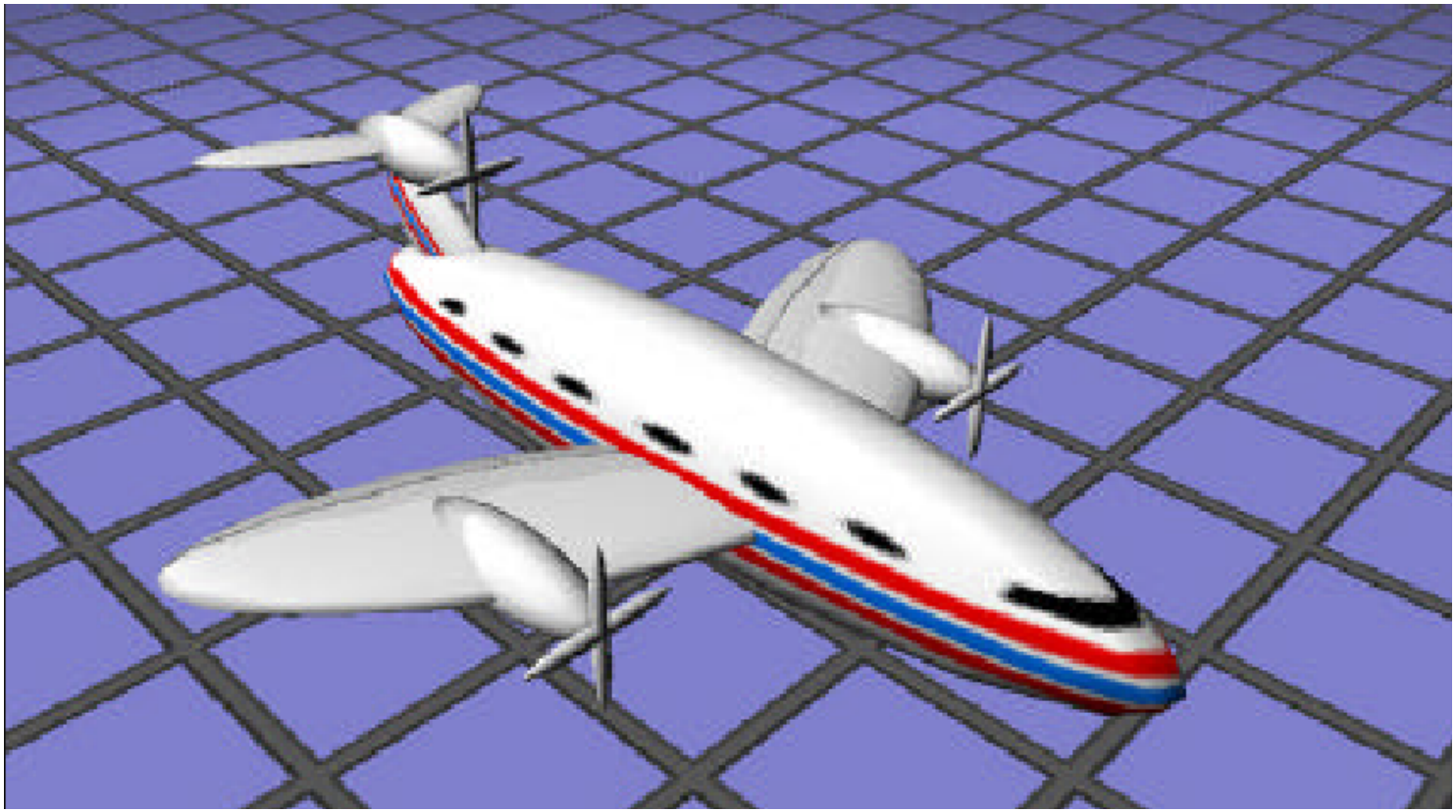
Motivation for Scene Graph

- Three-fold
  - Performance
  - Generality
  - Ease of use
- How to model a scene ?
  - Java3D, Open Inventor, Open Performer, VRML, etc.

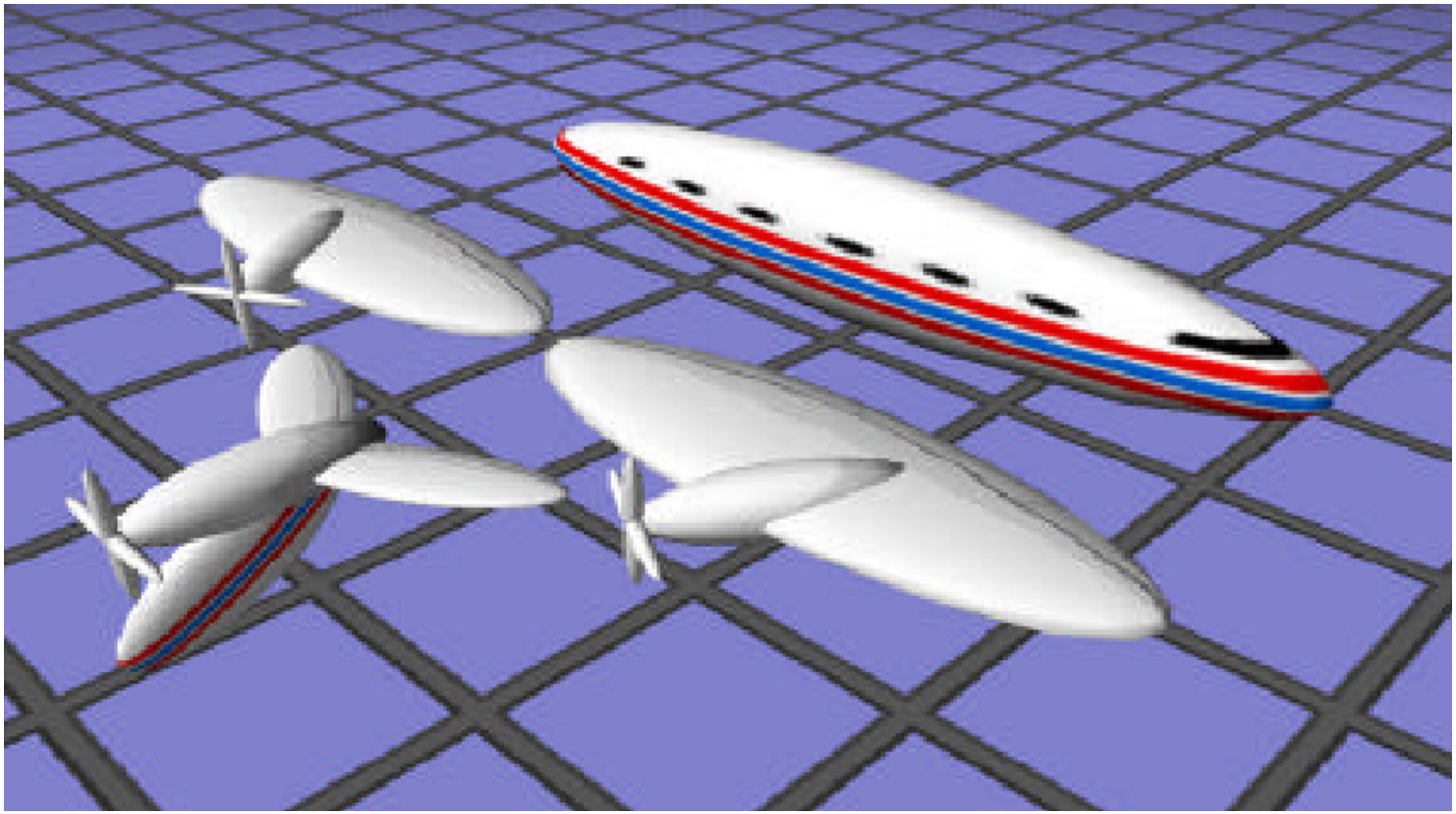
Scene Graph Example



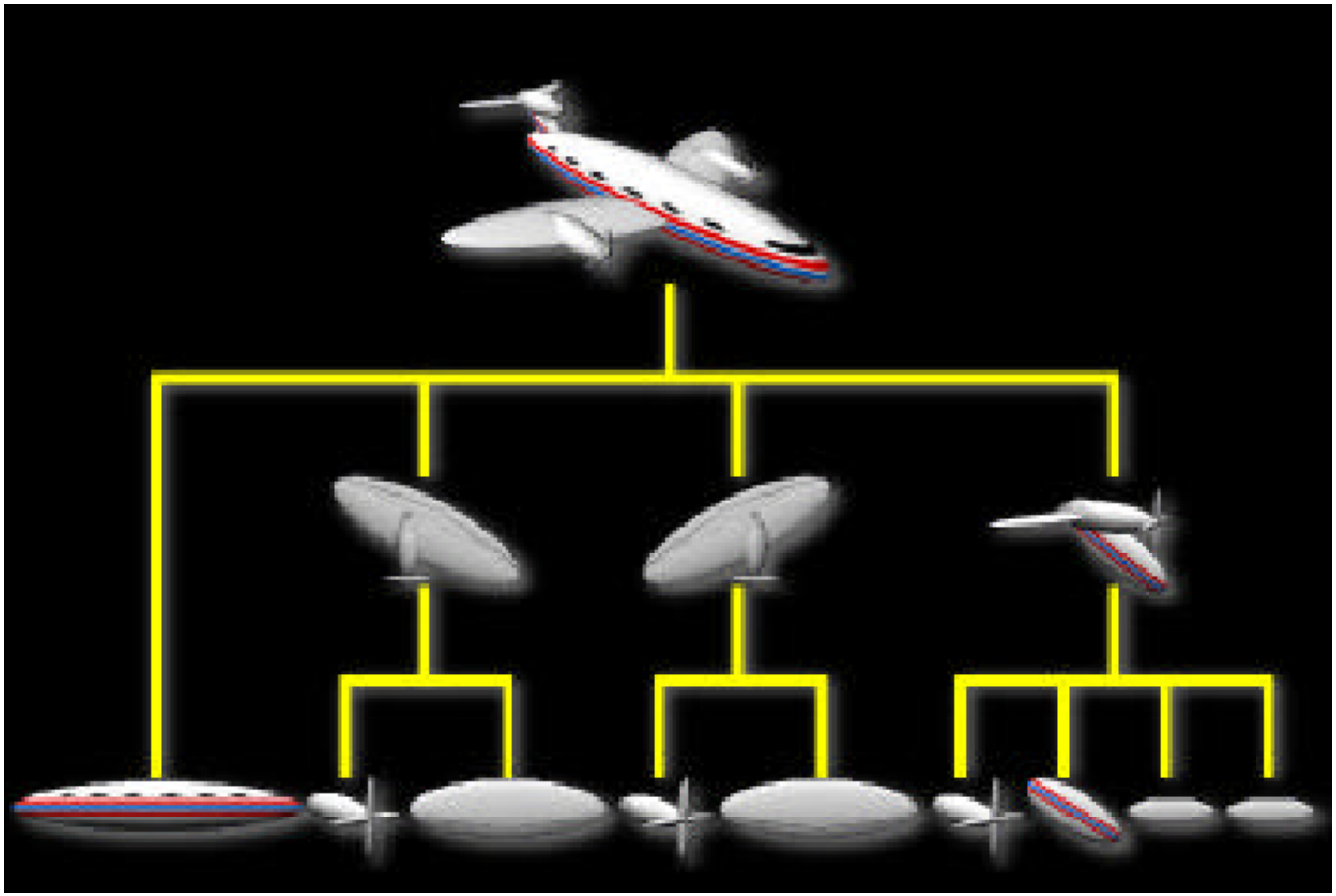
Scene Graph Example



Scene Graph Example



Scene Graph Example



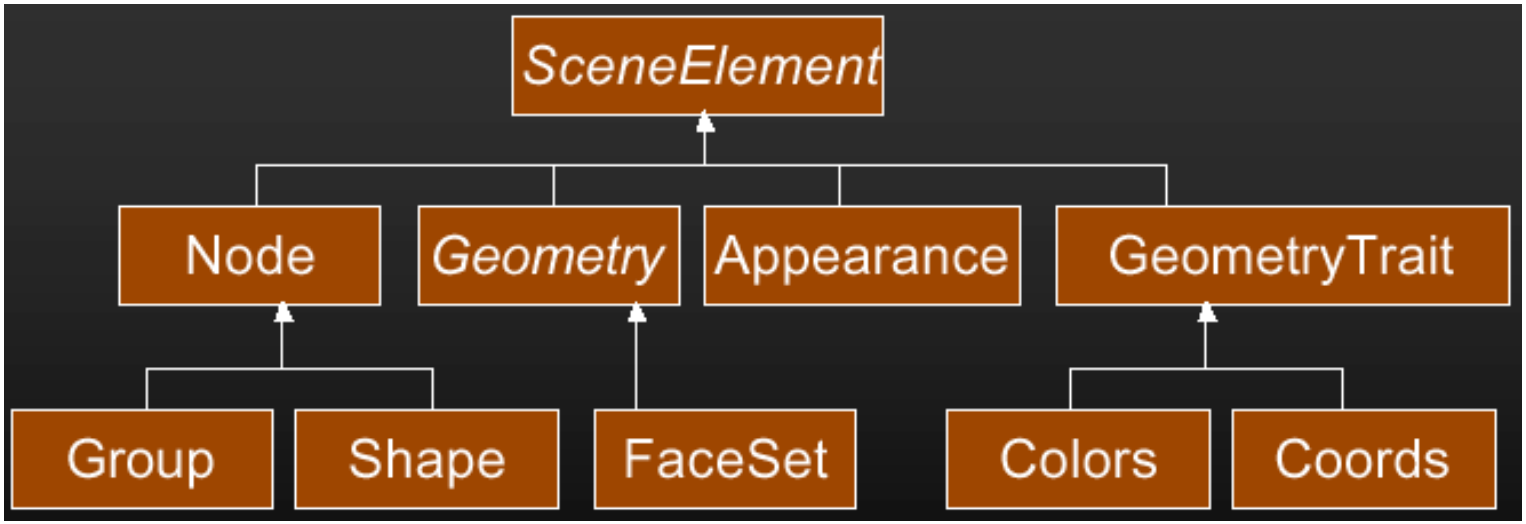
Scene Description

- Set of Primitives
- Specify for each primitive
  - Transformation
  - Lighting attributes
  - Surface attributes
    - Material (BRDF)
    - Texture
    - Texture transformation

Scene Graphs

- Scene Elements
  - Interior Nodes
    - Have children that inherit state
    - transform, lights, fog, color, …
  - Leaf nodes
    - Terminal
    - geometry, text
  - Attributes
    - Additional sharable state (textures)

Scene Element Class Hierarchy



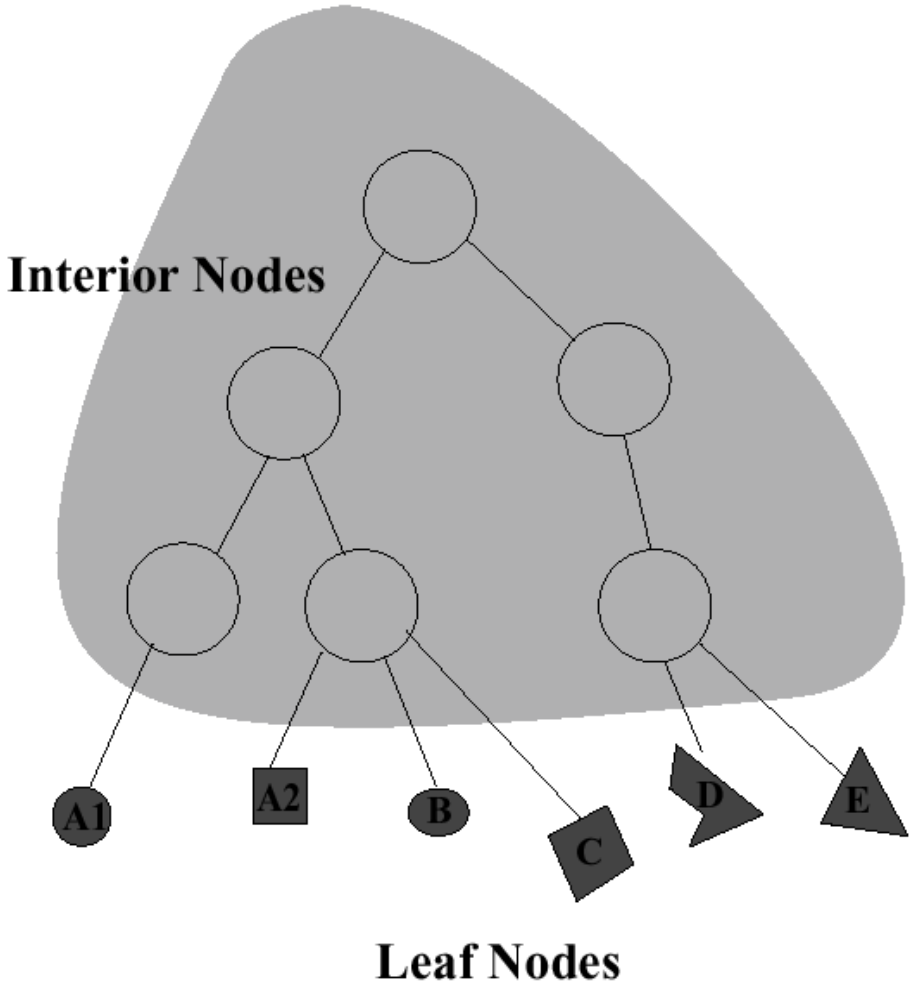
Scene Graph

- Graph Representation
  - What do edges mean?
  - Inherit state along edges
    - group all red object instances together
    - group logical entities together
      - parts of a car
  - Capture intent with the structure

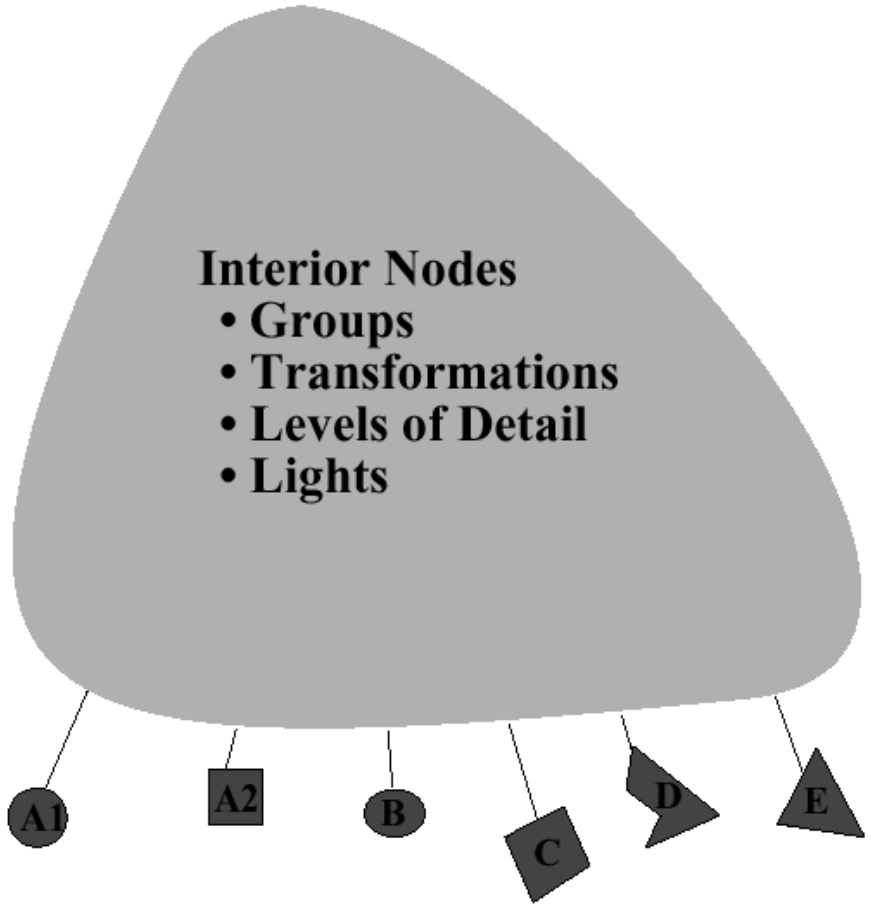
Scene Graph

- Inheritance -- Overloaded Term
  - Behavior inheritance (subclassing)
    - Benefit of OO design
  - Implementation inheritance
    - Perhaps provided by implementation language
    - *Not essential* for a good API design
  - Implied inheritance
    - Designed into the API

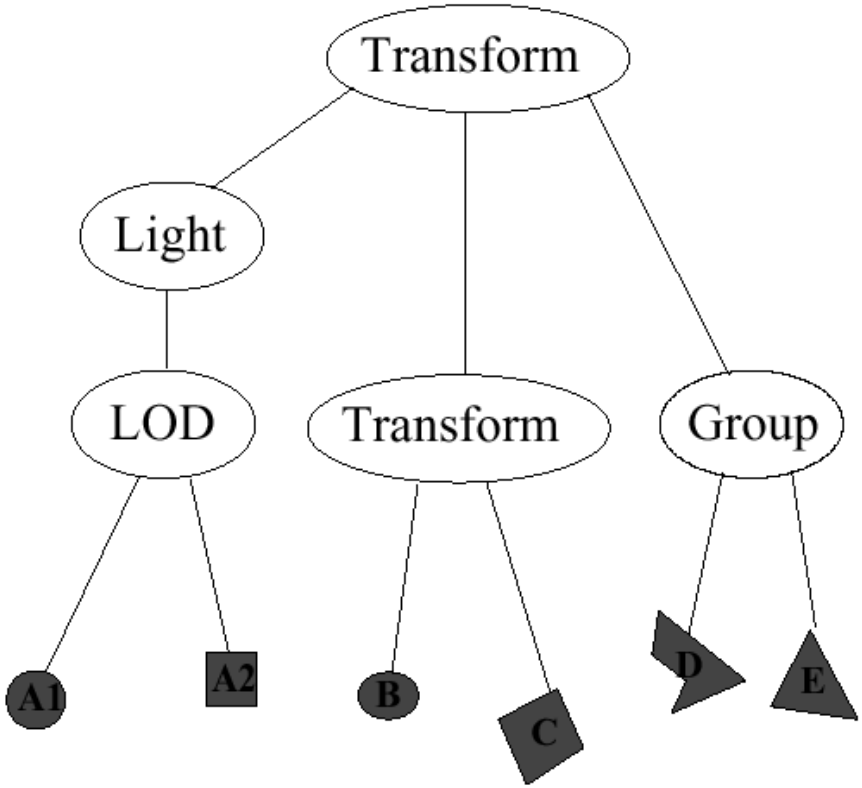
Scene Graph

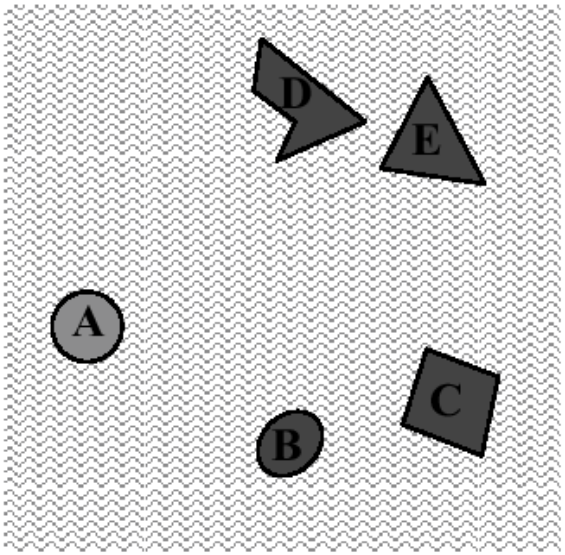


Scene Graph (VRML 2.0)



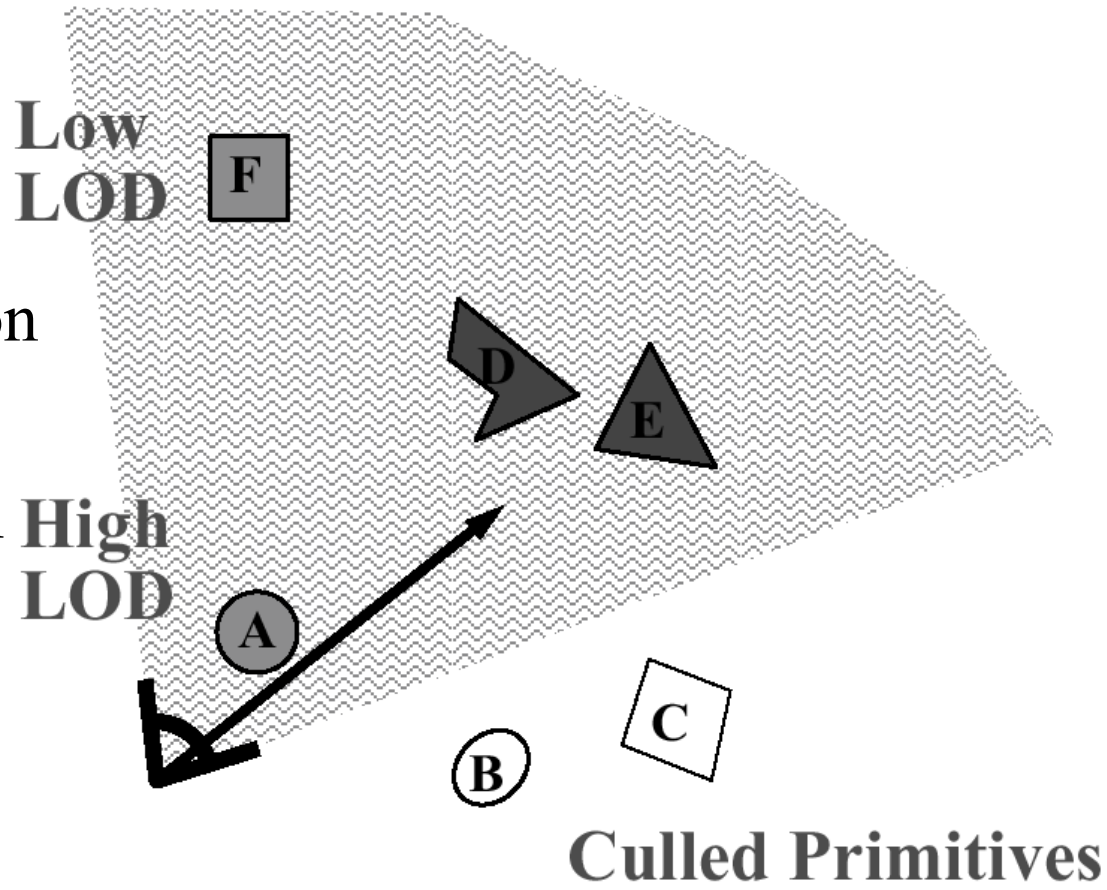
Example Scene Graph





Scene Graph Traversal

- Simulation
  - Animation
- Intersection
  - Collision detection
  - Picking
- Image Generation
  - Culling
  - Detail elision
  - Attributes

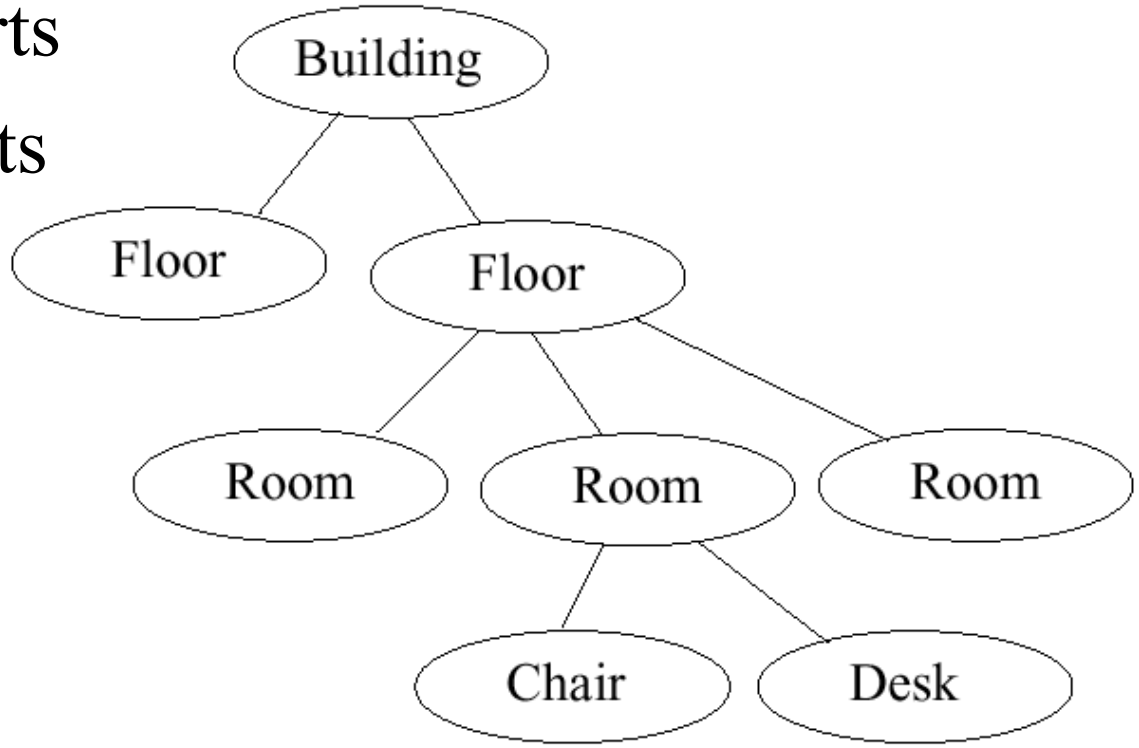


Scene Graph Considerations

- Functional Organization
  - Semantics
- Bounding Volumes
  - Culling
  - Intersection
- Levels of Detail
  - Detail elision
  - Intersection
- Attribute Management
  - Eliminate redundancies

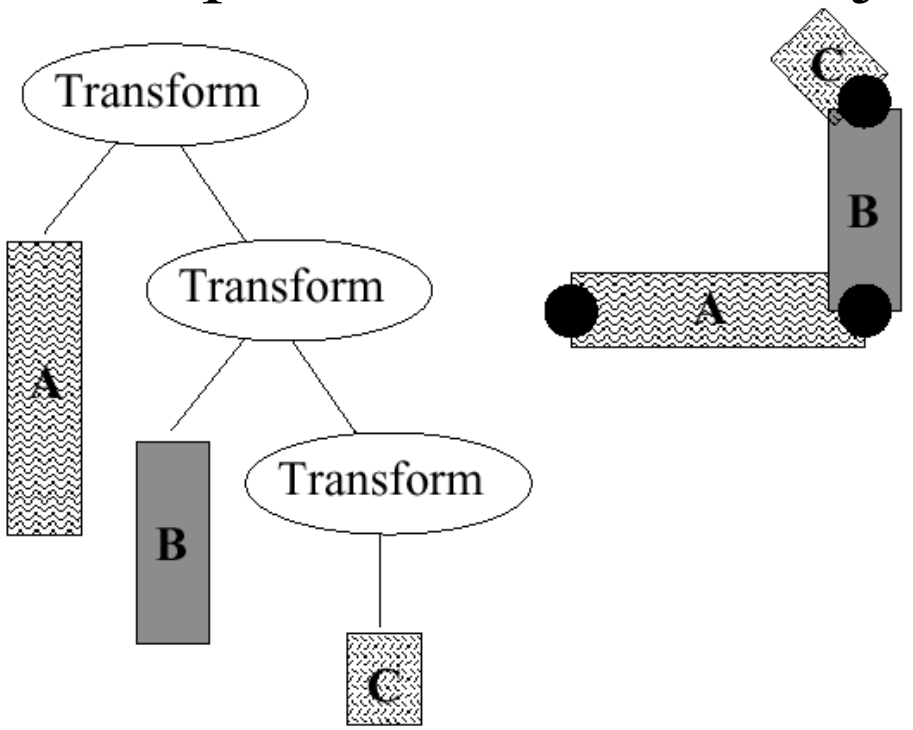
Functional Organization

- Semantics:
  - Logical parts
  - Named parts

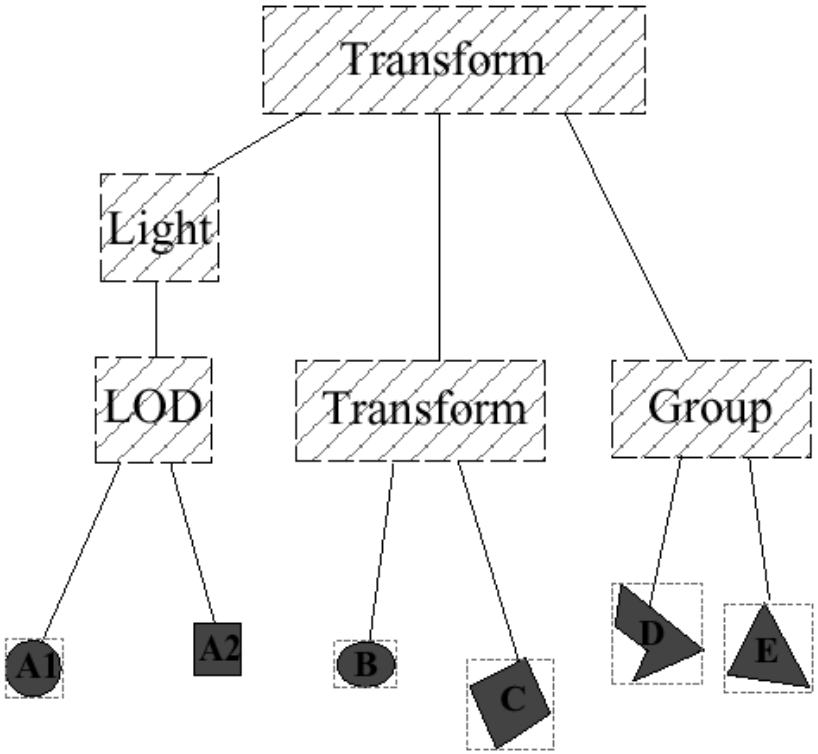


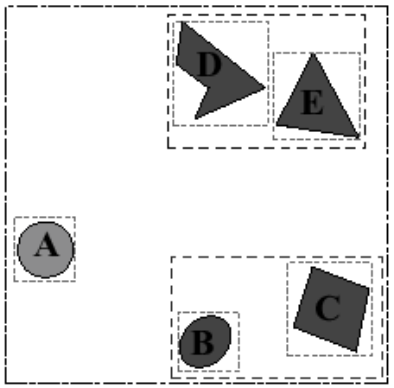
Functional Organization

- Articulated Transformations
  - Animation
  - Difficult to optimize animated objects

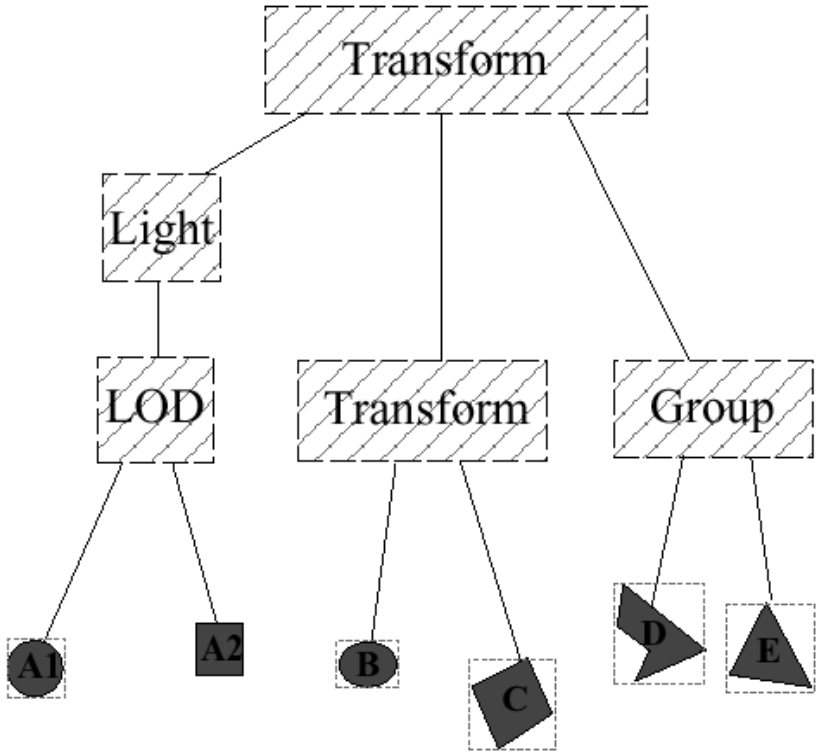


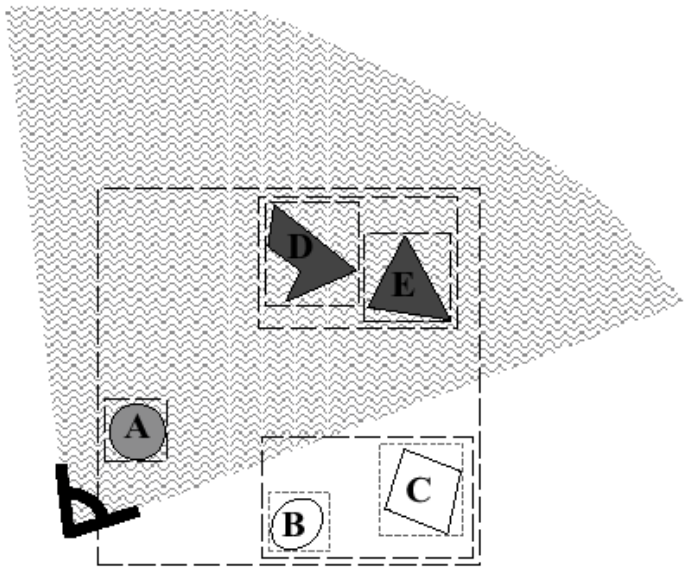
Bounding Volume Hierarchies





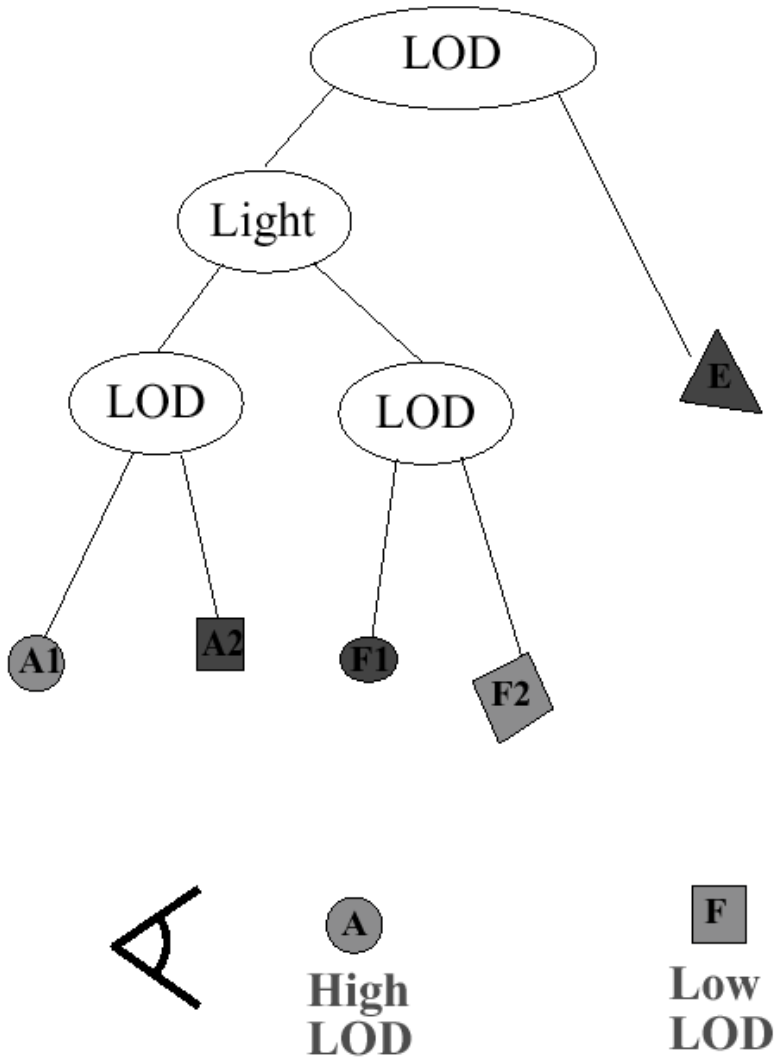
View Frustum Culling





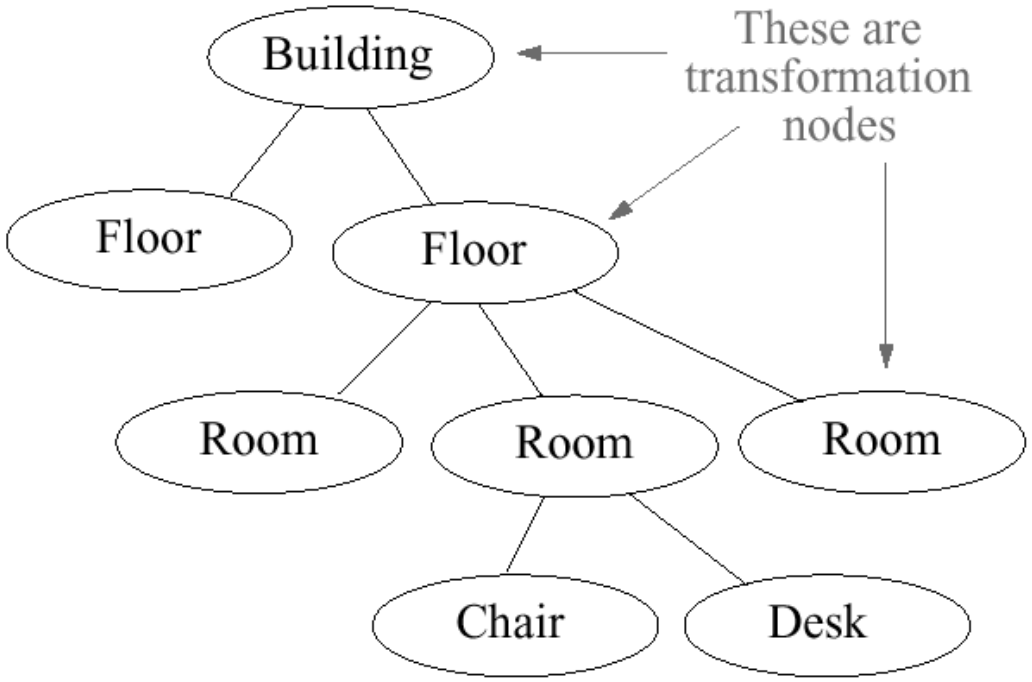
Level Of Detail (LOD)

- Each LOD nodes have distance ranges



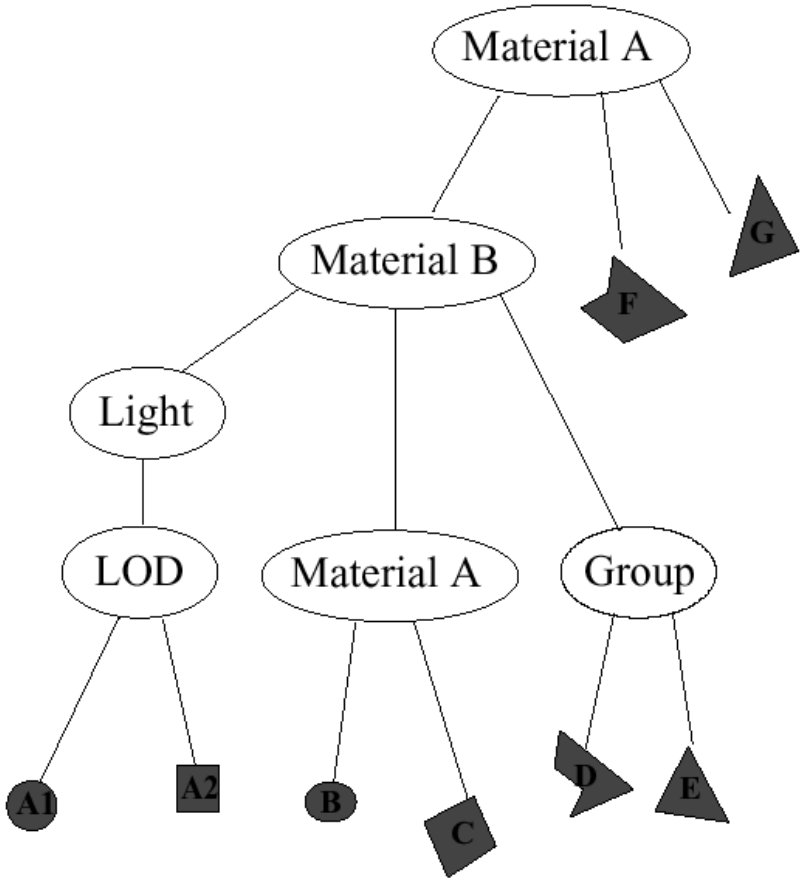
Attribute Management

- Minimize transformations
  - Each transformation is expensive during rendering, intersection, etc. Need automatic algorithms to collapse/adjust transform hierarchy.



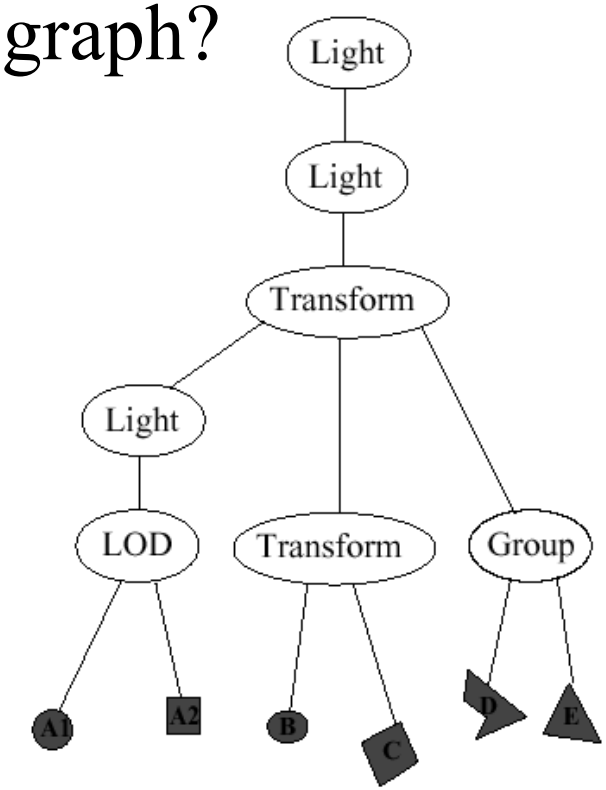
Attribute Management

- Minimize attribute changes
  - Each state change is expensive during rendering

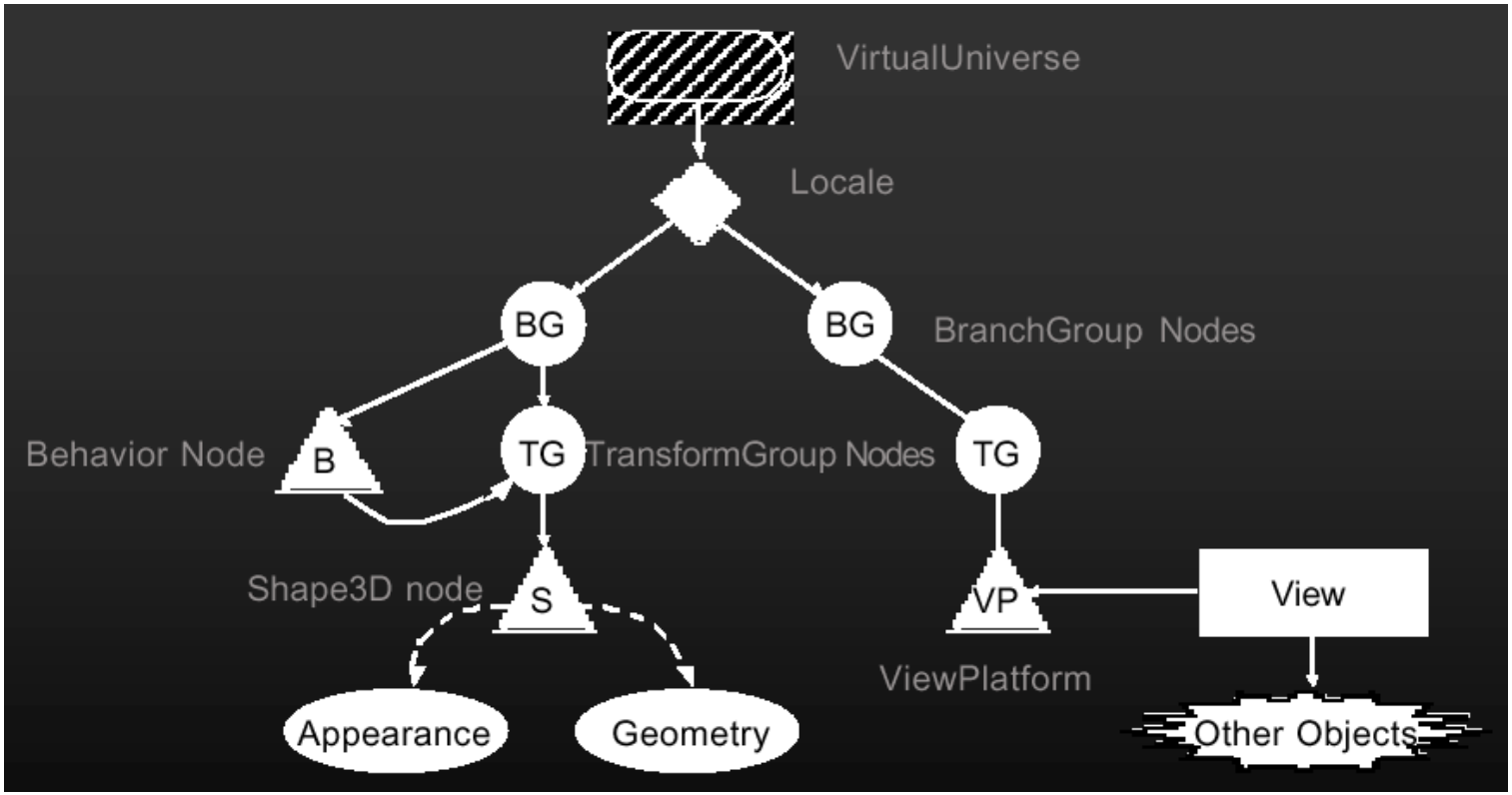


Question: How do you manage your light sources?

- OpenGL supports only 8 lights. What if there are 200 lights? The modeler must ‘scope’ the lights in the scene graph?



Sample Scene Graph



Think!

- How to handle optimization of scene graphs with multiple competing goals
  - Function
  - Bounding volumes
  - Levels of Detail
  - Attributes

Scene Graphs Traversal

- Perform operations on graph with traversal
  - Like STL iterator
  - Visit all nodes
  - Collect inherited state while traversing edges
- Also works on a sub-graph

Typical Traversal Operations

- Typical operations
  - Render
  - Search (pick, find by name)
  - View-frustum cull
  - Tessellate
  - Preprocess (optimize)

Scene Graphs Organization

- Tree structure best
  - No cycles for simple traversal
  - Implied depth-first traversal (not essential)
  - Includes lists, single node, etc as degenerate trees
- If allow multiple references (instancing)
  - Directed acyclic graph (DAG)
- Difficult to represent cell/portal structures

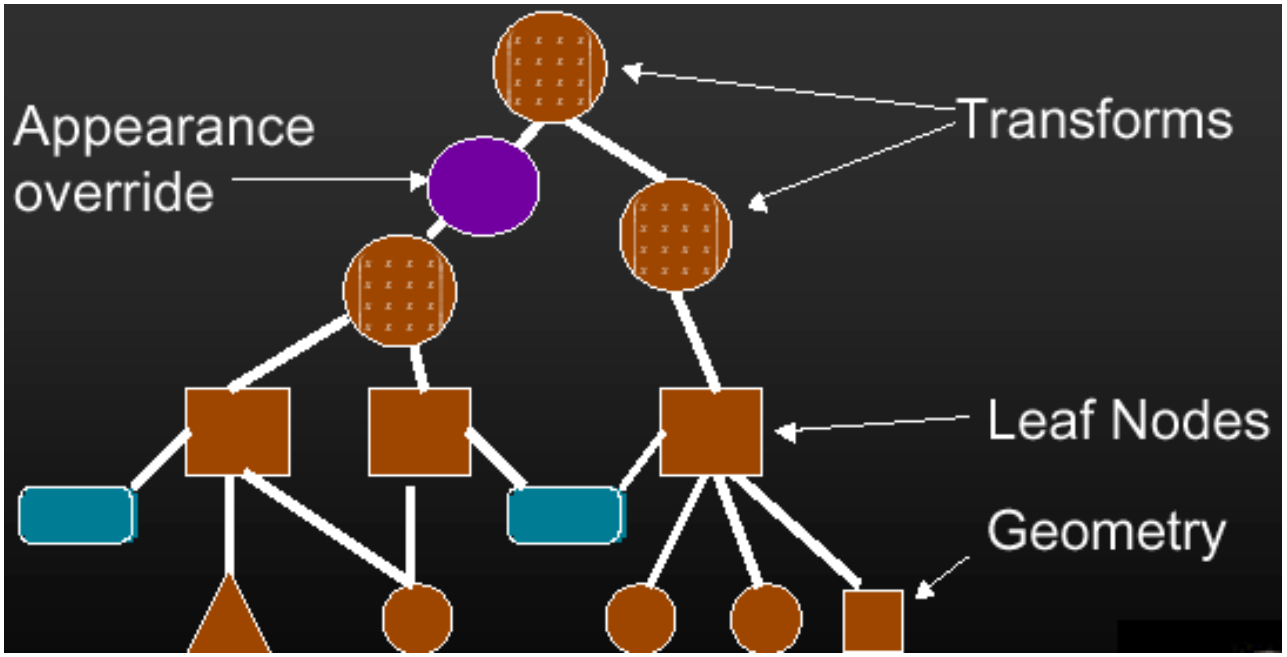
State Inheritance

- General (left to right, top to bottom, all state)
  - Open Inventor
  - Need Separator node to break inheritance
  - Need to visit all children to determine final state
- Top to bottom only
  - IRIS Performer, Java3D, …
  - State can be determined by traversing path to node

Scene Graphs Appearance Overrides

- One attempt to solve the "highlighting" problem
  - After picking an object, want to display it differently
  - Don't want to explicitly edit and restore its appearance
  - Use override node in the scene graph to override appearance of children
- Only works if graph organization matches model organization

Appearance Override

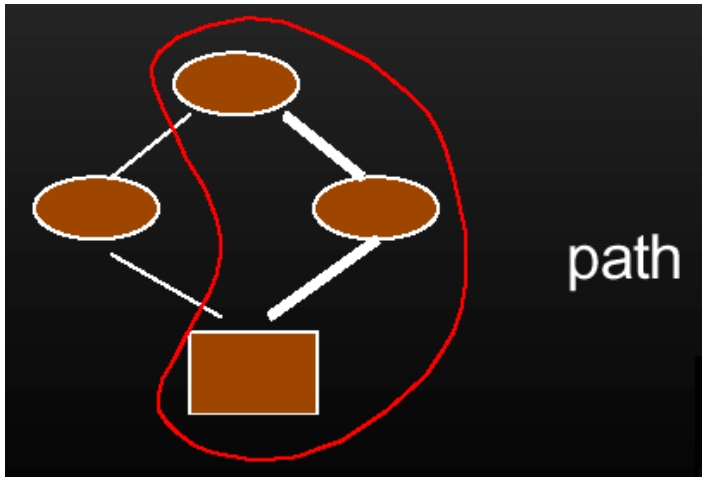


Multiple Referencing (Instancing)

- Convenient for representing multiple instances of an object
  - rivet in a large assembly
- Save memory
- Need life-time management
  - is the object still in use
  - garbage collection, reference counts

Multiple Referencing

- Changes trees into DAGs
- Instance of an object represented by its *path,* (path is like a mini-scene)
- Difficult to attach instance specific properties
  - e.g., caching transform at leaf node



Other Scene Graph Organizations

- Logical structure (part, assembly, etc.)
  - Used by modeling applications
- Topology structure, e.g., boundary
  - surfaces, faces, edges, vertices
  - Useful for CAD applications
- Behaviors, e.g., engine graph
- Environment graph (fog, lights, etc.)
- Scene graph is not just for rendering!!

Specifying Rotation

- How to parameterize rotation
  - Traditional way: use Euler angles, rotation is specified by using angles with respect to three mutually perpendicular axes
    - Roll, pitch and yaw angles (one matrix for each Euler angle)
    - Difficult for an animator to control all the angles (practically unworkable)
      - With a sequence of key frames, how to interpolate??
      - Separating motion from path
- Better to use parameterized interpolation of quaternions

Quaternion

- A way to specify rotation
- As an extension of complex numbers
- Quaternion:

  $u = (u_0, u_1, u_2, u_3) = u_0 + iu_1 + ju_2 + ku_3 = u_0 + \boldsymbol{u}$
- Pure quaternion: $u_0 = 0$
- Conjugate: $u^* = u_0 - \boldsymbol{u}$
- Addition: $u + v = (u_0 + v_0, u_1 + v_1, u_2 + v_2, u_3 + v_3)$
- Scalar multiplication: $c.u = (cu_0, cu_1, cu_2, cu_3)$

Quaternion multiplication

- u x v

  $= (u_0 + iu_1 + ju_2 + ku_3)x(v_0 + iv_1 + jv_2 + kv_3)$

  $= [u_0 v_0 - (\boldsymbol{u}.\boldsymbol{v})]+(\boldsymbol{u}x\boldsymbol{v}) + u_0\boldsymbol{v} + v_0\boldsymbol{u}$

- The result is still a quaternion, this operation is not commutative, but is associative
- u x u = - ($\boldsymbol{u}$. $\boldsymbol{u}$)
- u x u* = $u_0^2 + u_1^2 + u_2^2 + u_3^2 = |u|^2$
- Norm(u) = u/|u|
- Inverse quaternion:

  $u^{-1} = u^*/|u|^2$, u x u^{-1} = u^{-1} x u = 1

Polar Representation of Quaternion

- Unit quaternion: $|u|^2 = 1$, normalize with norm(u)
- For some theta, -pi < theta < pi, unit quaternion, u:

$$|u|^2 = \cos^2(\text{theta}) + \sin^2(\text{theta})$$

$$u = u_0 + |\boldsymbol{u}|\boldsymbol{s},\ \boldsymbol{s} = \boldsymbol{u}/|\boldsymbol{u}|$$

$$u = \cos(\text{theta}) + \boldsymbol{s}\sin(\text{theta})$$

Quaternion Rotation

- Suppose $\boldsymbol{p}$ is a vector (x,y,z), p is the corresponding quaternion: $p = 0 + \boldsymbol{p}$
- To rotate p about axis s (unit quaternion: u = cos(theta) + $\boldsymbol{s}$sin(theta)), by an angle of 2*theta, all we need is : upu* (u x p x u*)
- A sequence of rotations:
  - Just do: $u_n u_{n-1} \ldots u_1 p u^*_1 \ldots u^*_{n-1} u^*_n = 0 + \boldsymbol{p}'$
  - Accordingly just concatenate all rotations together: $u_n u_{n-1} \ldots u_1$

Quaternion Interpolation

- Quaternion and rotation matrix has a strict one-to-one mapping (pp. 489, 3D Computer Graphics, Watt, 3rd Ed)
- To achieve smooth interpolation of quaternion, need spherical linear interpolation (slerp), (on pp. 489-490, 3D Computer Graphics, Watt, 3rd Ed)
  - Unit quaternion form a hyper-sphere in 4D space
  - Play with the hyper-angles in 4D
- Gotcha: you still have to figure out your up vector correctly

More

- If you just need to consistently rotate an object on the screen (like in your lab assignments), can do without quaternion
  - Only deal with a single rotation that essentially corresponds to an orientation change
  - Maps to a ‘hyper-line’ in a ‘transformed 4D space’
  - Be careful about the UP vector
  - Use the Arcball algorithm proposed by Ken Shoemaker in 1985