Note that some problems are from Oppenheim’s book and the answers were given at the end of the book. So if you just provide the answer without detailed steps, you will get zero point for that problem.

1. On how to use the analysis or synthesis equations to calculate the Fourier transform of signal $x[n]$ or the inverse Fourier transform of $X(e^{j\omega})$.

   (a) $x[n] = \left(\frac{1}{2}\right)^{n-1}$
   (b) $x[n] = \delta[n + 2] - \delta[n - 2]$
   (c) $x[n] = 2 + \cos\left(\frac{\pi}{6}n + \frac{\pi}{8}\right)$
   (d) $X(e^{j\omega}) = \sum_{k=\infty}^{\infty} [2\pi\delta(\omega - 2\pi k) + \pi\delta(\omega - \frac{\pi}{2} - 2\pi k) + \pi\delta(\omega + \frac{\pi}{2} - 2\pi k)]$
   (e) $|X(e^{j\omega})| = \begin{cases} 1, & 0 \leq |\omega| \leq \frac{\pi}{4} \\ 0, & \frac{\pi}{4} \leq |\omega| \leq \pi \end{cases}$ and $\angle X(e^{j\omega}) = -\frac{3\omega}{2}$

2. On how to use the Fourier transform properties and transform pairs.

   (a) The following four facts are given about a real signal $x[n]$ with Fourier transform $X(e^{j\omega})$:
   i. $x[n] = 0$ for $n > 0$
   ii. $x[0] > 0$
   iii. $Im\{X(e^{j\omega})\} = \sin \omega - \sin 2\omega$
   iv. $\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3$

   Determine $x[n]$.

   (b) Let the inverse Fourier transform of $Y(e^{j\omega})$ be $y[n] = \left(\frac{\sin \omega \cdot n}{\pi n}\right)^2$, where $0 < \omega_c < \pi$. Determine the value of $\omega_c$ which ensures that $Y(e^{j\pi}) = \frac{1}{2}$

3. On LCDE. A causal and stable LTI system $S$ has the following property

   $\left(\frac{4}{5}\right)^nu[n] \rightarrow n\left(\frac{4}{5}\right)^nu[n]$
(a) Determine the frequency response $H(e^{j\omega})$ for the system $S$.
(b) Determine a difference equation relating any input $x[n]$ and the corresponding output $y[n]$. 