The objective of the homework is to be able to calculate convolution using pencil and paper.

Calculate the output signal using convolution.

1. (15 pts) \[ x[n] = -\delta[n+1] + 0.5\delta[n] + 2\delta[n-1] \]
   \( h[n] = 2\delta[n] + \delta[n-1] \)

2. (25 pts) \[
   x[n] = \begin{cases} 
   1 & 0 \leq n \leq 4 \\
   0 & \text{otherwise}
   \end{cases}, 
   h[n] = \begin{cases} 
   \alpha^n & 0 \leq n \leq 6 (\alpha > 1) \\
   0 & \text{otherwise}
   \end{cases}
\]

3. (25 pts) \[ x(t) = e^{-at}u(t) \ (a > 0) \]
   \( h(t) = u(t) \)

4. (10 pts) Let
   \[ x[n] = \begin{cases} 
   1 & 0 \leq n \leq 9 \\
   0 & \text{otherwise}
   \end{cases} \quad \text{and} \quad h[n] = \begin{cases} 
   1 & 0 \leq n \leq N \\
   0 & \text{otherwise}
   \end{cases} \]
   where \( N \leq 9 \) is an integer. Determine the value of \( N \), given that \( y[n] = x[n] * h[n] \) and \( y[4] = 5, y[14] = 0 \).

5. (25 pts) Calculate and sketch the convolution of the following two signals
   \[
   x(t) = \begin{cases} 
   t + 1 & 0 \leq t \leq 1 \\
   2 - t & 1 < t \leq 2 \\
   0 & \text{otherwise}
   \end{cases} \quad \text{and} \quad h(t) = \delta(t + 2) + 2\delta(t + 1)
   \]

6. (0 pt, Self Study Problem) The impulse response, \( h[n] \), of an LTI system is known to be zero, except in the interval \( N_0 \leq n \leq N_1 \). The input \( x[n] \) is known to be zero, except in the interval \( N_2 \leq n \leq N_3 \). As a result, the output is constrained to be zero, except in some interval \( N_4 \leq n \leq N_5 \). Determine \( N_4 \) and \( N_5 \) in terms of \( N_0, N_1, N_2, \) and \( N_3 \).