ECE315 - Signals and Systems, Spring 2018

Homework 6, Due 03/20/18

Note that some problems are from Oppenheim’s book and the answers were given at the end of the book. So if you just provide the answer without detailed steps, you will get zero point for that problem.

1. The following two problems are on representation. In specific, trignometric representation of real periodic signals.

   (a) \( x(t) \) is real-valued and has a fundamental period \( T = 8 \). The nonzero Fourier series coefficients for \( x(t) \) are \( a_1 = a_{-1} = j \), \( a_5 = a_{-5} = 2 \). Express \( x(t) \) in the form of

   \[
   x(t) = \sum_{k=0}^{\infty} A_k \cos(k\omega_0 t + \theta_k)
   \]

   (b) A DT periodic signal \( x[n] \) is real-valued and has a fundamental period \( N = 5 \). The nonzero Fourier series coefficients for \( x[n] \) are \( a_0 = 2 \), \( a_2 = a_{-2} = 2e^{j\pi/6} \), \( a_4 = a_{-4} = e^{j\pi/3} \). Express \( x[n] \) in the form

   \[
   x[n] = a_0 + 2 \sum_{k=1}^{\infty} A_k \cos(\omega_k n + \phi_k)
   \]

2. The following two problems are on synthesis and analysis. Pay attention to the DT case where (b) and (c) actually are two different input signals since the period is different.

   (a) For the CT periodic signal \( x(t) = 2 + \cos(\tfrac{2\pi}{3}t) + 4 \sin(\tfrac{5\pi}{4} t) \), determine the fundamental frequency \( \omega_0 \) and the Fourier series coefficients \( a_k \) such that \( x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 t} \).

   (b) Determine the FS coefficients for each of the following DT periodic signals. (choose 2)

   i. \( x[n] = \sin(2\pi n/3) \cos(\pi n/2) \)

   ii. \( x[n] \) periodic with period 4 and \( x[n] = 1 - \sin \tfrac{\pi n}{4} \) for \( 0 \leq n \leq 3 \).

   iii. \( x[n] \) periodic with period 12 and \( x[n] = 1 - \sin \tfrac{\pi n}{12} \) for \( 0 \leq n \leq 11 \).
3. The following two problems are on eigenvalue and eigenfunction. Note that
\( H(j\omega) \) is the notation that refers to the eigenvalue for CT signals and \( H(e^{j\omega}) \) is the notation that refers to the eigenvalue for DT signals; and \( \omega = k\omega_0 \) where \( \omega_0 \) is the fundamental frequency and \( \omega \) is the different harmonics.

(a) Consider a CT LTI system with \( H(j\omega) = \frac{\sin(4\omega)}{\omega} \). If the input to this system is a periodic signal

\[
x(t) = \begin{cases} 
  1 & 0 \leq t < 4 \\
  -1 & 4 \leq t < 8 
\end{cases}
\]

with period \( T = 8 \), determine the corresponding system output \( y(t) \).

(b) When the impulse train \( x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k] \) is the input to an LTI system with \( H(e^{j\omega}) \), the output of the system is found to be \( y[n] = \cos(\frac{5\pi}{2}n + \frac{\pi}{4}) \). Determine the value of \( H(e^{j\pi/2}) \) for \( k = 0, 1, 2, \) and \( 3 \).