Problem 1: (35/35) On Gradient Descent (GD). Find the global minimum of \( f(t) = 50 \cdot \sin(t) + t^2 \) over \(-10 \leq t \leq 10\). This problem intends to give you a hands-on experience on how gradient descent works and how it can get trapped at the local minima.

a) (5/5) Use MATLAB (or whatever toolbox) to plot this function. Visualize the multiple local minima and the global minimum.

b) Use gradient descent to find the global minimum. (you can use the code provided in the lecture note)

   i) (10/10) Pick a starting point at \( t=7 \). What's the minimum? Show the convergence path.

   ii) (10/10) Pick a starting point at \( t=1 \). What's the minimum? Show the convergence path.

   iii) (10/10) Change the step size and see what kind of step size will help overpass the local minimum when \( t=7 \).

Problem 2: (30/30) On Perceptron. Use Perceptron to implement the OR logic and the XOR logic. Show output from each iteration with the maximum number of iterations being 10. You can use either MATLAB or C/C++.

Problem 3: (35/35) Comparison between FLD, PCA, and Perceptron.

Note that FLD and PCA are dimensionality reduction methods that only output a projection direction. Additional classification methods need to be applied to find the decision boundary. Suppose the minimum (Euclidean) distance (MD) classifier is used. On the other hand, Perceptron is a linear classifier that outputs the decision boundary directly. On the same figure, plot the four samples of the AND gate, and compare the decision boundary from FLD+MD, PCA+MD, and Perceptron. You can use whichever language that you feel comfortable (pencil & paper, MATLAB, C/C++).