ECE 472/572 - Digital Image Processing

Lecture 5 - Image Enhancement - Frequency Domain Filters

09/13/11

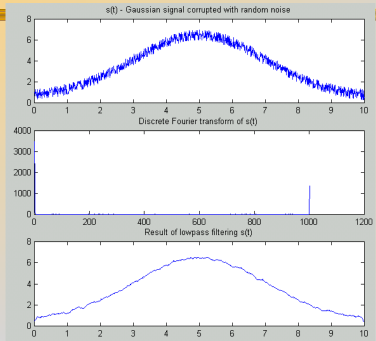
Roadmap

- Introduction
  - Image format (vector vs. bitmap)
  - IP vs. CV vs. CG
  - HLIP vs. LLIP
  - Image acquisition
- Perception
  - Structure of human eye
    - rods vs. conss (Scotopic vision vs. photopic vision)
    - Fovea and blind spot
    - Flexible lens (near-sighted vs. far-sighted)
  - Brightness adaptation and Discrimination
    - Weber ratio
    - Dynamic range
  - Image resolution
    - Sampling vs. quantization
- Image enhancement
  - Enhancement vs. restoration
  - Spatial domain methods
    - Point-based methods
      - Log trans. vs. Power-law
        - Gamma correction
        - Dynamic range compression
      - Contrast stretching vs. HE
        - What is HE?
        - Derivation of tran. func.
      - Gray-level vs. Bit plane slicing
      - Image averaging (principle)
    - Mask-based (neighborhood-based) methods - spatial filter
      - Smoothing vs. Sharpening filter
      - Linear vs. Non-linear filter
      - Smoothing
        - Average vs. weighted average
        - Average vs. Median
      - Sharpening
        - UM vs. High boosting
        - 1st vs. 2nd derivatives
  - Frequency domain methods

Questions

- In-depth understanding
  - Why do we need to conduct image processing in the frequency domain?
  - What does Fourier series do?
  - What does the Fourier spectrum of an image tell you?
  - How to calculate the fundamental frequency?
  - Why is padding necessary?
- Properties
  - Is FT a linear or nonlinear process?
  - What would the FT of a rotated image look like?
  - When implementing FFT, what kind of properties are used?
  - What does the autocorrelation of an image tell you?
  - What is F(0,0)? Or Why is the center of the FT extremely bright?

Why FT? – 1



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Why FT? – 2

$$g(x,y) = h(x,y) * f(x,y) \Leftrightarrow G(u,v) = H(u,v)F(u,v)$$
$$g(x,y) = F^{-1}\{H(u,v)F(u,v)\}$$

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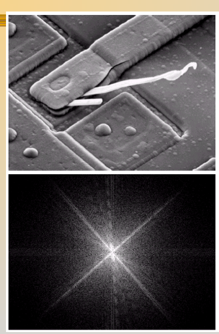
Another example



a
b

FIGURE 4.4 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

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Fourier series

- Fourier series (S_F) can represent any function over a finite interval T_F
- Outside T_F, S_F repeats periodically with period T_F.

$$\text{complex form}: \quad s(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2n\pi f_F t}$$

$$c_n = \frac{1}{T_F}\int_{-T_F/2}^{T_F/2} s(t) e^{-j2n\pi f_F t}\,dt$$

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Fourier series (cont')

- T_F is the interval of signal s(t) over which the Fourier series represents
- $f_F = 1/T_F$ is the fundamental frequency of the Fourier series representation
- n is called the "harmonic number"
  - E.g., $2f_F$ is the second harmonic of the fundamental frequency f_F.
- The Fourier series representation is always periodic and is linear combinations of sinusoids at f_F and its harmonics.

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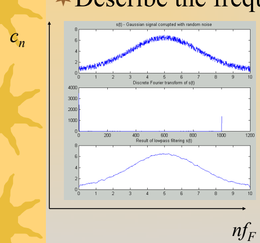
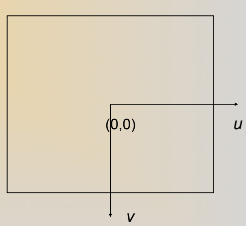
Fourier transform

- Describe the frequency distribution





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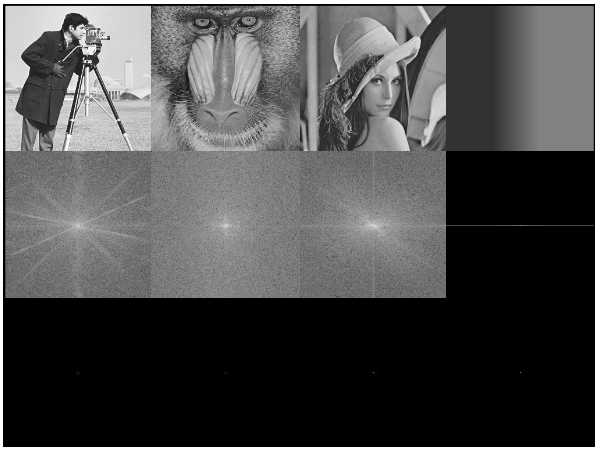

1-D Fourier transform

- Fourier transform: $F(u)=\int_{-\infty}^{\infty} f(t)\exp(-j2\pi ut)\,dt$
- Inverse FT: $f(t)=\int_{-\infty}^{\infty} F(u)\exp(j2\pi ut)\,du$

DFT

$$F(u)=\frac{1}{N}\sum_{x=0}^{N-1} f(x)\exp(-j2\pi ux/N)$$

$$f(x)=\sum_{u=0}^{N-1} F(u)\exp(j2\pi ux/N)$$

- Complex form
  $F(u)=R(u)+jI(u)$
  $F(u)=|F(u)|e^{j\phi(u)}$
- Fourier spectrum $|F(u)|=\sqrt{R^2(u)+I^2(u)}$
- Power spectrum (spectral density) $P(u)=|F(u)|^2$
- Phase angle $\phi(u)=\tan^{-1}\left[\frac{I(u)}{R(u)}\right]$

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2-D Fourier transform

- CFT

$$F(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp[-j2\pi(ux+vy)]\,dxdy$$

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\exp[j2\pi(ux+vy)]\,dudv$$

- DFT

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\exp[-j2\pi(ux/M+vy/N)]$$

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\exp[j2\pi(ux/M+vy/N)]$$

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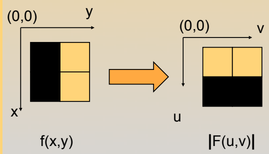
Understanding and implementing Fourier transform

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)\exp[-j2\pi(ux/M+vy/N)]$$

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)\exp[j2\pi(ux/M+vy/N)]$$

$$\Delta u=\frac{1}{M\Delta x},\Delta v=\frac{1}{N\Delta y}$$



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$$F(0,0)=\frac{1}{2*2}(f(0,0)+f(0,1)+f(1,0)+f(1,1))=127.5$$

$$F(0,1)=\frac{1}{2*2}(f(0,0)*e^{-j2\pi(0*0/2+1*0/2)}+f(0,1)*e^{-j2\pi(0*0/2+1*1/2)}$$
$$+f(1,0)*e^{-j2\pi(0*1/2+1*0/2)}+f(1,1)*e^{-j2\pi(0*1/2+1*1/2)})=-127.5$$

$$F(1,0)=\frac{1}{2*2}(f(0,0)*e^{-j2\pi(1*0/2+0*0/2)}+f(0,1)*e^{-j2\pi(1*0/2+0*1/2)}$$
$$+f(1,0)*e^{-j2\pi(1*1/2+0*0/2)}+f(1,1)*e^{-j2\pi(1*1/2+0*1/2)})=0$$

$$F(1,1)=\frac{1}{2*2}(f(0,0)*e^{-j2\pi(1*0/2+1*0/2)}+f(0,1)*e^{-j2\pi(1*0/2+1*1/2)}$$
$$+f(1,0)*e^{-j2\pi(1*1/2+1*0/2)}+f(1,1)*e^{-j2\pi(1*1/2+1*1/2)})=0$$

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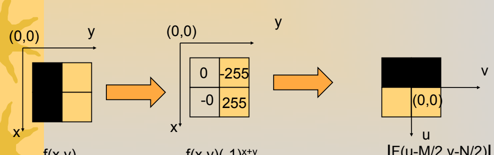
Understanding and implementing Fourier transform

- According to "translation"

  For one complete period,

  $$f(x,y)(-1)^{x+y}\Leftrightarrow F(u-M/2,v-N/2)$$



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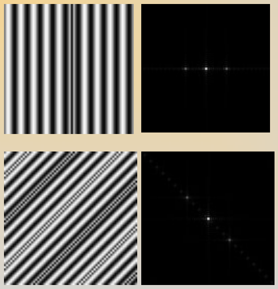
Understanding and implementing Fourier transform



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Block diagram of FT←→IFT

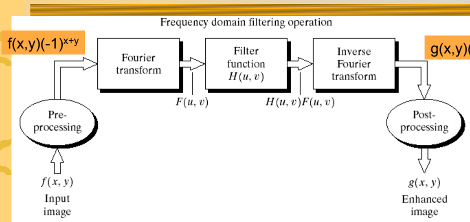

FIGURE 4.5 Basic steps for filtering in the frequency domain.

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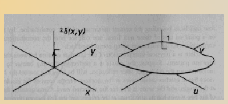
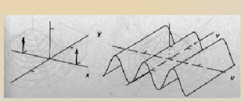
Impulse transforms





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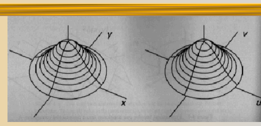
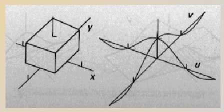
Typical transforms

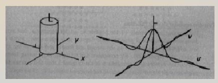
- Gaussian hump ←→ Gaussian hump



- Rectangular (square aperture) ←→sinc



- Pillbox (circular aperture) ←→jinc



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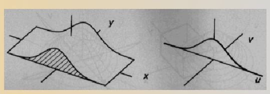
Typical transforms

- Gaussian ridge

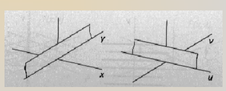
  $\exp(-\pi x^2)$



- Line impulse

  $\tau^{-1}\exp(-\pi x^2/\tau^2)$

  when $\tau \to 0$



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Reference

- All figures scanned from R. N. Bracewell's "Two-Dimensional Imaging," Prentice Hall, 1995.

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2D FT pairs

$f(x,y)$	$F(u,v)$
$\delta(x,y)$	1
$\delta(x \pm x_0, y \pm y_0)$	$\exp(\pm j2\pi x_0 u)\exp(\pm j2\pi y_0 v)$
$rect(x,y)$	$\sin c(u,v)$
$tri(x,y)$	$\sin c^2(u,v)$
$comb(x,y)$	$comb(u,v)$
$\exp\left[-\pi\left(x^2+y^2\right)\right]$	$\exp\left[-\pi\left(u^2+v^2\right)\right]$

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Important properties of FT

- Linearity (distributivity && scaling)
- Separability
- Translation
- Periodicity
- Conjugate symmetry
- Rotation
- Convolution
- Correlation
- Sampling

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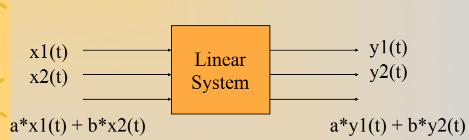
Linearity



- FT is a linear image processing method

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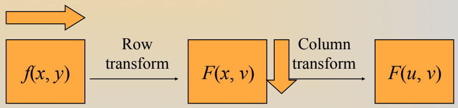
Separability

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\exp[-j2\pi(ux/M+vy/N)]$$



Translation

$$f(x,y)\exp[j2\pi(u_0x+v_0y)/N] \Leftrightarrow F(u-u_0,v-v_0)$$
$$f(x-x_0,y-y_0) \Leftrightarrow F(u,v)\exp[-j2\pi(ux_0+vy_0)/N]$$

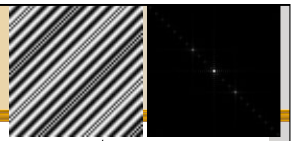
To show one complete period,

$$f(x,y)(-1)^{x+y} \Leftrightarrow F(u-N/2,v-N/2)$$

Periodicity and Conjugate symmetry

- Periodicity
  $$F(u,v)=F(u+N,v)=F(u,v+N)=F(u+N,v+N)$$
- Conjugate symmetry
  $$F(u,v)=F^*(-u,-v)$$
  $$|F(u,v)|=|F(-u,-v)|$$

Example



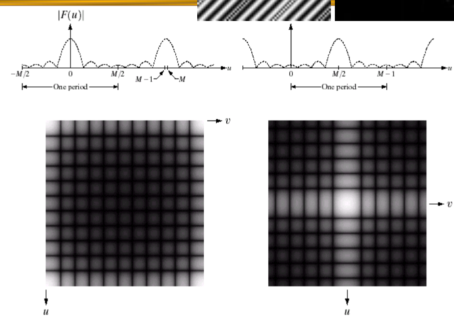


FIGURE 4.34 (a) Fourier spectrum showing back-to-back half periods in the interval [0, $M-1$]. (b) Shifted spectrum showing a full period in the same interval. (c) Fourier spectrum of an image, showing the same back-to-back properties as (a), but in two dimensions. (d) Centered Fourier spectrum.

Rotation

$$f(r,\theta+\theta_0)=F(\omega,\phi+\theta_0)$$

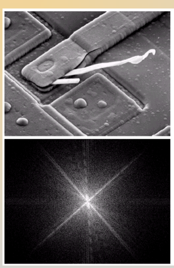


FIGURE 4.4 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

Averaging

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\exp[-j2\pi(ux/M+vy/N)]$$

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\exp[j2\pi(ux/M+vy/N)]$$

$$\bar{f}(x,y)=F(0,0)$$

Convolution

- Continuous and discrete convolution

$$f(x,y)*g(x,y)=\int\int_{-\infty}^{\infty} f(\alpha,\beta)g(x-\alpha,y-\beta)d\alpha d\beta$$

$$f_e(x,y)*g_e(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f_e(m,n)g_e(x-m,y-n)$$

- The convolution theorem

$$f(x,y)*g(x,y)\Leftrightarrow F(u,v)G(u,v)$$
$$f(x,y)g(x,y)\Leftrightarrow F(u,v)*G(u,v)$$

- Practically, computing the discrete convolution in the frequency domain often is more efficient than doing it in the spatial domain directly

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Correlation

- Continuous and discrete correlation

$$f(x,y)\circ g(x,y)=\int\int_{-\infty}^{\infty} f^*(\alpha,\beta)g(x+\alpha,y+\beta)d\alpha d\beta$$

$$f_e(x,y)\circ g_e(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f_e(m,n)g_e(x+m,y+n)$$

- The correlation theorem

$$f(x,y)\circ g(x,y)\Leftrightarrow F^*(u,v)G(u,v)$$
$$f^*(x,y)g(x,y)\Leftrightarrow F(u,v)\circ G(u,v)$$

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Correlation (cont')

- Autocorrelation vs. cross correlation
- Autocorrelation theorem

$$F\{f(x,y)\circ f(x,y)\}=|F(u,v)|^2$$

- Application: template or prototype matching

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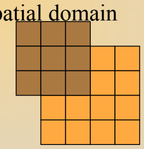
Practical issues – Implement convolution in frequency domain

- In spatial domain



- In frequency domain
  - f*g ←→ F(f)G(g)
  - Phase? Mag?
  - How to pad?

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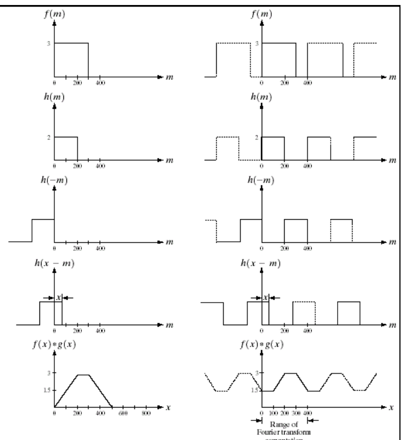
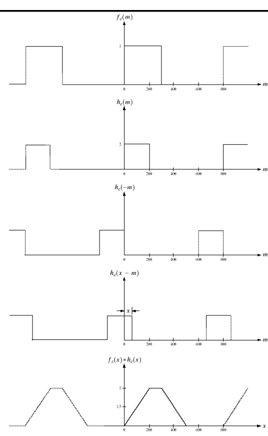
a f
b g
c h
d i
e j

FIGURE 4.36 Left: convolution of two discrete functions. Right: convolution of the same functions, taking into account the implied periodicity of the DFT. Note in (j) how data from adjacent periods corrupt the result of convolution.



a
b
c
d
e

FIGURE 4.37 Result of performing convolution with extended functions. Compare Figs. 4.37(e) and 4.36(e).



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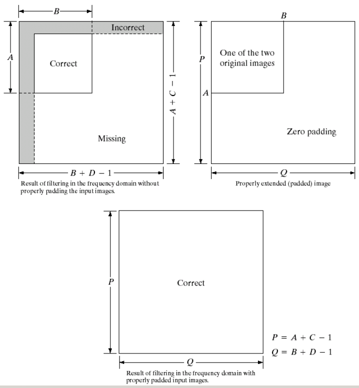


a b
c

FIGURE 4.38 Illustration of the need for function padding. (a) Result of performing 2-D convolution without padding. (b) Proper function padding. (c) Correct convolution result.

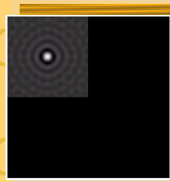
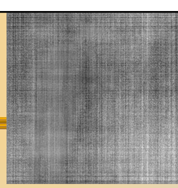


FIGURE 4.39 Padded lowpass filter is the spatial domain (only the real part is shown).

FIGURE 4.40 Result of filtering with padding. The image is usually cropped to its original size since there is little valuable information past the image boundaries.



Difference image from convolution in the spatial domain

Convolution in the frequency domain

No padding

With padding

Conv. spatially

Different enhancement approaches

- Lowpass filter
- Highpass filter
- Homomorphic filter

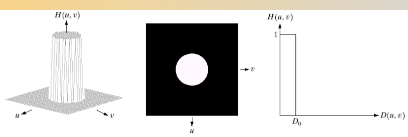
Lowpass filtering

- Ideal filter $H(u,v)=\begin{cases}1 & \text{if } D(u,v)\le D_0\\ 0 & \text{if } D(u,v)> D_0\end{cases}$
  - $D(u, v)$: distance from point (u, v) to the origin
  - cutoff frequency (D_0)
  - nonphysical
  - radially symmetric about the origin
- Butterworth filter

$$H(u,v)=\frac{1}{1+\left[D(u,v)/D_0\right]^{2n}}$$

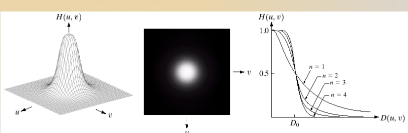
- Gaussian lowpass filter

$$H(u,v)=e^{-D^2(u,v)/2D_0^2}$$



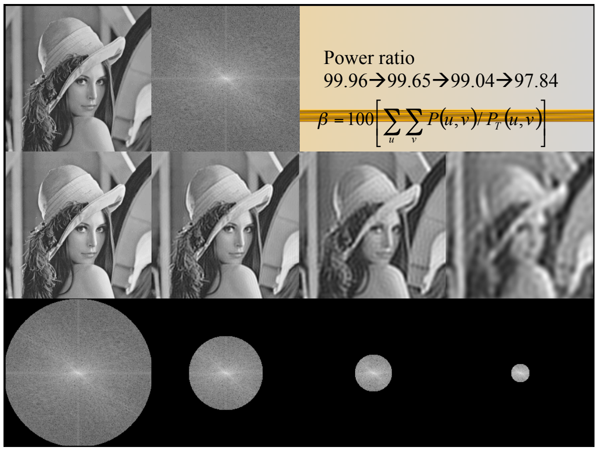
a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.



a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



Highpass filter

- Ideal filter $H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$
- Butterworth filter
$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$
- Gaussian highpass filter
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

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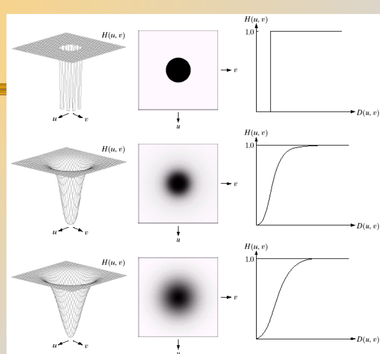

a b c
d e f
g h i

FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

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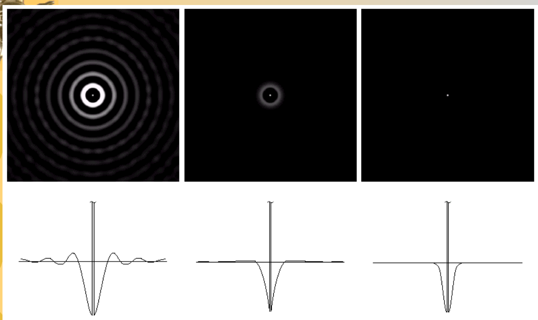

a b c

FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

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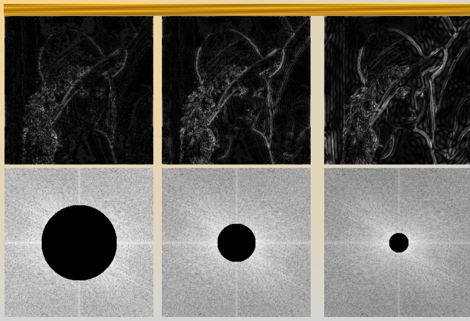
Example



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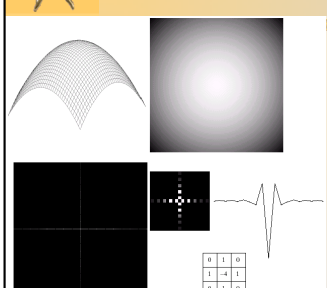
The Laplacian in the frequency domain



0	1	0
1	–4	1
0	1	0

a b c d e f

FIGURE 4.27 (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$H(u,v) = -4\pi^2 \left(u^2 + v^2\right)$$

Pay attention to the scaling factor

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UM in the frequency domain

$$g(x,y) = f(x,y) + k * g_{mask}(x,y) = f(x,y) + k * (f(x,y) - f_{LP}(x,y))$$
$$g(x,y) = \Im^{-1}\{[1 + k * H_{HP}(u,v)]F(u,v)\}$$

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Homomorphic filtering

- A simple image model
  - $f(x,y)$: the intensity is called the **gray level** for monochrome image
  - $f(x, y) = i(x, y).r(x, y)$
  - $0 < i(x, y) <$ inf, the illumination
  - $0 < r(x, y) < 1$, the reflectance

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Homomorphic filter (cont')

$$f(x,y) = i(x,y) \cdot r(x,y)$$
$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$
$$F\{z(x,y)\} = F\{\ln i(x,y)\} + F\{\ln r(x,y)\}$$
$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

$$S(u,v) = H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$

$$s(x,y) = i'(x,y) + r'(x,y)$$

$$g(x,y) = \exp[s(x,y)] = \exp[i'(x,y)]\exp[r'(x,y)]$$

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Homomorphic filter (cont')

- The illumination component
  - Slow spatial variations
  - Low frequency
- The reflectance component
  - Vary abruptly, particularly at the junctions of dissimilar objects
  - High frequency
- Homomorphic filters
  - Affect low and high frequencies differently
  - Compress the low frequency dynamic range
  - Enhance the contrast in high frequency

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Homomorphic Filter (cont')

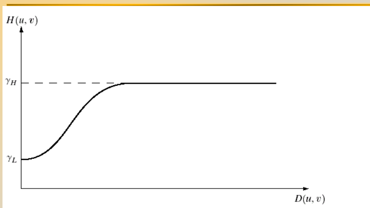

FIGURE 4.32 Cross section of a circularly symmetric filter function. $D(u, v)$ is the distance from the origin of the centered transform.

$\gamma_H > 1$

$\gamma_L < 1$

$$H(u,v) = (\gamma_H - \gamma_L)[1 - e^{-c(D^2(u,v)/D_0^2)}] + \gamma_L$$

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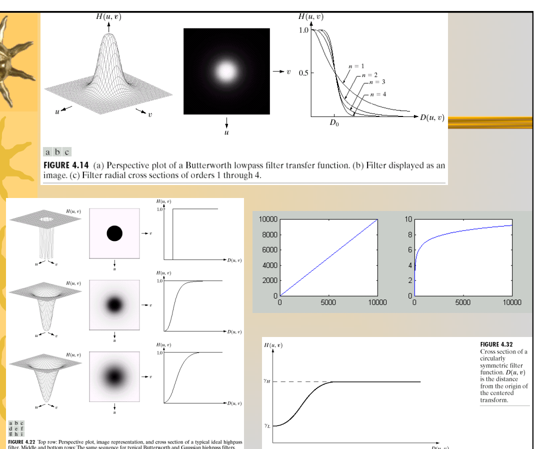

Homomorphic filter - example

a b

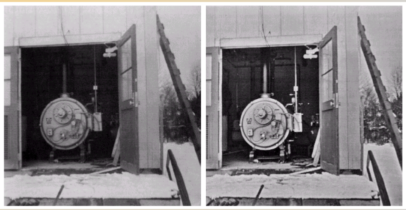

FIGURE 4.33 (a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)

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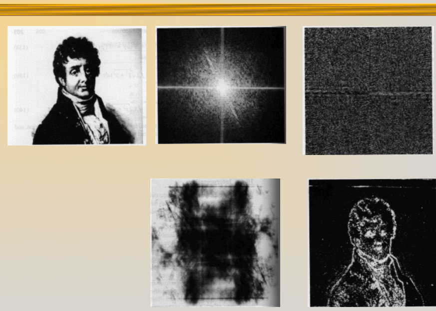
- Point processing
- Simple gray level transformations
  - Image negatives
  - Log transformations
  - Power-law transformations
  - Contrast stretching
  - Gray-level slicing
  - Bit-plane slicing
- Histogram processing
  - Histogram equalization
  - *Histogram matching (specification)
- Arithmetic/logic operations
  - Image averaging

- Mask processing (spatial filters)
- Smoothing filters (blur details)
  - Average, weighted average
  - Order statistics (e.g. median)
- Sharpening filters (highlight details)
  - Unsharp masking
  - High-boost filters
  - Derivative filters
    - The Laplacian
    - The Gradient

- Frequency domain filters
- Smoothing filters (blur details)
  - Ideal lowpass filter
  - Butterworth lowpass
  - Gaussian lowpass
- Sharpening filters (highlight details)
  - Unsharp masking
  - High-boost filters
  - Derivative filters - The Laplacian
  - Ideal highpass filter
  - Butterworth highpass filter
  - Gaussian highpass filter
- Homomorphic filtering

FFT and IFFT



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