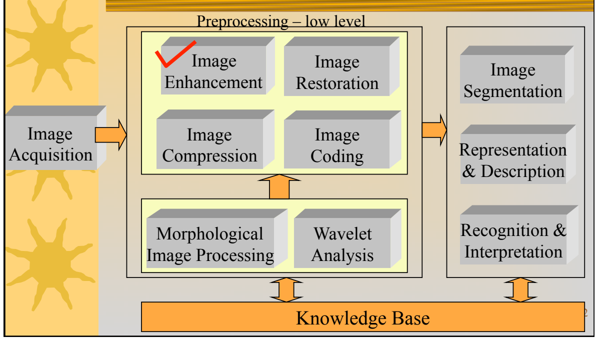
ECE 472/572 - Digital Image Processing

Lecture 7 - Image Restoration - Noise Models

10/04/11

DIP Components



Roadmap

- Introduction
  - Image format (vector vs. bitmap)
  - IP vs. CV vs. CG
  - HLIP vs. LLIP
  - Image acquisition
- Perception
  - Structure of human eye
  - Brightness adaptation and Discrimination
  - Image resolution
- Image enhancement
  - Enhancement vs. restoration
  - Spatial domain methods
    - Point-based methods
    - Mask-based (neighborhood-based) methods - spatial filter
  - Frequency domain methods
- Geometric correction
  - Affine vs. Perspective transformation
  - Homogeneous coordinates
  - Inverse vs. Forward transform
  - Composite vs. Concatenate transformation
  - General transformation
- Image restoration
  - Analyze the noise
    - Type of noise
      - Spatial invariant
        - SAP
        - Gaussian
      - Periodic noise
    - How to identify the type of noise?
      - Test pattern
      - Histogram
    - How to evaluate noise level?
      - RMSE
      - PSNR
  - Noise removal
    - Spatial domain
      - Mean filters
      - Order-statistics filters
      - Adaptive filters
    - Frequency domain
      - Band-pass
      - Band-reject
      - Notch filters
      - Optimal notch filter
  - Analyze the blur
  - Deblurring

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Questions

- What's the different objectives between image enhancement and image restoration?
- How to estimate noise?
- Arithmetic mean vs. geometric mean
- Contraharmonic filter and different parameter values vs. the type of noise removed
- Mean filters vs. order statistics filters
- What's the philosophy of the adaptive filters?
- Understand adaptive median filter
- How to design a notch filter?

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Image Enhancement vs. Restoration

- Image enhancement: process image so that the result is more suitable than the original image for a specific application
- Image restoration: recover image from distortions to its original image

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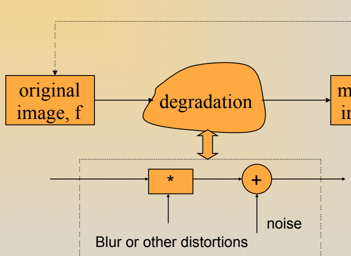
The degradation



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Solving the problem

- Model the degradation
- Apply the inverse process to recover the original image

$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

$$f(x,y) = H^{-1}[g(x,y) - \eta(x,y)]$$

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Different approaches

- Noise
  - Noise models and denoising (5.2, 5.3, 5.4)
- Blur (linear, position-invariant degradations)
  - Estimate the degradation and inverse filters (5.5, 5.6, 5.7, 5.8, 5.9, 5.10)

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Noise sources

- Image acquisition
- Image transmission

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Noise models

- Spatially independent noise models
  - Gaussian noise
  - Rayleigh noise
  - Erlang (Gamma) noise
  - Exponential noise
  - Impulse (salt-and-pepper) noise
- Spatially dependent noise model
  - Periodic noise

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The test pattern

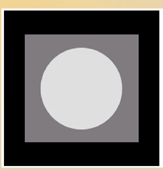

FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

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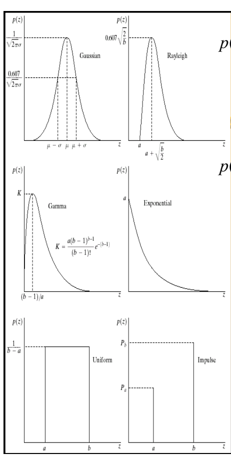

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \ge a \\ 0 & \text{for } z < a \end{cases}$$

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$$p(z) = \begin{cases} \frac{1}{\text{b - a}} & \text{for } a \le z \le \text{b} \\ 0 & \text{otherwise} \end{cases}$$

$$p(z) = \begin{cases} P_a & z = a \\ P_b & z = b \\ 0 & \text{otherwise} \end{cases}$$

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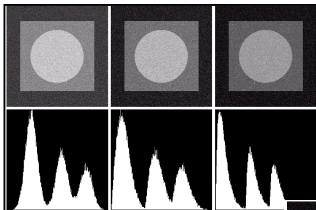
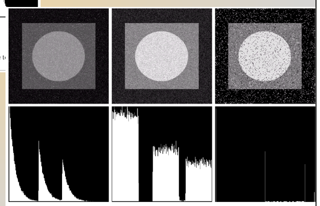


a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise t… in Fig. 5.3.



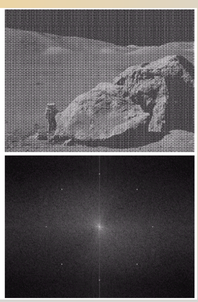
g h i
j k l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

The periodic noise

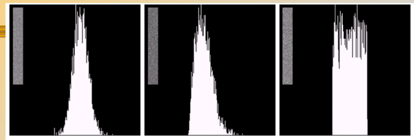
a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)



How to estimate noise parameters?

- Periodic noise
  - Analyze the FT
- Noise PDFs
  - Use "flat" images if we can acquire them
  - What if it's salt-and-pepper noise?
  - Use small strips of uniform intensity if we only have the images but not the acquisition system



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

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Restoration from noise

- Spatial
  - $g(x,y) = f(x,y) + \eta(x,y)$
- Frequency
  - $G(u,v) = F(u,v) + N(u,v)$

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Spatial domain – Neighborhood-based

- Mean filters
  - Arithmetic mean filter (AMF, average)
    - Local smooth
    - Results in blur
  - Geometric mean filter
    - Keeps more detail than AMF
  - Harmonic mean filter
    - Works well for salt noise, but fails for pepper noise
    - Does well on Gaussian noise
  - Contraharmonic mean filter
    - Reducing or virtually eliminating the salt (negative Q)-and-pepper (positive Q) noise

$$f(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

$$f(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

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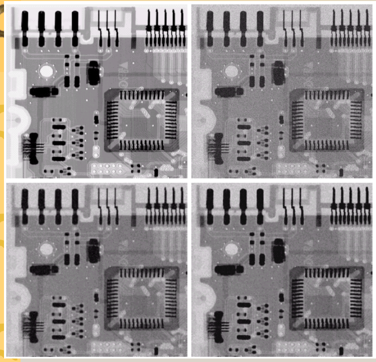

a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

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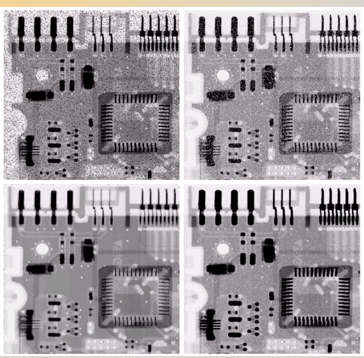
a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.



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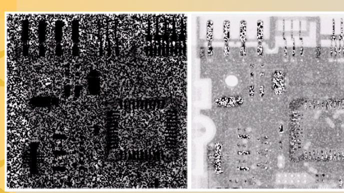

a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3 × 3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

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Spatial domain – Neighborhood-based

- Order-statistics filters
  - Median filter
    - Particularly well on salt-and-pepper noises
  - Max and min filters
    - Max: reduces what noise?
    - Min: reduces what noise?
  - Midpoint filter
    - Average the max and min intensity values
    - Combines order statistics and averaging
    - Works best for Gaussian or uniform noise
  - Alpha-trimmed mean filter
    - Delete the d/2 lowest and the d/2 highest, average the remaining

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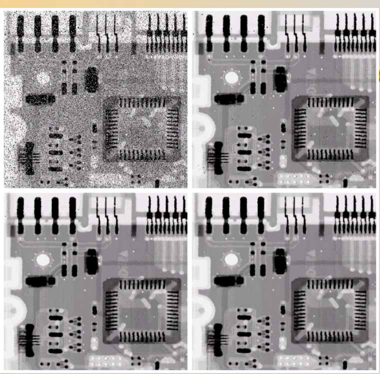
a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3 × 3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.



Example of repetitive application of the same filter

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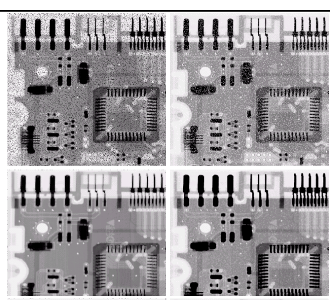
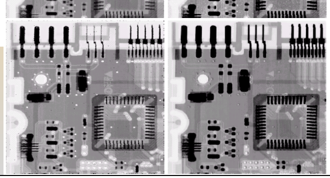
a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.





a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.

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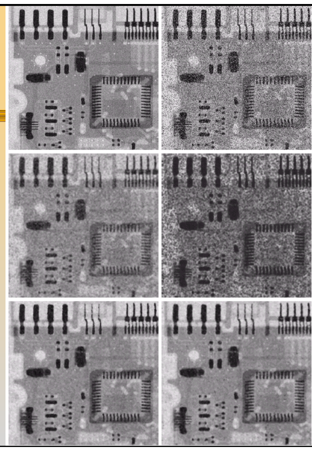
Additive Uniform Noise



S-a-p

5x5 AMF

5x5 geometric mean

5x5 median

alpha-trimmed mean with d=5

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Spatial domain – Neighborhood-based

- Adaptive filters
  - Adaptive local noise reduction filter
  - Local variance, variance of noise, g(x,y), and local mean
- Behavior of filter
  - If global variance is zero, return g(x,y)
  - If the local variance is high compared to the global variance, return a value close to g(x,y)
  - If the two variances are equal, return the arithmetic mean value

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}[g(x,y) - m_L]$$

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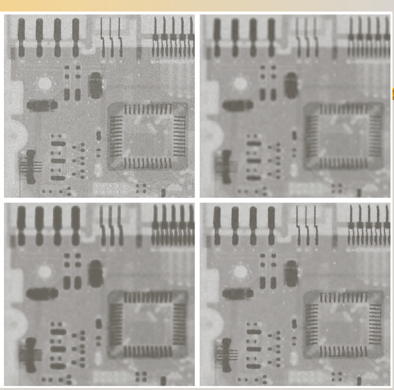

a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7 × 7.

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Adaptive median filter

- Stage A:
  - A1 = zmed - zmin
  - A2 = zmed - zmax
  - If A1>0 AND A2<0, go to stage B
  - Else increase the window size
  - If window size <= Smax, repeat stage A
  - Else output zmed
- Stage B:
  - B1 = zxy - zmin
  - B2 = zxy - zmax
  - If B1>0 AND B2<0, output zxy
  - Else output zmed

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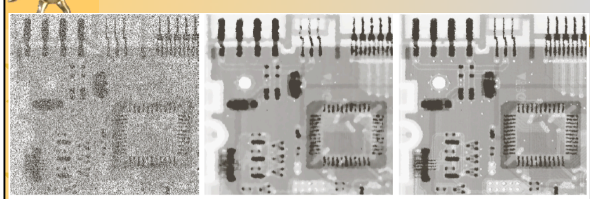

a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

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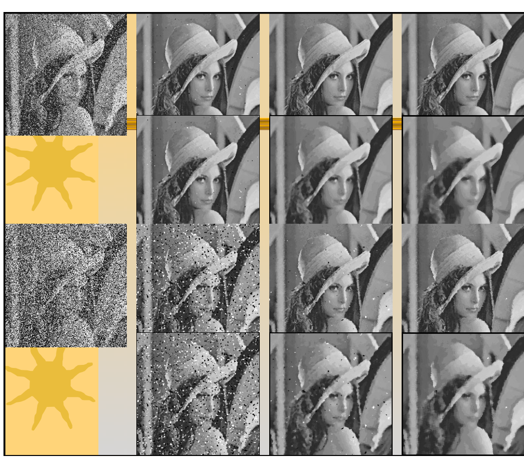

Adaptive Median (3, 5, 7)

median

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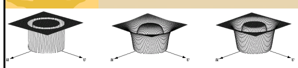
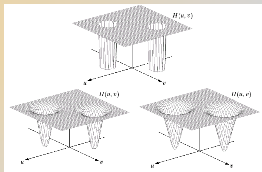
Frequency domain

- Bandreject filters
- Bandpass
- Notch filters



a b c

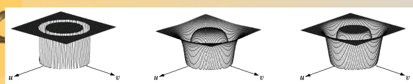
FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.



a
b c

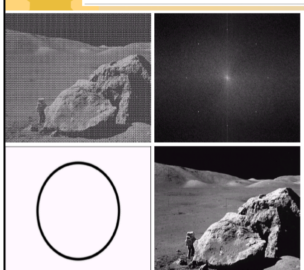
FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v)H_{-k}(u,v)$$



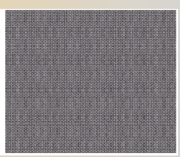
a b c

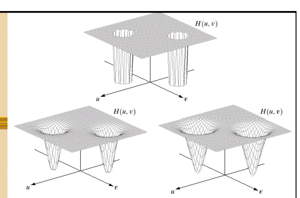
FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.



a b
c d

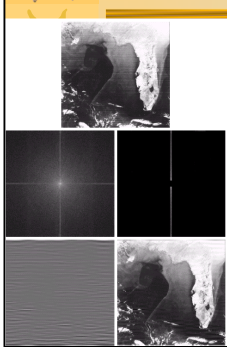
FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

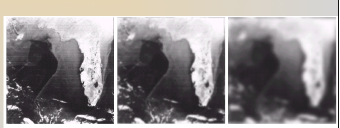




a
b c

FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.



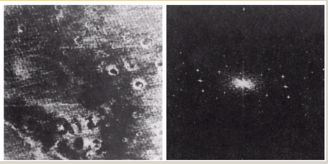


a b c

FIGURE 4.21 (a) Image showing prominent scan lines. (b) Result of using a GLPF with $D_0 = 30$. (c) Result of using a GLPF with $D_0 = 10$. (Original image courtesy of NOAA.)

Optimal notch filters (572)

- Section 5.4.4



a b
FIGURE 5.20 (a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)

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Optimal notch - Derivation

$$N(u,v) = H_{NP}(u,v)G(u,v)$$
$$\eta(x,y) = F^{-1}\{H_{NP}(u,v)G(u,v)\}$$

$$\hat{f}(x,y) = g(x,y) - \eta(x,y)$$
$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y)$$

Select w(x,y) so that the variance of the estimate f(x,y)^
is minimized over a specified neighborhood of every point (x,y)

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \overline{g}(x,y)\overline{\eta}(x,y)}{\overline{\eta^2}(x,y) - \overline{\eta}^2(x,y)}$$

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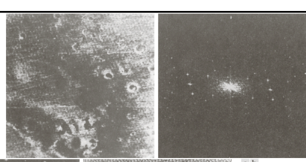
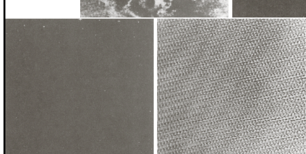
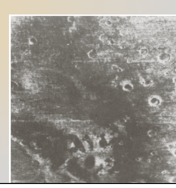
a b
FIGURE 5.20 (a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)





a b
FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

FIGURE 5.23 Processed image. (Courtesy of NASA.)



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Discussion

- Can we apply adaptive frequency domain filters and how?

Evaluating the noise level

- (Root) Mean Square Error (MSE)
  - $E\{||g(x,y) - f(x,y)||^2\}$ or
  - $E\{||(g(x,y)-\underline{g(x,y)}) - (f(x,y)-\underline{f(x,y)})||^2\}$
- Peak Signal to Noise Ratio (PSNR)

  $10\log_{10}[(L-1)/\text{sqrt}(MSE)](dB)$

How to add SAP noise?

```
/**
 * Add salt-and-pepper noise to an image
 * @param inimg The input image.
 * @param q The probability. 0<q<1.
 * For each pixel in the image, generate a random number, say r.
 *   If r<q, change the pixel's intensity to zero.
 *   If r>1-q, change the pixel's intensity to L
 * The higher the q, the worse the noise
 * @return Image corrupted by salt and pepper noise.
 */
Image sapNoise(Image &inimg, float q) {
  // add SAP noise
  srand(time(0));         // so that a different seed nr is generated
  for (i=0; i<nr; i++)
    for (j=0; j<nc; j++)
      for (k=0; k<nchan; k++) {
        r = rand()/(RAND_MAX+1.0);
        outimg(i,j,k) = inimg(i,j,k);
        if (r < q)
          outimg(i,j,k) = 0;
        if (r > 1-q)
          outimg(i,j,k) = L;
      }
}
```

Summary

- Type of noise
  - Spatial invariant
    - SAP
    - Gaussian
  - Periodic noise
- How to identify the type of noise?
  - Test pattern
  - Histogram
- How to evaluate noise level?
  - RMSE
  - PSNR
- Noise removal
  - Spatial domain
    - Mean filters
    - Order-statistics filters
  - Frequency domain
    - Band-pass
    - Band-reject
    - Notch filters
    - Optimal notch filter

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