Digital Signal Processing
Lecture 12 - Spectral Analysis Using DFT

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Overview
Roadmap - 1

- Introduction
- Discrete-time signals and systems - LTI systems
  - Unit sample response $h[n]$: uniquely characterizes an LTI system
  - Linear constant-coefficient difference equation
  - Frequency response: $H(e^{i\omega})$
  - Complex exponentials being eigenvalues of an LTI system: $y[n] = H(e^{i\omega})x[n]$
  - Fourier transform
- $z$ transform
  - The $z$-transform, $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$
  - Region of convergence - the $z$-plane
  - System function, $H(z)$
  - Properties of the $z$-transform
  - The significance of zeros
  - The inverse $z$-transform, $x[n] = \frac{1}{2\pi j} \oint_{C} X(z)z^{n-1}dz$: inspection, power series, partial fraction expansion
Relationships between the $n$, $\omega$, and $z$ domains: Knowing the correspondence between $h[n]$, $H(e^{j\omega})$, and the pole-zero plot

Design structures

- Block diagram vs. Signal flow graph: Knowing how to determine system function, unit sample response, or difference equation from the graphs
- Different design structures: Knowing pros and cons of each form [Direct form I (zeros first), Direct form II (poles first) - Canonic structure, Transposed direct form II (zeros first), Cascade form, Parallel form, Coupled form]
- Specific to IIR or FIR: Feedback in IIR (computable vs. noncomputable), Linear phase in FIR
- Metrics: computational resource and precision
- Sources of errors: Knowing the concept of pole sensitivity of 2nd-order structures leading to coupled form design, and coefficient quantization examples between direct form vs. cascade form
Lecture 12

Roadmap - 3

- Filter design
  - IIR: CT → DT (impulse invariance vs. bilinear transformation)
  - FIR
    - Knowing the characteristics of the four types of causal linear phase FIR filters
    - Window method - Kaiser window: must use minimum specs; the approximation error is scaled by the size of the jump that produces them
    - Optimal method (Alternation theorem <knowing how to determine the number of alternations>, PM algorithm)
- DTFT vs. DFS vs. DFT
- FFT
DTFT vs. DFT

- Analyzing frequency content of signals
- Special consideration when using DFT to “approximate” DTFT
- For finite-length signals, the DFT provides frequency-domain samples of the DTFT
Finite vs. Infinite length

- Windowing
- Time-dependent Fourier transform
  - Stationary signal: All frequency components exist at all time
  - Non-stationary signal: Frequency components do not exist at all time
Example
Example - the Doppler effect

stationary source

moving source
Short-time Fourier transform (STFT)

\[ X(f) = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi ft) dt \]

\[ STFT^w_X(t, f) = \int_{-\infty}^{\infty} x(t)w(t - t') \exp(-j2\pi ft) dt \]
Problems of STFT

- The Heisenberg uncertainty principle
  - One cannot know the exact time-frequency representation of a signal (instance of time)
  - What one can know are the time intervals in which certain band of frequencies exist
  - This is a resolution problem

- Dilemma
  - If we use a window of infinite length, we get the FT, which gives perfect frequency resolution, but no time information.
  - In order to obtain the stationarity, we have to have a short enough window, in which the signal is stationary. The narrower we make the window, the better the time resolution, and better the assumption of stationarity, but poorer the frequency resolution

- Compactly supported: The width of the window is called the support of the window
MRA is designed to give good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies.

This approach makes sense especially when the signal at hand has high frequency components for short durations and low frequency components for long durations.

The signals that are encountered in practical applications are often of this type.
Example
Example - cont’d
Continuous wavelet transform

\[ X(f) = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi ft) \, dt \]

\[ \text{STFT}^w_X(t, f) = \int_{-\infty}^{\infty} x(t) w(t - t') \exp(-j2\pi ft) \, dt \]

\[ \text{CWT}^\psi_X(\tau, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} x(t) \psi^* \left( \frac{t - \tau}{s} \right) \, dt \]

where \( \psi(t) \) is the mother wavelet
Examples of mother wavelets
Discrete wavelet transform

The continuous wavelet transform was computed by changing the scale of the analysis window, shifting the window in time, multiplying by the signal, and integrating over all times.

In the discrete case, filters of different cutoff frequencies are used to analyze the signal at different scales. The signal is passed through a series of highpass filters to analyze the high frequencies, and it is passed through a series of lowpass filters to analyze the low frequencies.

- The resolution of the signal, which is a measure of the amount of detail information in the signal, is changed by the filtering operations.
- The scale is changed by upsampling and downsampling operations.
The process halves time resolution, but doubles frequency resolution.

Quadrature Mirror Filters (QMF),

\[ g[L - 1 - n] = (-1)^n h[n] \]
Example
Acknowledgement

- The instructor thanks the contribution from Dr. Robi Polikar for an excellent tutorial on wavelet analysis, the most readable and intuitive so far.

http://engineering.rowan.edu/~polikar/WAVELETS/WTtutorial.html