ECE644 – Lecture 10

The Network Information Theory

Communication Problem
- A system with many senders and receivers will cause
  - Interference
  - Cooperation
  - Feedback
- Given many senders and receivers and a channel transition matrix which describes the effects of the interference and the noise in the network, decide whether or not the sources can be transmitted over the channel
- Problems to be solved
  - Distributed source coding (data compression)
  - Distributed communication (finding the capacity region of the network) – Unsolved problem

Large Communication Networks
- Computer networks
- Satellite networks
- Phone systems
- Different components in a computer
- A complete theory of network information would have wide implications for the design of communication and computer networks
### Gaussian Multiple User Channels

- Single user Gaussian channel
- Gaussian multiple access channel with m users
- Gaussian broadcast channel
- Gaussian relay channel
- Gaussian interference channel
- Gaussian two-way channel

### A Gaussian Channel

\[ Y_i = X_i + Z_i \]

### Capacity of Gaussian Channel

\[ C_f = \max_{p(x)} I(X;Y) \]

\[ = h(Y) - h(Y|X) = h(Y) - h(Z|X) = h(Y) - h(Z) \]

\[ \leq \frac{1}{2} \log 2\pi e (\frac{P+N}{N}) - \frac{1}{2} \log 2\pi e N \]

\[ = \frac{1}{2} \log \left(1 + \frac{P}{N}\right) \quad \text{(Single user Gaussian channel)} \]

- Codebook \( (2^n, n) \), \( n \) is sufficiently large. \( R < C \)
Gaussian Multiple Access Channel

- m transmitters, each with a power P
- m codebooks, the ith book having $2^{nR_i}$ codewords of power P.
- Optimal decoding
  - Look for m codewords from each codebook, such that the vector sum is closest to Y in Euclidean distance
  $$\sum_{i=1}^{m} R_i < \frac{1}{2} \log \left(1 + \frac{mP}{N}\right) \times C \left(\frac{mP}{N}\right)$$

Gaussian Broadcast Channel

- A sender of power P and two distant receivers, one with Gaussian noise power N1 and the other with N2, N1≠N2
- Y1=X+Z1, Y2=X+Z2
- Encoding
  - 2 codebooks. The transmitter takes one codeword from each and computes the sum and sends the sum over the channel
- Decoding
  - The bad receiver Y2. Look into codebook 2 and find the closest codeword (X2'), and subtract it from Y1. Then looks for the codeword in the first book that's closest to Y1-X2'.
  $$\min \left\{ d(Y1-X2', Y1-Y2') \right\}$$
ECE644 – Lecture 10
The Network Information Theory

Communication Problem
- A system with many senders and receivers will cause
  - Interference
  - Cooperation
  - Feedback
- Given many senders and receivers and a channel transition matrix which describes the effects of the interference and the noise in the network, decide whether or not the sources can be transmitted over the channel
- Problems to be solved
  - Distributed source coding (data compression)
  - Distributed communication (finding the capacity region of the network) – Unsolved problem.

Large Communication Networks
- Computer networks
- Satellite networks
- Phone systems
- Different components in a computer
- A complete theory of network information would have wide implications for the design of communication and computer networks
Gaussian Multiple User Channels

- Single user Gaussian channel
- Gaussian multiple access channel with m users
- Gaussian broadcast channel
- Gaussian relay channel
- Gaussian interference channel
- Gaussian two-way channel

A Gaussian Channel

\[ X_i \rightarrow Z \rightarrow Y_i \]

Capacity of Gaussian Channel

\[ C_f = \max_{P(\cdot)} I(X;Y) \]
\[ = h(Y) - h(Y | X) = h(Y) - h(Z + X | X) \]
\[ = h(Y) - h(Z | X) = h(Y) - h(Z) \]
\[ \leq \frac{1}{2} \log 2 \pi e (P + N) - \frac{1}{2} \log 2 \pi e N \]
\[ = \frac{1}{2} \log \left( 1 + \frac{P}{N} \right) \]

- Single user Gaussian channel
- Codebook \((2^n, n)\). n is sufficiently large. \( R < C \)
Gaussian Multiple Access Channel

- $m$ transmitters, each with a power $P$
- $m$ codebooks, the $i$th book having $2^{nR_i}$ codewords of power $P$.
- Optimal decoding
  - Look for $m$ codewords from each codebook, such that the vector sum is closest to $Y$ in Euclidean distance

\[
\sum_{i=1}^{m} R_i < \frac{1}{2} \log \left( 1 + \frac{mP}{N} \right) = \log \left( \frac{mP}{N} \right)
\]

Gaussian Broadcast Channel

- A sender of power $P$ and two distant receivers, one with Gaussian noise power $N_1$ and the other with $N_2$, $N_1 < N_2$
- $Y_1 = X + Z_1$, $Y_2 = X + Z_2$
- Encoding
  - 2 codebooks. The transmitter takes one codeword from each and computes the sum and sends the sum over the channel
- Decoding
  - The bad receiver $Y_2$. Look into codebook 2 and find the closest codeword from each and computes the sum and sends the sum over the channel
  - The good receiver $Y_1$ first decodes $Y_2$'s codeword $(X_2')$, and subtract it from $Y_1$. Then looks for the codeword in the first book that's closest to $Y_1 - X_2'$
Communication Problem

- A system with many senders and receivers will cause
  - Interference
  - Cooperation
  - Feedback
- Given many senders and receivers and a channel transition matrix which describes the effects of the interference and the noise in the network, decide whether or not the sources can be transmitted over the channel
- Problems to be solved
  - Distributed source coding (data compression)
  - Distributed communication (finding the capacity region of the network) – Unsolved problem

Large Communication Networks

- Computer networks
- Satellite networks
- Phone systems
- Different components in a computer
- A complete theory of network information would have wide implications for the design of communication and computer networks
Gaussian Multiple User Channels
- Single user Gaussian channel
- Gaussian multiple access channel with m users
- Gaussian broadcast channel
- Gaussian relay channel
- Gaussian interference channel
- Gaussian two-way channel

A Gaussian Channel

Capacity of Gaussian Channel
\[ C_f = \max_{P(x)} I(X;Y) \]
\[ = h(Y) - h(Y|X) = h(Y) - h(Z + X|X) \]
\[ = h(Y) - h(Z|X) = h(Y) - h(Z) \]
\[ \leq \frac{1}{2} \log 2\pi e (P + N) - \frac{1}{2} \log 2\pi e N \]
\[ = \frac{1}{2} \log \left( 1 + \frac{P}{N} \right) \]
- Single user Gaussian channel

- 1 codebook (2^n, n). n is sufficiently large. R<C
Gaussian Multiple Access Channel

- m transmitters, each with a power P
- m codebooks, the ith book having $2^{nR_i}$ codewords of power P.
- Optimal decoding
  - Look for m codewords from each codebook, such that the vector sum is closest to Y in Euclidean distance
  $$\sum_{i=1}^{m} R_i \leq \frac{1}{2} \log \left(1 + \frac{mP}{N}ight) \times C \left(\frac{mP}{N}\right)$$

Gaussian Broadcast Channel

- A sender of power P and two distant receivers, one with Gaussian noise power $N_1$ and the other with $N_2$, $N_1 < N_2$
- $Y_1 = X + Z_1$, $Y_2 = X + Z_2$
- Encoding
  - 2 codebooks. The transmitter takes one codeword from each and computes the sum and sends the sum over the channel
- Decoding
  - The bad receiver $Y_2$. Look into codebook 2 and find the closest codeword to $Y_2$.
  - The good receiver $Y_1$ first decodes $Y_2$’s codeword ($X_2'$), and subtract it from $Y_1$. Then looks for the codeword in the first book that’s closest to $Y_1 - X_2'$.
Communication Problem

- A system with many senders and receivers will cause
  - Interference
  - Cooperation
  - Feedback
- Given many senders and receivers and a channel transition matrix which describes the effects of the interference and the noise in the network, decide whether or not the sources can be transmitted over the channel
- Problems to be solved
  - Distributed source coding (data compression)
  - Distributed communication (finding the capacity region of the network) – Unsolved problem

Large Communication Networks

- Computer networks
- Satellite networks
- Phone systems
- Different components in a computer
- A complete theory of network information would have wide implications for the design of communication and computer networks
Gaussian Multiple User Channels

- Single user Gaussian channel
- Gaussian multiple access channel with m users
- Gaussian broadcast channel
- Gaussian relay channel
- Gaussian interference channel
- Gaussian two-way channel

A Gaussian Channel

\[
\begin{align*}
X_i & \rightarrow Z_i \\
\rightarrow & \rightarrow Y_i
\end{align*}
\]

Capacity of Gaussian Channel

\[
C_f = \max_{p(x|z)} I(X;Y) = h(Y) - h(Y|X) = h(Y) - h(Z + X|X) = h(Y) - h(Z) \leq \frac{1}{2} \log 2 \pi e (P + N) - \frac{1}{2} \log 2 \pi e N = \frac{1}{2} \log \left(1 + \frac{P}{N}\right)
\]

- Single user Gaussian channel
- Codebook (2^n, n), n is sufficiently large. R < C
Gaussian Multiple Access Channel

- m transmitters, each with a power P
- m codebooks, the ith book having $2^{nR_i}$ codewords of power $P_i$
- Optimal decoding
  - Look for m codewords from each codebook, such that the vector sum is closest to $Y$ in Euclidean distance
  
  \[
  \sum_{i} R_i \leq \frac{1}{2} \log \left( 1 + \frac{mP_i}{N} \right) + C \left( \frac{mP_i}{N} \right)
  \]

Gaussian Broadcast Channel

- A sender of power $P$ and two distant receivers, one with Gaussian noise power $N_1$ and the other with $N_2$, $N_1 < N_2$
- $Y_1 = X + Z_1$, $Y_2 = X + Z_2$
- Encoding
  - 2 codebooks. The transmitter takes one codeword from each and computes the sum and sends the sum over the channel
- Decoding
  - The bad receiver $Y_2$. Look into codebook 2 and find the closest codeword from each and computes the sum and sends the sum over the channel
  - The good receiver $Y_1$ first decodes $Y_2$'s codeword ($X_2'$), and subtract it from $Y_1$. Then looks for the codeword in the first book that's closest to $Y_1 - X_2'$

\[

\sum_{i} R_i \leq C \left( \frac{mP_i}{N_i} \right) \quad \text{or} \quad \text{Decoding}
\]