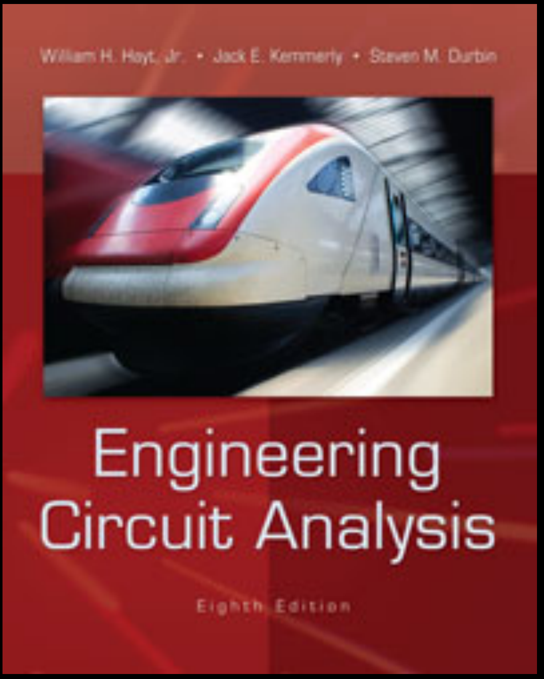
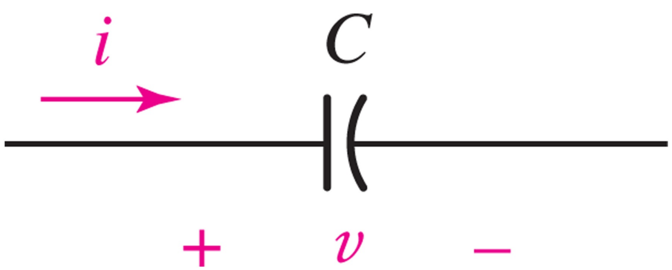
Capacitors and Inductors



The Capacitor

- the ideal capacitor is a passive element with circuit symbol



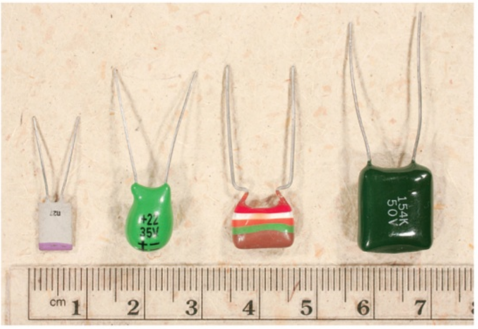
- the current-voltage relation is

$$i = C\frac{dv}{dt}$$

- the capacitance C is measured in farads (F)

Some Capacitors

- capacitors can be bulky and typical values range from pF to μF



(a)

(b)

(c)

Capacitors Store Energy

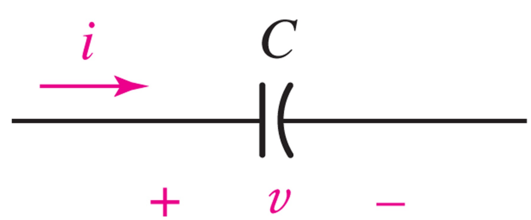
Since $$p(t) = i(t)v(t) = \left(C\frac{dv}{dt}\right)v = \frac{dw}{dt}$$

then the energy stored in a capacitor is

$$w = \tfrac{1}{2}Cv^2$$

Key Capacitor Behaviors

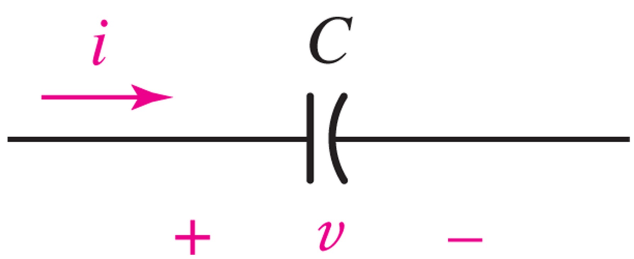
- capacitors are open circuits to dc voltages
- the voltage on a capacitor *cannot* jump
- capacitors store energy (*iv>0)* or deliver energy (*iv<0)*

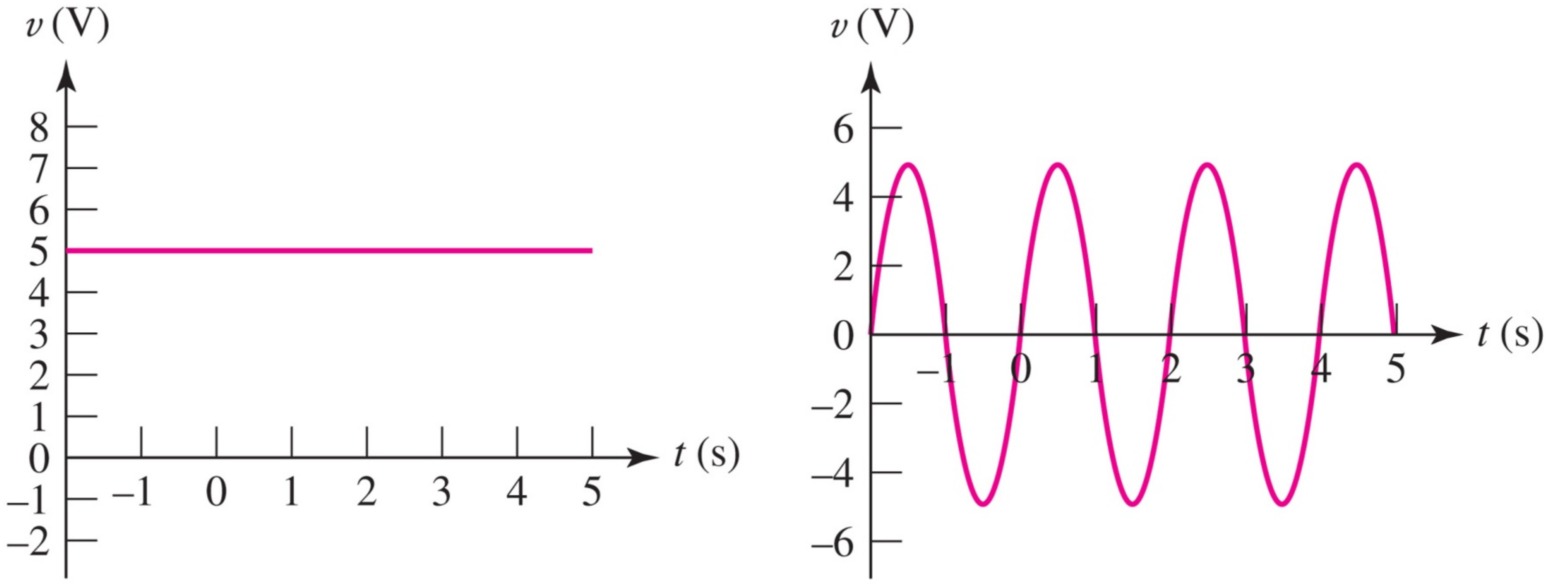


$$i = C\frac{dv}{dt}$$

Example: i-v Curves (part 1 of 2)

Find *i(t)* for the voltages shown, if C=2 F.

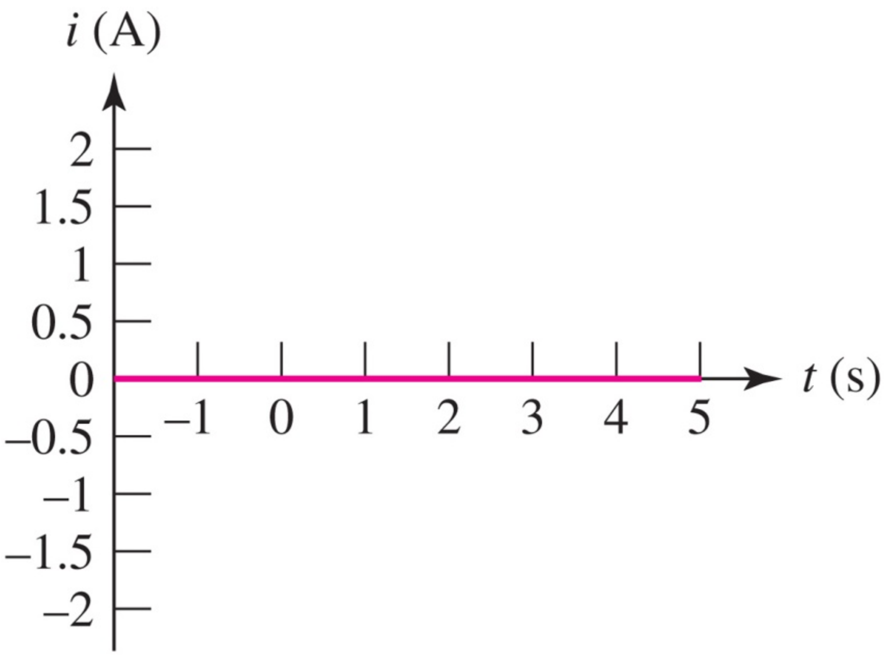




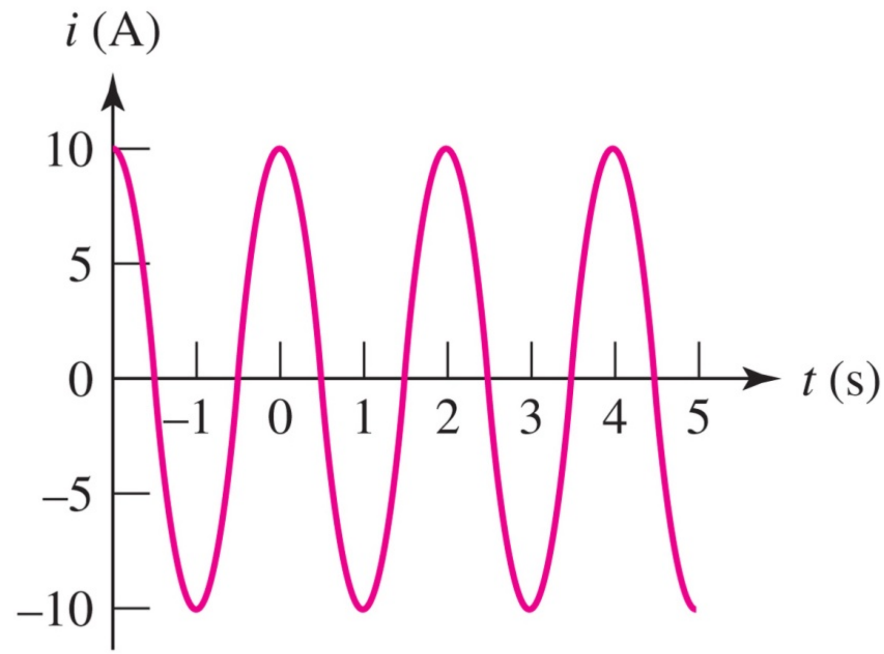
(*a*) (*b*)

Example: i-v Curves (part 2 of 2)

Solution: apply $i(t)=2dv/dt$ and graph:



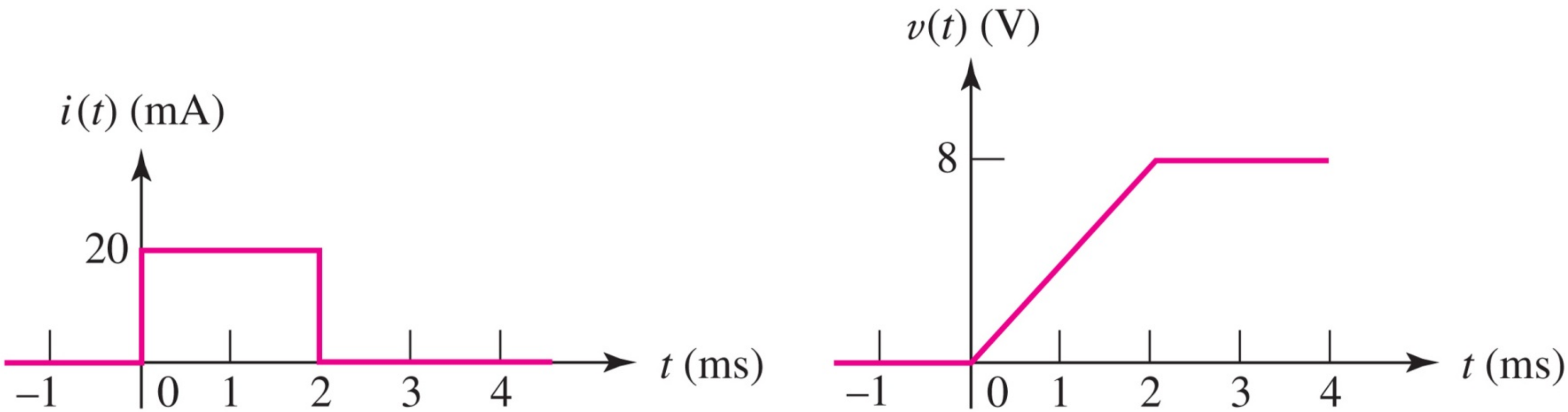
(a)



(b)

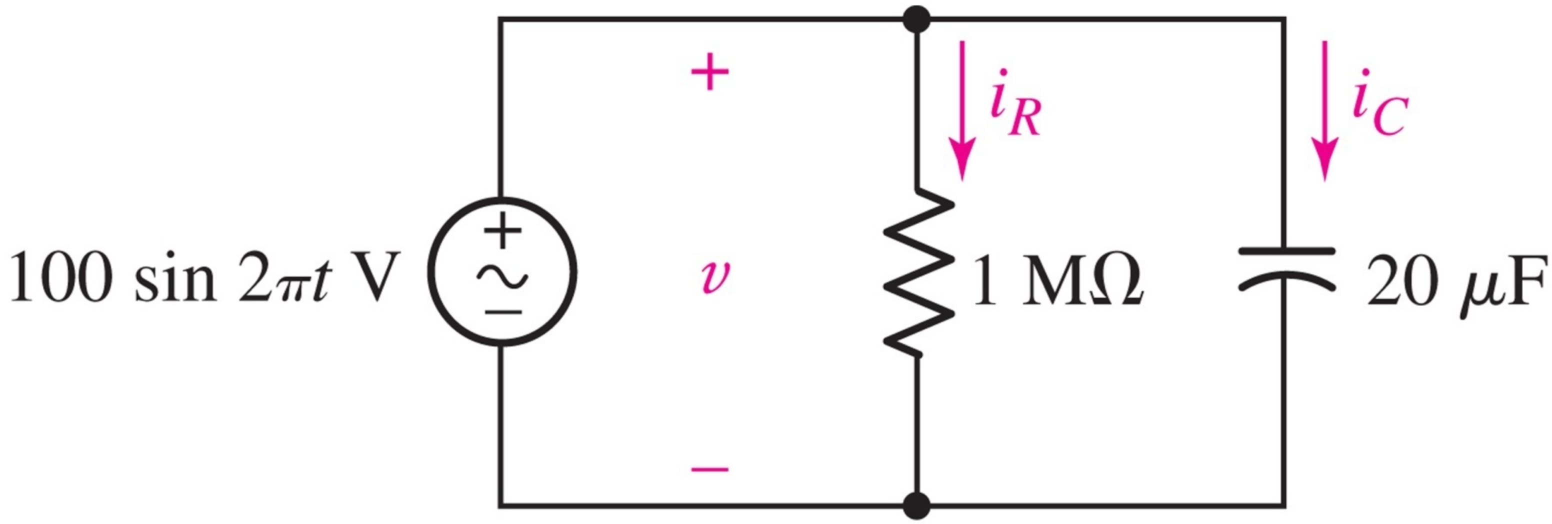
Example: i-v Curves

Show that the following graphs are matching voltage and current graphs for a capacitor of C=5μF.



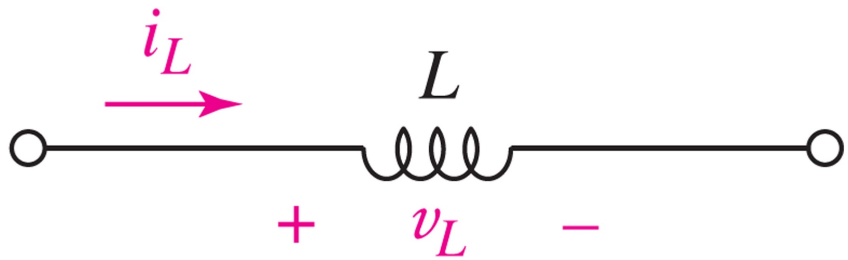
Example: Capacitor Energy

Determine the maximum energy stored in the capacitor, and plot i_R and i_C.



The Inductor

- the ideal inductor is a passive element with circuit symbol



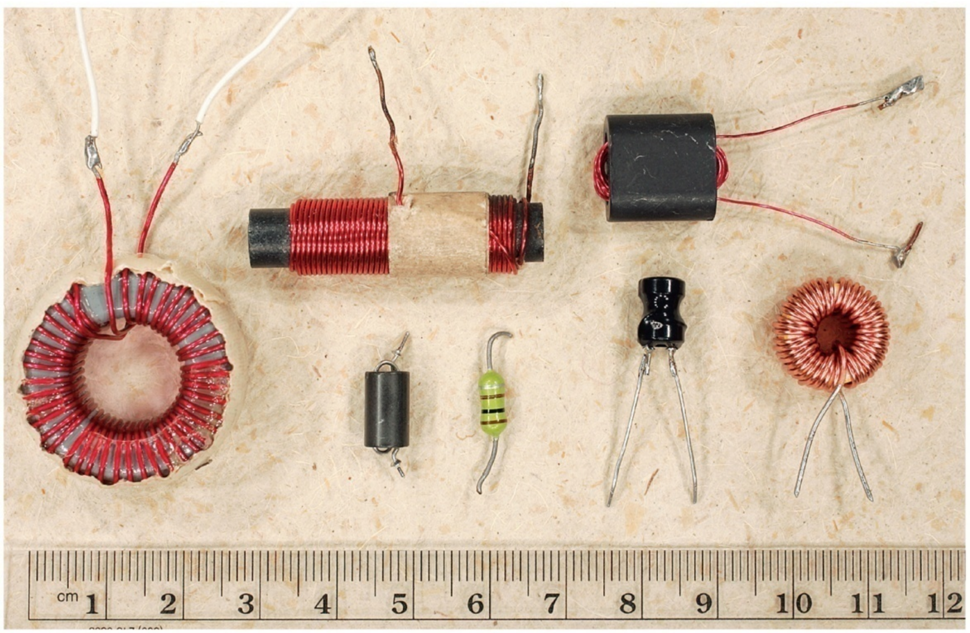
- the current-voltage relation is

$$v = L\frac{di}{dt}$$

- the unit of inductance L is henry (H)

Some Inductors

- inductors can be bulky and typical values range from μH to H



(a)

(b)

Inductors Store Energy

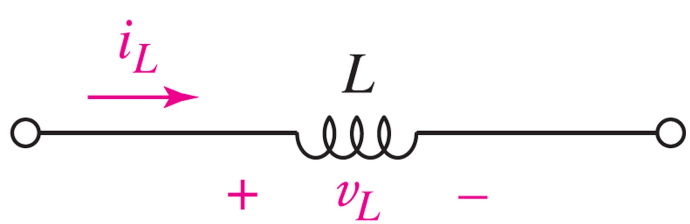
Since $$p(t) = i(t)v(t) = \left(L\frac{di}{dt}\right)i = \frac{dw}{dt}$$

then the energy stored in a inductor is

$$w = \tfrac{1}{2}Li^2$$

Key Inductor Behaviors

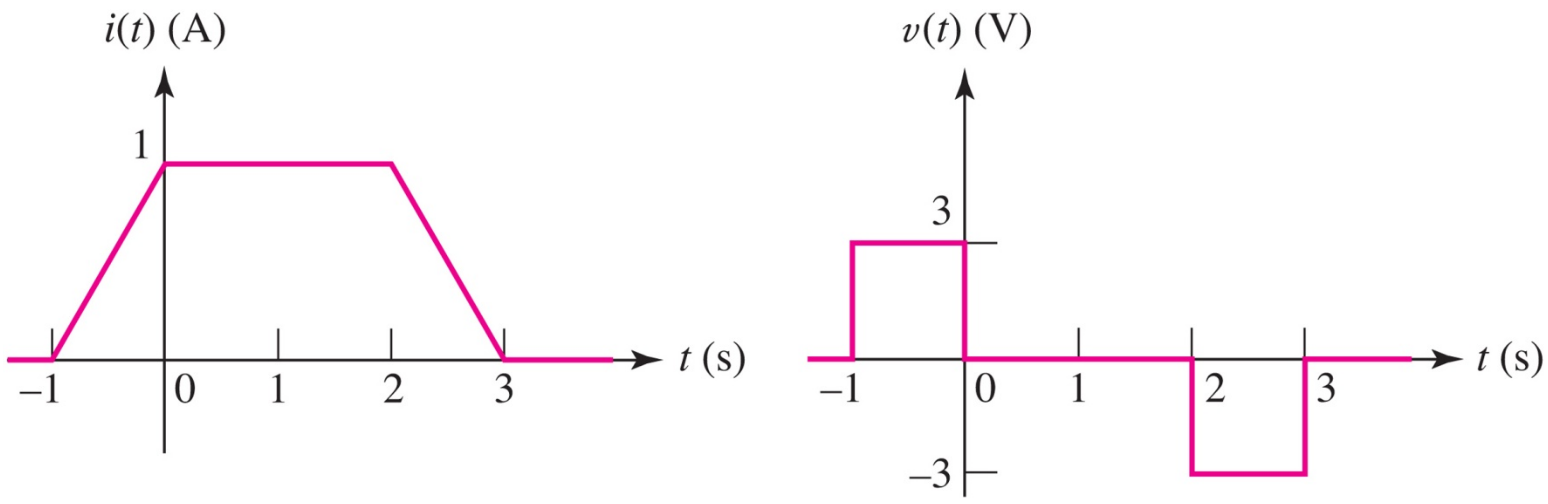
- inductors are short circuits to dc voltages
- the current through an inductor *cannot* jump
- inductors store energy (*iv>0*) or deliver energy (*iv<0*)



$$v = L\frac{di}{dt}$$

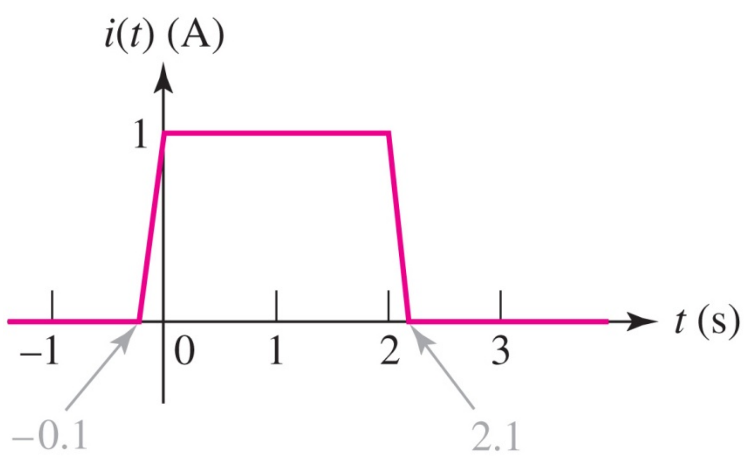
Example: i-v Curves for L

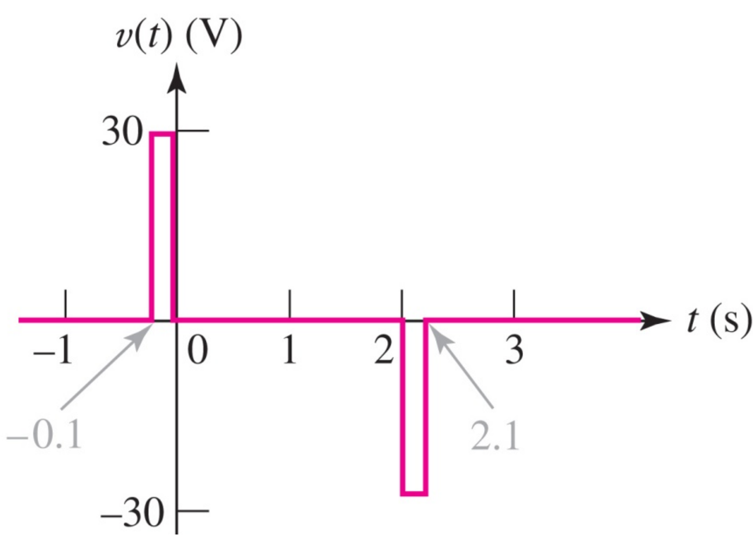
Show that the following graphs are matching voltage and current graphs for an inductor of L=3 H.



Example: i-v Curves for L

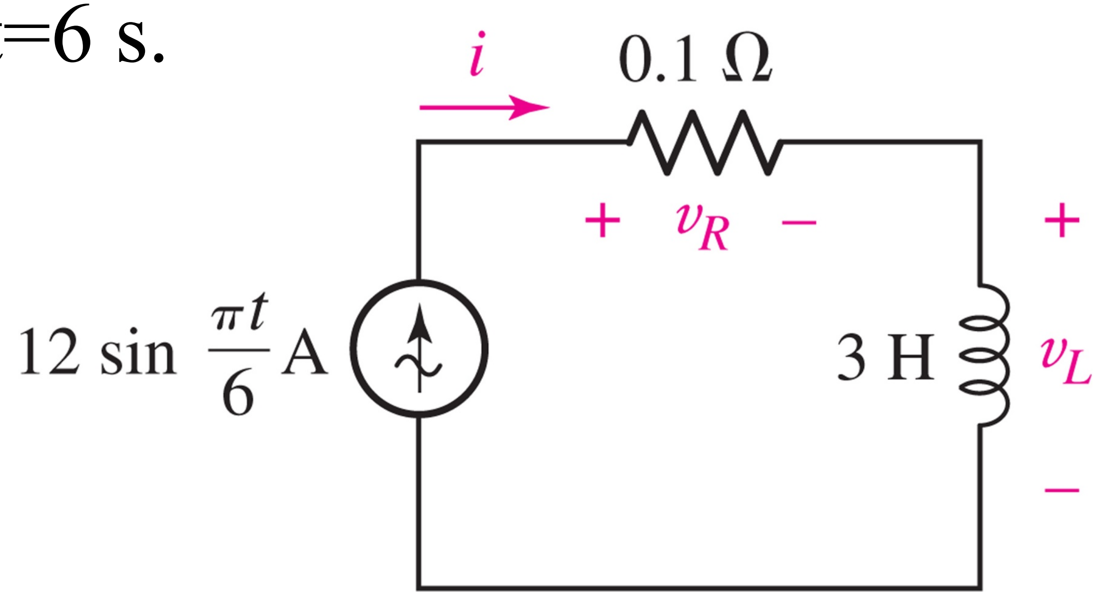
For the same 3-H inductor, the voltages are 10 times larger when the current is ramped 10 times faster:





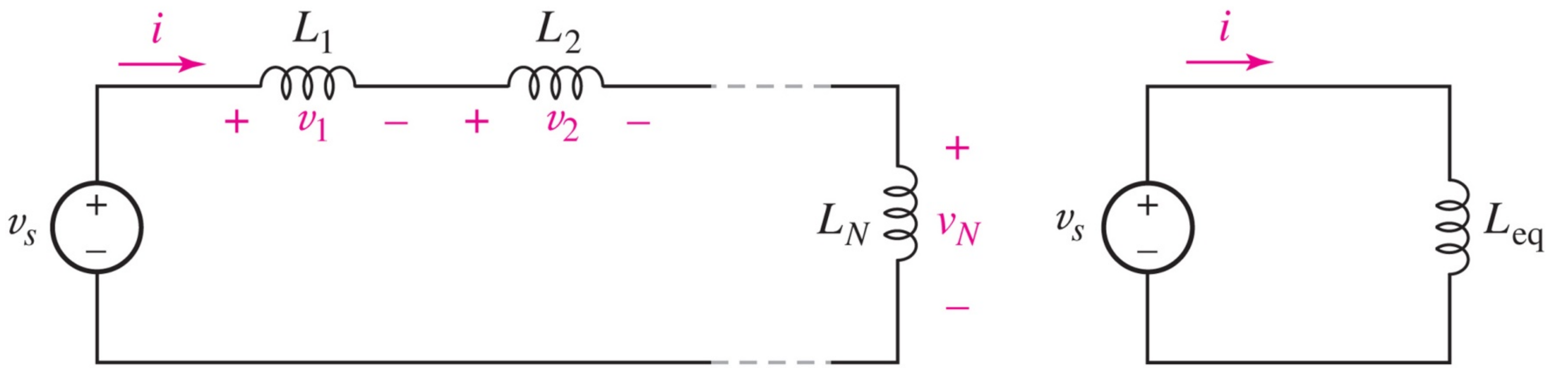
Example: Energy in L

Determine the maximum energy stored in the inductor, and find the energy lost to resistor from t=0 to t=6 s.



Answer: 216 J, 43.2 J

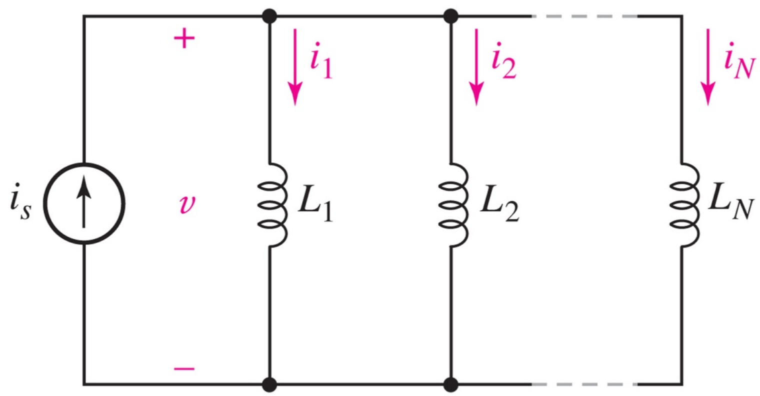
Inductors in Series

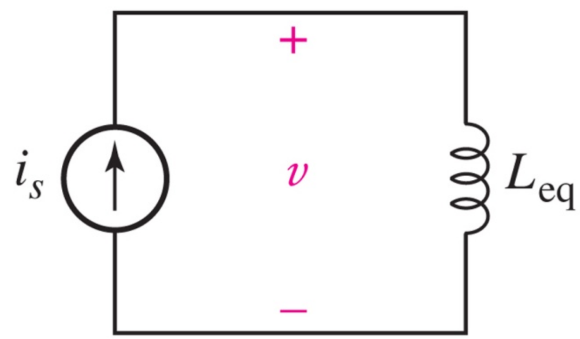


Apply KVL to show:

$$L_{eq} = L_1 + L_2 + \cdots + L_N$$

Inductors in Parallel

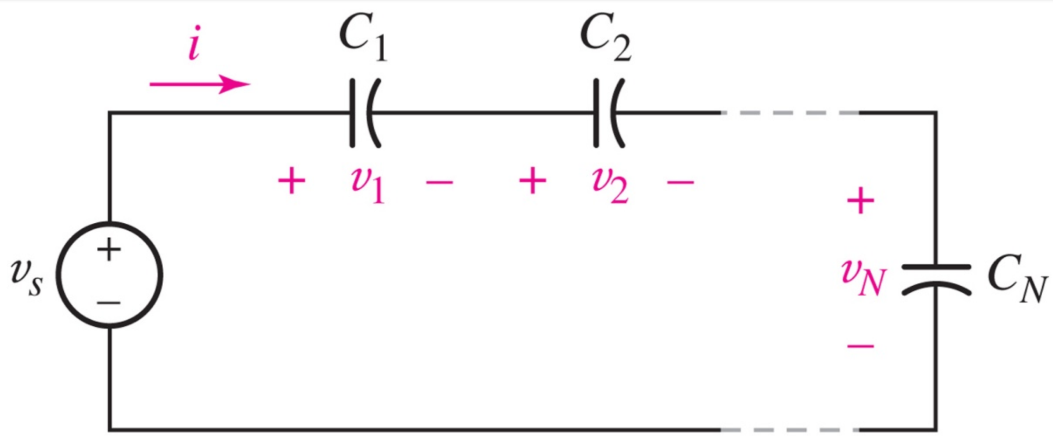


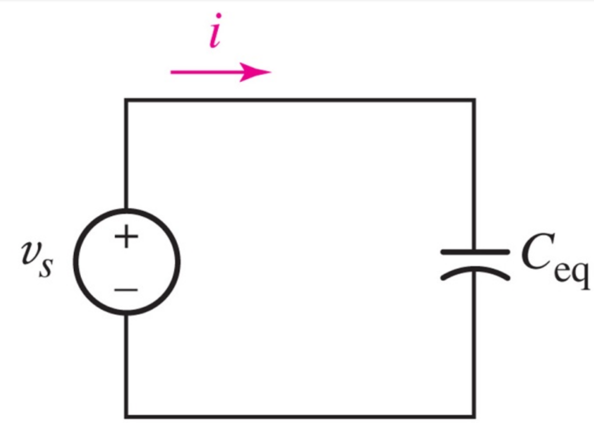


Apply KCL to show

$$L_{eq} = \frac{1}{1/L_1 + 1/L_2 + \cdots + 1/L_N}$$

Capacitors in Series

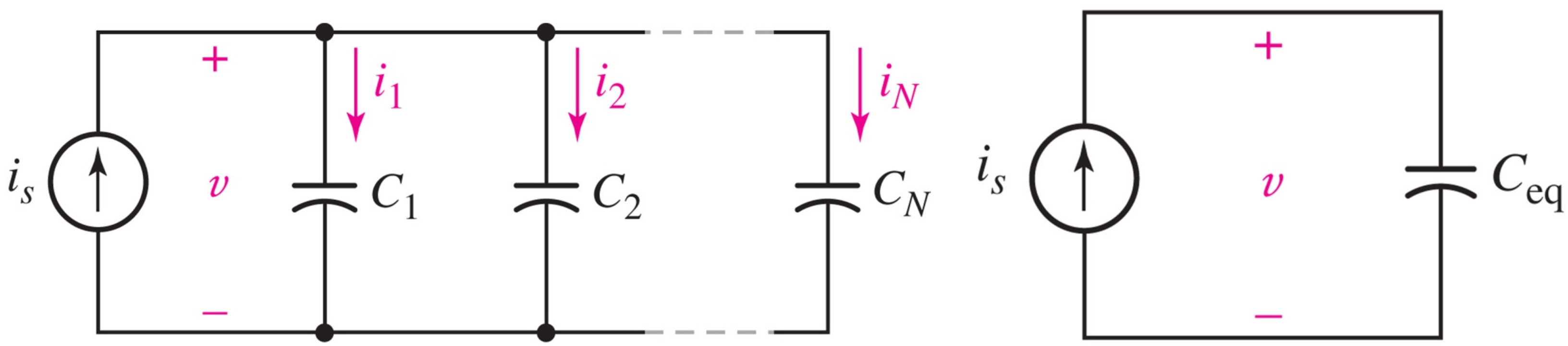




Apply KVL to show:

$$C_{eq} = \frac{1}{1/C_1 + 1/C_2 + \cdots + 1/C_N}$$

Capacitors in Parallel



Apply KCL to show:

$$C_{eq} = C_1 + C_2 + \cdots + C_N$$

Two-Element Shortcuts

Two capacitors in ***series***:

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

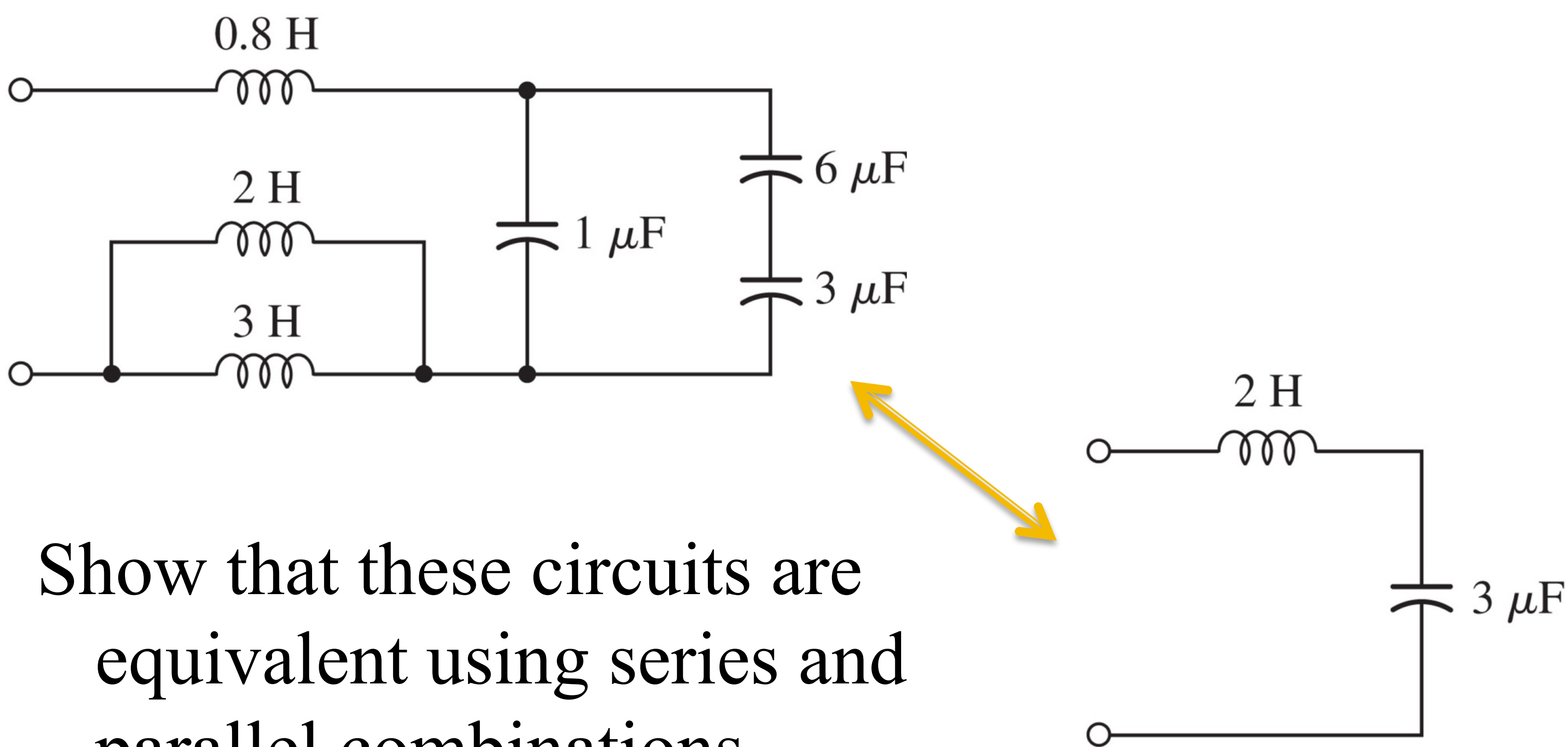
Two inductors in parallel:

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Two resistors in parallel:

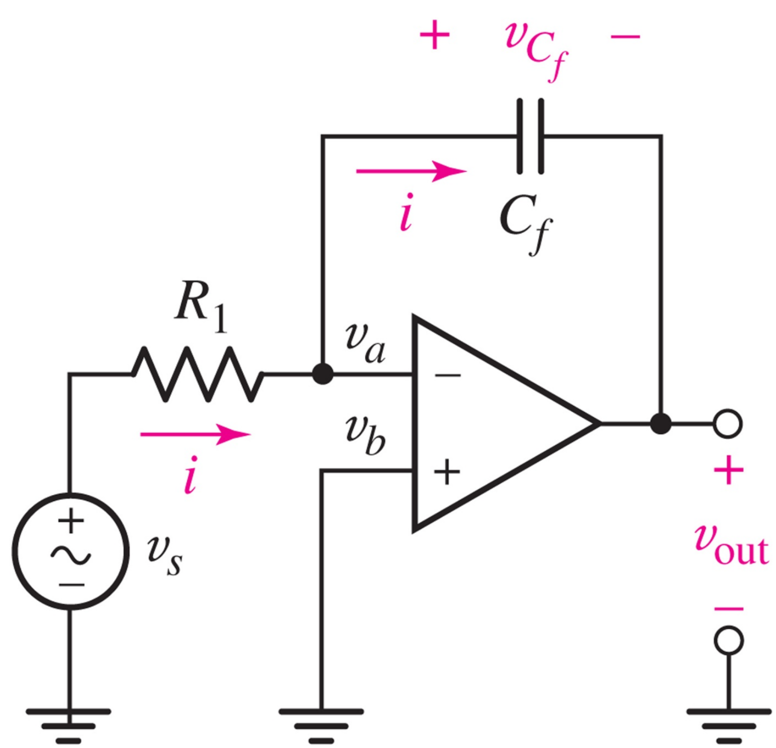
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Example: Simplifying LC



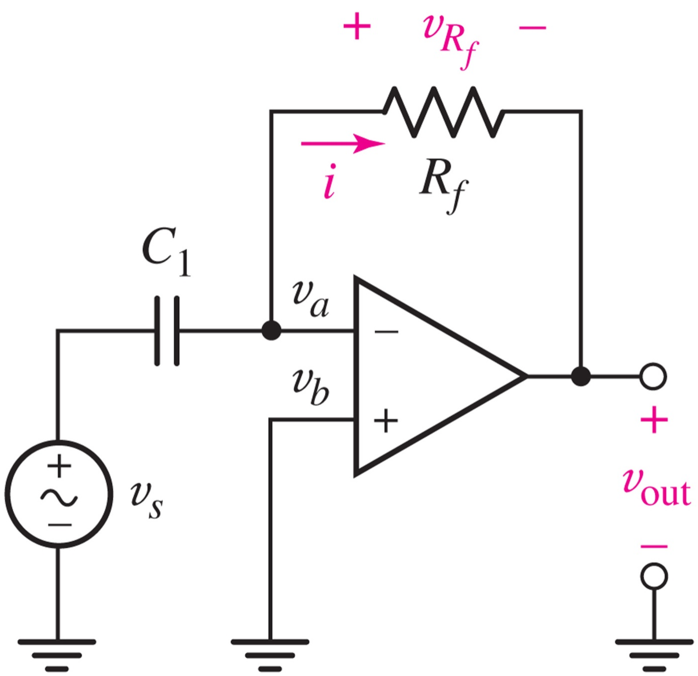
Show that these circuits are equivalent using series and parallel combinations.

Op Amp Integrator



$$v_{out} = -\frac{1}{R_1 C_f} \int_0^t v_s \, dt' - v_{C_f}(0)$$

Op Amp Differentiator



$$v_{out} = -C_1 R_f \frac{dv_s}{dt}$$