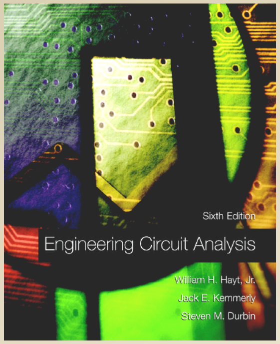

Chapter 8 Basic RL and RC Circuits

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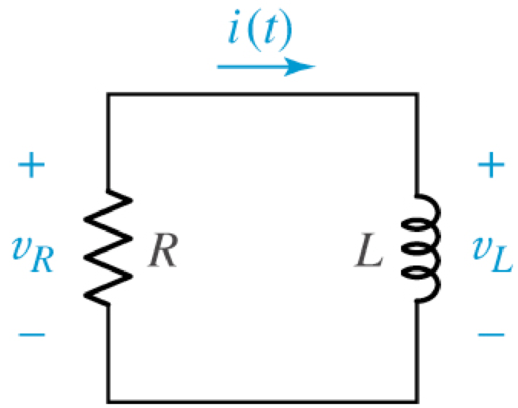

A series RL circuit for which i(t) is to be determined, subject to the initial condition that i(0) = I_0.

$$\text{KVL:}\;\; R\,\mathrm{i}(t) + L\frac{d}{dt}\big(\mathrm{i}(t)\big) = 0$$

1st-order, linear, constant-coefficient, ordinary, homogeneous differential equation. The solution form is $\mathrm{i}(t) = Ae^{\lambda t}$.

$$RAe^{\lambda t} + L\lambda Ae^{\lambda t} = 0 \Rightarrow R + \lambda L = 0 \Rightarrow \lambda = -R/L \Rightarrow \mathrm{i}(t) = Ae^{-Rt/L}.$$

To find A we need a boundary condition. The most common boundary condition is $\mathrm{i}(0) = I_0$. Then

$$I_0 = A \Rightarrow \mathrm{i}(t) = I_0 e^{-Rt/L}$$

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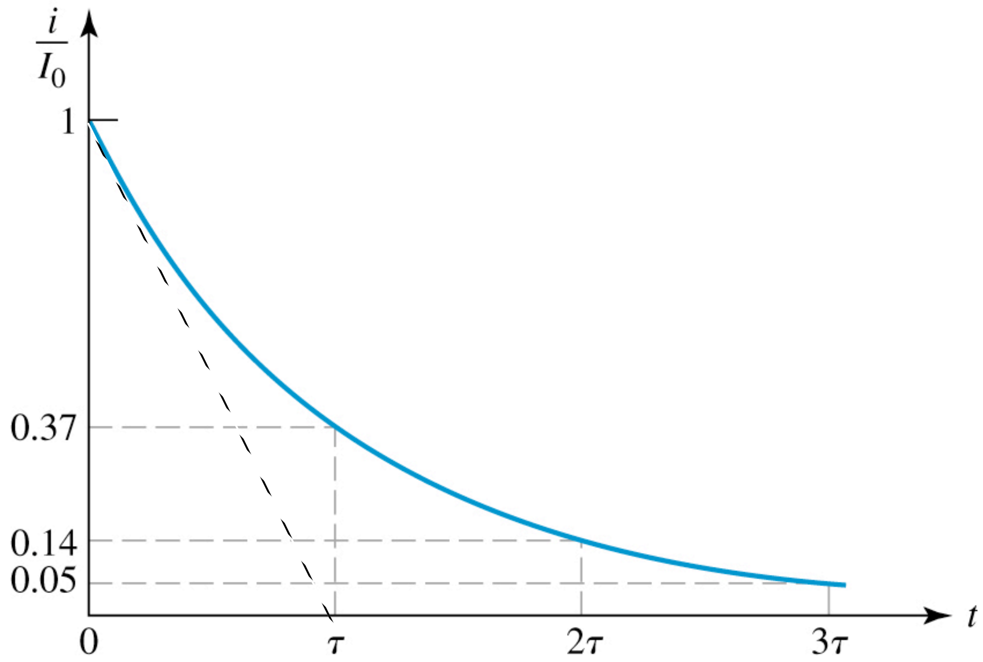

A plot of the exponential response versus time.

$$i(t) = I_0 e^{-Rt/L} = I_0 e^{-t/\tau} \text{ where } \tau = L/R$$

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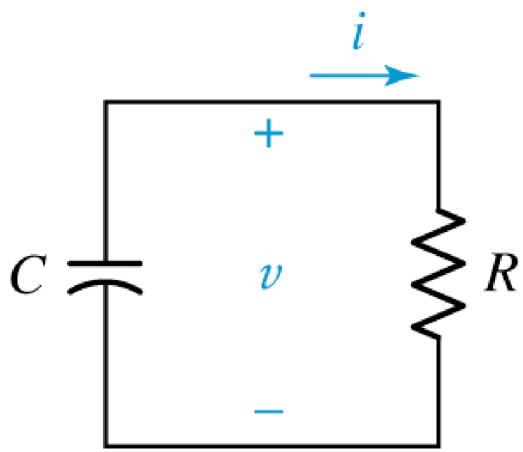

A parallel RC circuit for which *v*(t) is to be determined, subject to the initial condition that *v*(0) = V_0.

$$\text{KCL: } C\frac{d}{dt}\big(\mathrm{v}(t)\big)+\frac{\mathrm{v}(t)}{R}=0\ ,\ \mathrm{v}(t)=Ae^{\lambda t}\Rightarrow CA\lambda e^{\lambda t}+\frac{Ae^{\lambda t}}{R}=0$$

$$C\lambda+1/R=0\Rightarrow\lambda=-1/RC\Rightarrow\mathrm{v}(t)=Ae^{-t/RC}$$

$$\text{If } \mathrm{v}(0)=V_0,\ \text{then } \mathrm{v}(t)=V_0e^{-t/RC}=V_0e^{-t/\tau}\ ,\ \text{where } \tau=RC.$$

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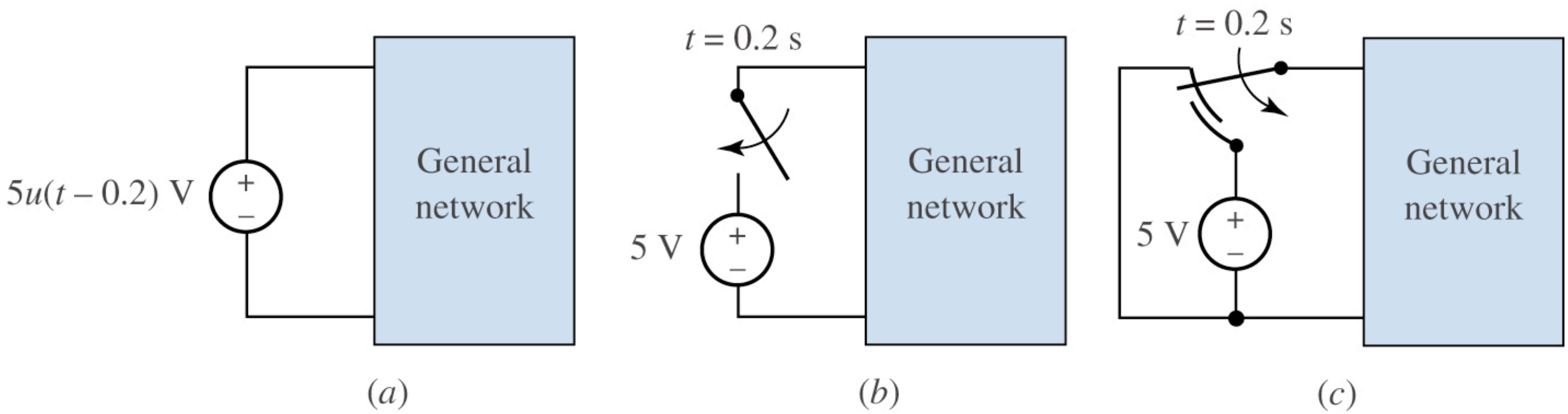

(a) A voltage-step forcing function is shown as the source driving a general network. (b) A simple circuit which, although not the exact equivalent of part (a), may be used as its equivalent in many cases. (c) An exact equivalent of part (a).

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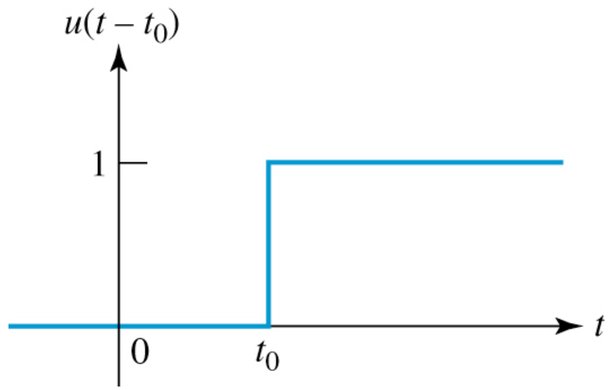
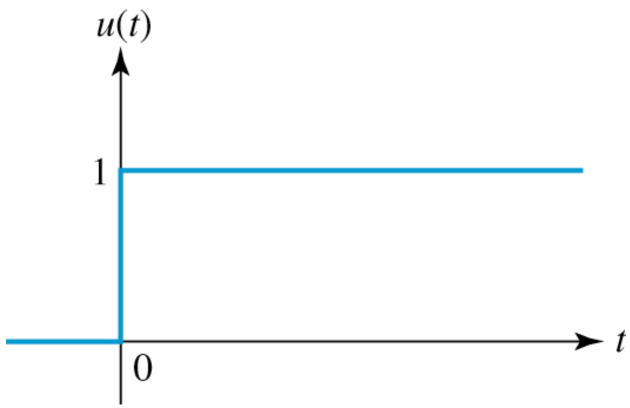


The unit-step forcing function $u(t - t_0)$.



The unit-step forcing function $u(t)$.

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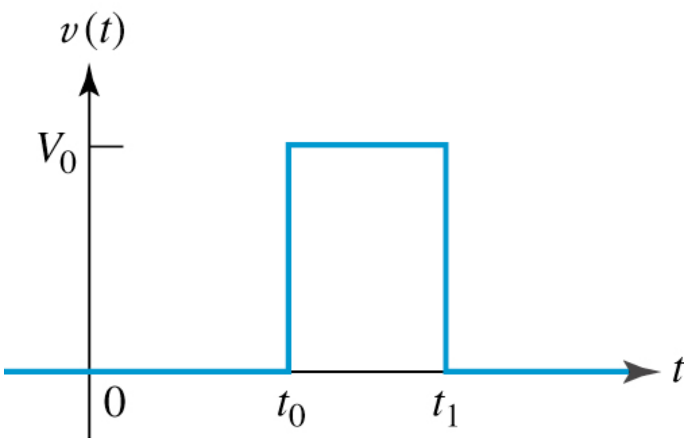
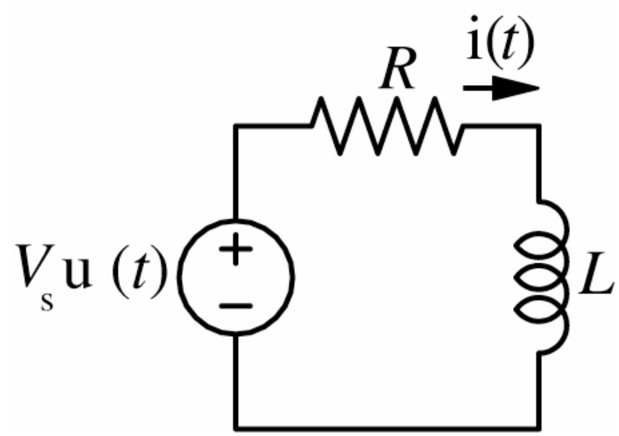


The rectangular pulse function

$$v(t - t_0) = \begin{cases} 0 & t < t_0 \\ V_0 & t_0 < t < t_1 \\ 0 & t > t_1 \end{cases}$$

Driven *RL* Circuit



$$R\,\mathrm{i}(t) + L\frac{d}{dt}\left(\mathrm{i}(t)\right) = V_s\,\mathrm{u}(t)$$

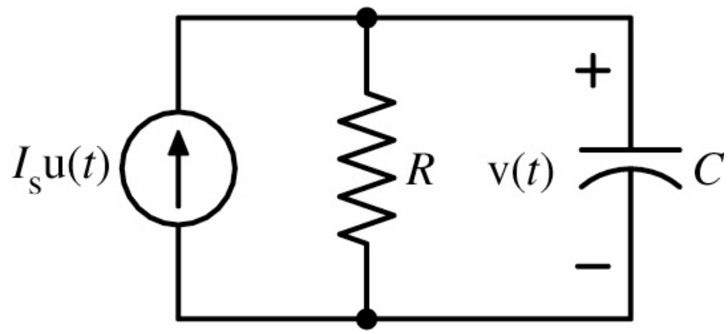
Solution form is $\mathrm{i}(t) = Ae^{-t/\tau} + I_f \;,\; t > 0$ where $I_f = \mathrm{i}(\infty)$.

$$\tau = L/R \;\;,\;\; I_f = V_s/R \Rightarrow \mathrm{i}(t) = Ae^{-Rt/L} + V_s/R$$

$$\mathrm{i}(0^+) = \mathrm{i}(0^-) = 0 = A + V_s/R \Rightarrow A = -V_s/R$$

$$\mathrm{i}(t) = (V_s/R)\left(1 - e^{-Rt/L}\right) \;\;,\;\; t > 0$$

Driven *RC* Circuit



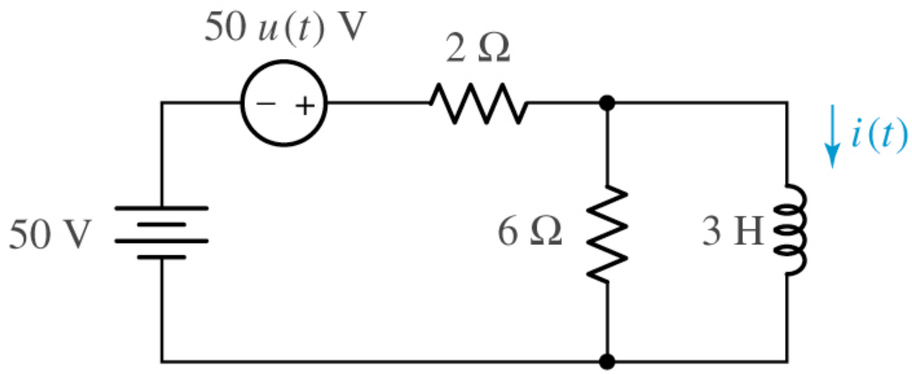
$$\frac{\mathrm{v}(t)}{R}+C\frac{d}{dt}\left(\mathrm{v}(t)\right)=I_s\,\mathrm{u}(t)$$

Solution form is $\mathrm{v}(t)=Ae^{-t/\tau}+V_f$, $t>0$ where $V_f=\mathrm{v}(\infty)$.

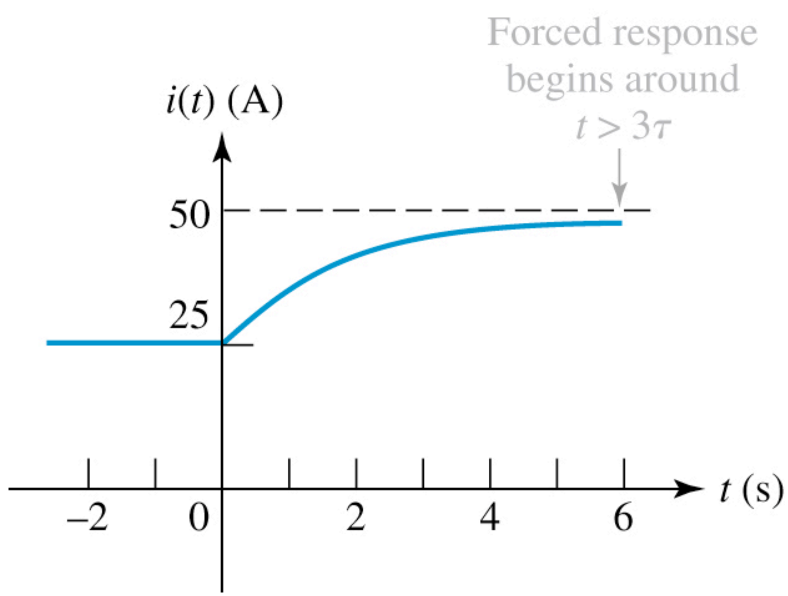
$$\tau=RC\ \ ,\ \ V_f=RI_s\Rightarrow\mathrm{v}(t)=Ae^{-t/RC}+RI_s$$

$$\mathrm{v}(0^+)=\mathrm{v}(0^-)=0=A+RI_s\Rightarrow A=-RI_s$$

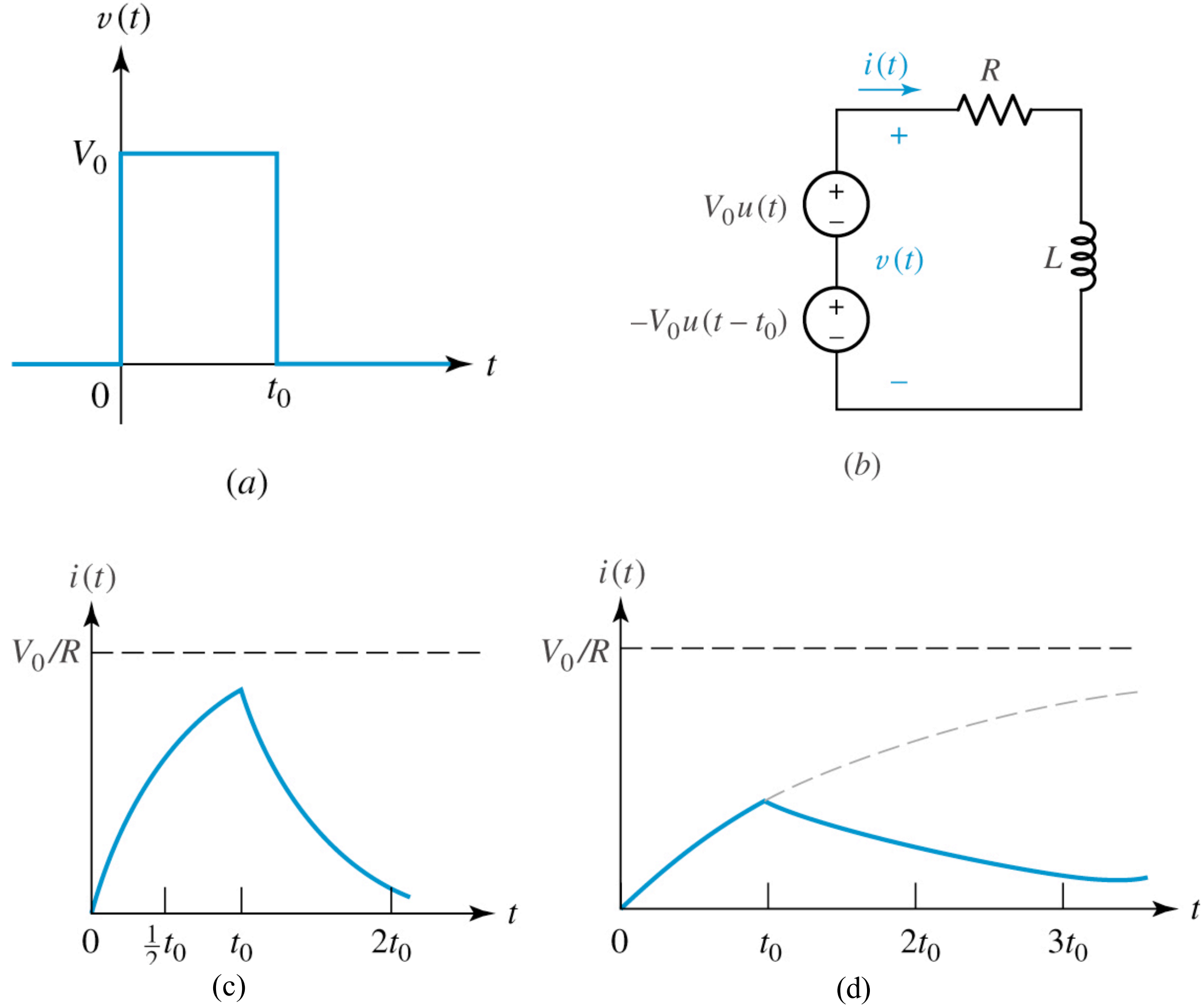
$$\mathrm{v}(t)=RI_s\left(1-e^{-t/RC}\right)\ \ ,\ \ t>0$$



Circuit for which a complete response *i*(t) is desired.



The desired current response as a function of time.



(a) Pulse waveform; (b) Circuit driven by waveform (a); (c) $\tau = t_0/2$; (d) $\tau = 2t_0$.

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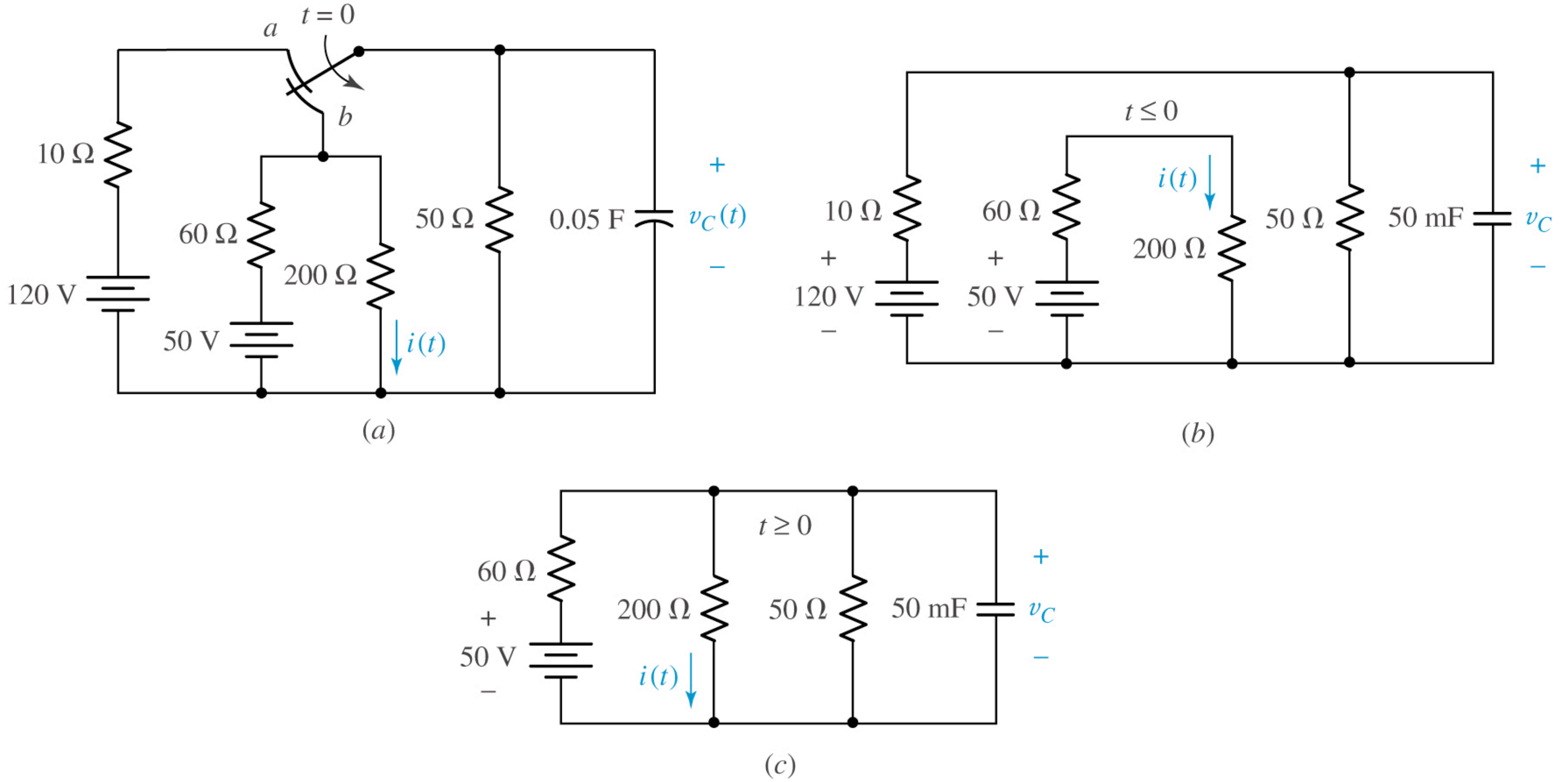
Find the capacitor voltage $v_C(t)$ and the current $i(t)$ in the 200-Ω resistor of the circuit shown in (a).



(a) Original circuit; (b) circuit valid for $t \leq 0$; (c) circuit for $t \geq 0$.

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