Laboratory – 3

Introduction To
Two-Port Networks

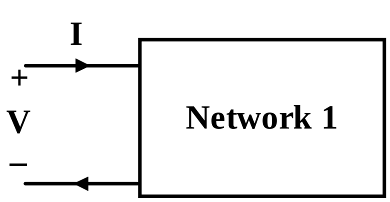
Objectives

The objectives of this laboratory are as follows:

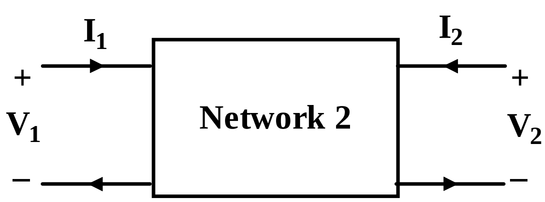
- to become familiar with the equations that are used to describe two-port networks,
- measure currents and voltages of a two-port network and learn to use these measurements to calculate any of the two-port parameters,
- to learn the use of the table for converting from one set of two-port parameters to another set.

Background

In theory, a network may have either one port, two-ports, or N ports, depending on the number of circuit mesh. The idea of a port is illustrated in Figure 7.1.



(a) a one port network



(b) a two port network

Figure 7.1: Diagram defining network ports.

Obviously, the network in Figure 7.1(a) is a one-port. This could be either an input port or an output port, but not both. The network in Figure 7.1(b) is described by a port on the left, called the input port. The port on the right is usually called the output port. This is a standard convention used in describing two-port networks. In this laboratory exercise you will be considering networks described as two-port networks.

There are four sets of parameters commonly used to describe two-port networks. There is a fifth set but most often it is omitted. The four to be consider in this laboratory are:

- the two-port described using Y (admittance) parameters
- the two-port described using Z (impedance) parameters
- the two-port described using H (hybrid) parameters
- the two-port described using ABCD (transmission) parameters

The network inside the "box" of Figure 7.1(b) can contain resistors, inductors, capacitors, transformers, transistors and in general any linear circuit device, including depending devices but ***no independent sources are allowed.***

Essentially, there are two ways to view the two-port network problem. First, view the problem as if you were in a laboratory and you actually had a "box" with an input port and output port as shown above. Depending on the parameters one desires to find, measurements are made of currents I_1 and I_2 with sources V_1 and V_2 present and with the sources replaced with short circuits. This becomes clear in the presentation below. The second way to view the problem is as if you knew the construction of the network and you determined the various open-circuited voltages and short-circuited currents. In both cases one uses open-circuited voltages, shorted terminals, and short-circuit currents to determine the parameters. This may sound confusing but the whole process is rather straightforward.

The Y parameters:

The equations used to describe the Y parameters of a two-port are:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \text{Eq. 7.1}$$

We observe the following from Eq. 7.1:

$$y_{11} = \frac{I_1}{V_1} \; \text{with } V_2 = 0 \quad \text{input admittance} \qquad \text{Eq.7.2}$$

$$y_{21} = \frac{I_2}{V_1} \; \text{with } V_2 = 0 \quad \text{transfer admittance} \qquad \text{Eq.7.3}$$

$$y_{22} = \frac{I_2}{V_2} \; \text{with } V_1 = 0 \quad \text{output admittance} \qquad \text{Eq.7.4}$$

$$y_{12} = \frac{I_1}{V_2} \; \text{with } V_1 = 0 \quad \text{transfer admittance} \qquad \text{Eq.7.5}$$

As an example of how to use the above equations, consider the network shown in Figure 7.2. We want to determine the Y parameters using Eq. 7.2 through 7.5.

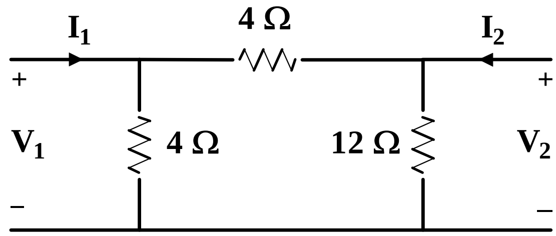


Figure 7.2: Circuit for determining two-port parameters.

We start by using the expression for I_1 from Equation 7.1:

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad \text{Eq. 7.6}$$

We want to find y_{11}. If we make $V_2 = 0$ then from Equation 7.6 we have,

$$y_{11} = \frac{I_1}{V_1} \qquad \text{Eq. 7.7}$$

To make $V_2 = 0$ we use the circuit of Figure 7.3.

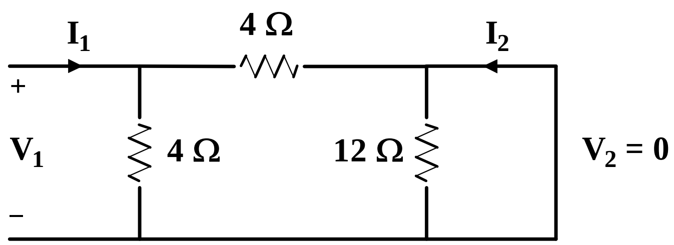


Figure 7.3: Circuit for finding y_{11} and y_{21}.

Obviously, from Figure 7.3 we have;

$$y_{11} = \frac{I_1}{V_1} = 0.5\,S \quad \text{and} \quad y_{21} = \frac{I_2}{V_1} = -0.25\,S \qquad \text{Eq. 7.8}$$

Similarly, we use the circuit shown in Figure 7.4 to find y_{12} and y_{22}.

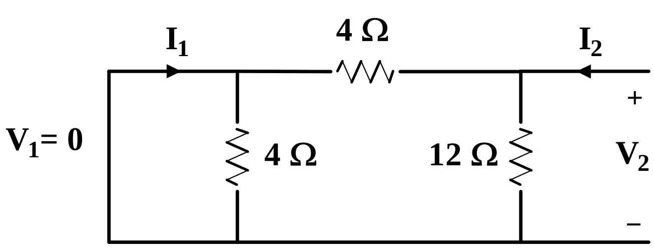


Figure 7.4: Circuit for finding y_{12} and y_{22}.

From Figure 7.4 we find,

$$y_{12} = \frac{I_1}{V_2} = -0.25\,S \qquad \text{and} \qquad y_{22} = \frac{I_2}{V_2} = \frac{1}{3}\,S \qquad \text{Eq. 7.9}$$

Note: in the laboratory if you want to determine, say $y_{22;}$ you short the terminals where V_1 is located; you <u>apply a known voltage for V_2,</u> say 5 V and measure I_2 with an ammeter connected in the circuit. Then y_{22} is the ratio of the I_2 current to the known input voltage V_2. Very easy to do. That is the power of the two-port network. The parameters are easy to measure.

We can summarize the above in matrix form as,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.333 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \text{Eq. 7.10}$$

The Z Parameters:

The Z parameters are defined from the equations shown below.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \text{Eq. 7.11}$$

We follow the same pattern and line of reasoning for finding the Z parameters as we did for finding the Y parameters. Thus,

$$z_{11} = \frac{V_1}{I_1} \quad \textit{with } I_2 = 0 \quad \textit{input impedance} \qquad \textit{Eq.}7.12$$

$$z_{21} = \frac{V_2}{I_1} \quad \textit{with } I_2 = 0 \quad \textit{transfer impedance} \qquad \textit{Eq.}7.13$$

$$z_{12} = \frac{V_1}{I_2} \quad \textit{with } I_1 = 0 \quad \textit{transfer impedance} \qquad \textit{Eq.}7.14$$

$$z_{22} = \frac{V_2}{I_2} \quad \textit{with } I_1 = 0 \quad \textit{output impedance} \qquad \textit{Eq.}7.15$$

Applying Equations 7.12 through 7.15 to the circuit of Figure 7.2 will give the following for the Z parameters.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{16}{5} & \frac{12}{5} \\ \frac{12}{5} & \frac{24}{5} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \text{Eq. 7.16}$$

Again, keep in mind that if you had the network of Figure 7.2, to determine z_{11} you would open the output terminals (I_2=0) , apply a know voltage for V_1, measure the resulting current I_1 and form the ratio of V_1 to I_1 and you have z_{11}. Easy!

The H Parameters:

The equations defining the H parameters are given below.

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad \text{Eq. 7.17}$$

The parameters are defined by the following conditions.

$$h_{11} = \frac{V_1}{I_1} \quad with\, V_2 = 0 \quad ss\ input\, impedance \quad (ss: short\, circuit) \qquad Eq. 7.18$$

$$h_{21} = \frac{I_2}{I_1} \quad with V_2 = 0 \quad ss\ forward\ current\ gain \qquad Eq. 7.19$$

$$h_{12} = \frac{V_1}{V_2} \quad with\, I_1 = 0 \quad os\, reverse\, voltage\ gain \qquad Eq. 7.20$$

$$h_{22} = \frac{I_2}{V_2} \quad with\, I_1 = 0 \quad os\, output\, admit\tan ce \quad (os: open\, circuit) \qquad Eq. 7.21$$

At one time these parameters were frequently used with transistor circuits. Manufactures would give the H parameters for particular devices.

The Transmission Parameters:

These parameters are the A, B, C, D parameters. The defining equations are given below.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad \text{Eq. 7.22}$$

Similar to previous schemes, the parameters are defined as follows:

$$A = \frac{V_1}{V_2} \quad with\, I_2 = 0 \qquad oc\ voltage\, ratio \qquad\qquad Eq.7.23$$

$$C = \frac{I_1}{V_2} \quad with\, I_2 = 0 \qquad -sc\ transfer\, impedance \qquad\qquad Eq.7.24$$

$$B = \frac{V_1}{-I_2} \quad with V_2 = 0 \qquad oc\ transfer\, admit\tan ce \qquad\qquad Eq.7.25$$

$$D = \frac{I_1}{-I_2} \quad with V_2 = 0 \quad -sc\ current\, ratio \qquad\qquad Eq.7.26$$

One can use mathematical analysis to relate one set of parameters to another set. A table showing how the sets are related is given below.

$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_H}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_H}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
$\begin{bmatrix} \frac{\Delta_Z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_Y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

Table 7.1: Two-port parameter conversion table.

As an illustration of how to use this table, consider the Y parameters that were determined in an earlier example for the circuit of Figure 7.2.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.333 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \text{Eq. 7.27}$$

Suppose we desire to convert these to Z parameters. We note from Table 7.1

$$\begin{matrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{matrix} \rightarrow \begin{matrix} \dfrac{y_{22}}{\Delta_y} & \dfrac{-y_{12}}{\Delta_y} \\ \dfrac{-y_{21}}{\Delta_y} & \dfrac{y_{11}}{\Delta_y} \end{matrix} \rightarrow \begin{matrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{matrix} \qquad \text{Eq. 7.28}$$

In other words,

$$z_{11} = \frac{y_{22}}{\Delta_y}, \qquad z_{12} = \frac{-y_{12}}{\Delta_y}, \qquad z_{21} = \frac{-y_{21}}{\Delta_y}, \qquad z_{22} = \frac{y_{11}}{\Delta_y} \qquad \text{Eq. 7.29}$$

In the above equations,

$$\Delta_y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{3} \end{vmatrix} = \frac{5}{48} \qquad \text{Eq. 7.30}$$

Then to convert to z_{11},

$$z_{11} = \frac{y_{22}}{\Delta_y} = \frac{\frac{1}{3}}{\frac{5}{48}} = \frac{1}{3} x \frac{48}{5} = \frac{16}{5} \qquad \text{Eq. 7.31}$$

Similar calculations are made for the remaining Z parameters.

As one last illustration of how one might use the two-port parameters, consider once again the circuit of Figure 7.2 except with sources added to the input and output terminals as shown in Figure 7.5. One is not restricted to adding sources. One might add a mix of source and resistor, for example. However, we add two sources for this illustration as shown below.

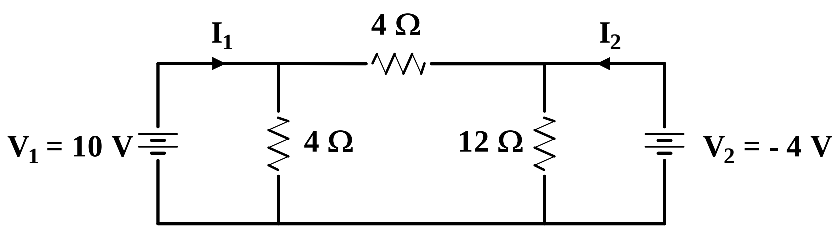


Figure 7.5: Circuit of previous illustration with sources added.

Earlier we saw that in terms of the Y parameters this network was described by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.333 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \text{Eq. 7.36}$$

We substitute the given values for V_1 and V_2 into Equation 7.36 and calculate I_1 and I_2. Thus,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.333 \end{bmatrix} \begin{bmatrix} 10 \\ -4 \end{bmatrix} \qquad \text{Eq. 7.37}$$

$$I_1 = \frac{5}{2}\ A \quad \text{and} \quad I_2 = \frac{-23}{6}\ A \qquad \text{Eq. 7.38}$$

EASY !!

Prelab Exercises

Complete the following exercises prior to coming to the lab. As usual, turn-in your prelab work to the lab instructor before starting the Laboratory Exercises.

Part IPE:

a) Calculate the Y parameters for the circuit given in Figure 7.6 by circuit analysis.

b) Use the conversion table provided to find the Z and ABCD parameters from the Y parameters.

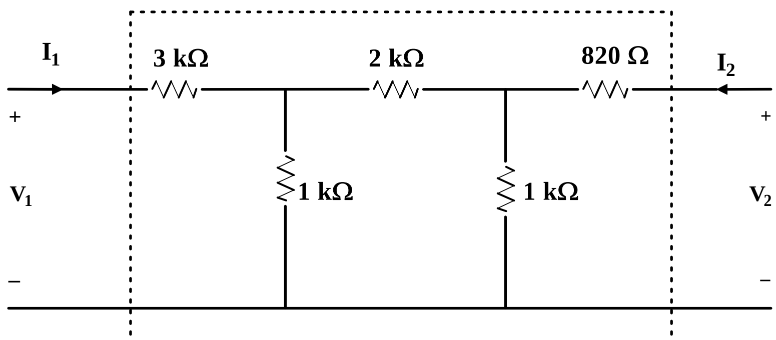


Figure 7.6: Circuit used for determining Y parameters, Part 1PE.

Part 2PE:

(a) In the circuit of Figure 7.6, make V_1 = 10 V and V_2 = 10 V. Use the Y parameters you determined in Part 1PE and calculate I_1 and I_2.

(b) Use the ABCD parameters to determine I_1 and I_2 with V_1 and V_2 as given in above.

Laboratory Exercises

In this laboratory exercise you will determine the Y, Z and ABCD parameters of the network of Figure 7.6 by direct measurement.

Part 1LE:

Place a short across the output port as shown in Figure 7.7. Measure and record V_1, I_1 and I_2. Use $V_1 = 10$ V.

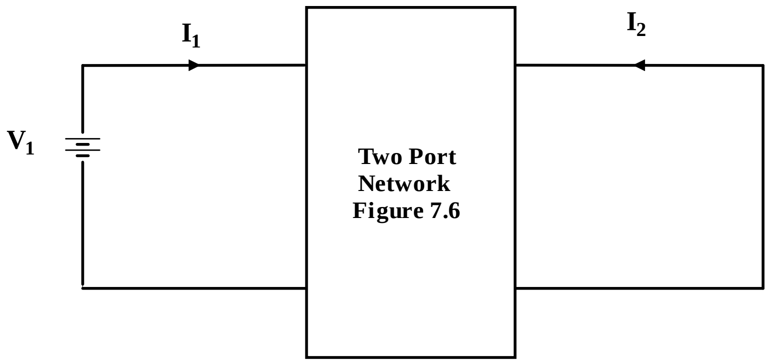


Figure 7.7: Measuring I_1 and I_2 for Part 1LE.

Part 2LE
Leave the output port open as shown in Figure 7.8. Measure and record V_2, and I_1. Use $V_1 = 10$ V.

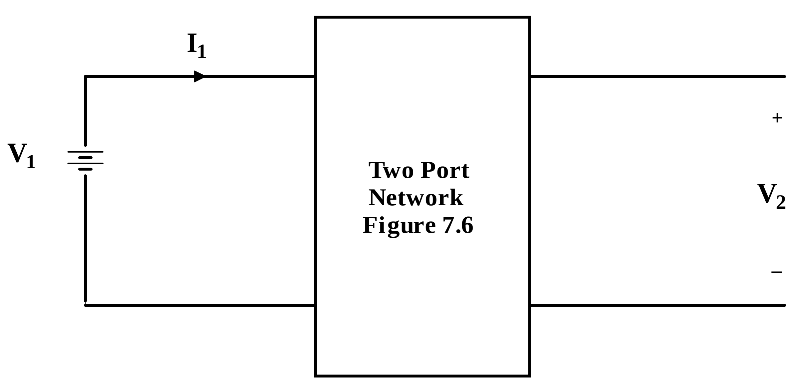


Figure 7.8: Measuring voltage and current for Part 2LE.

Part 3LE
Leave the input port open as shown in Figure 7.9. Measure and record V_1 and I_2.
Use V_2 = 10 V.

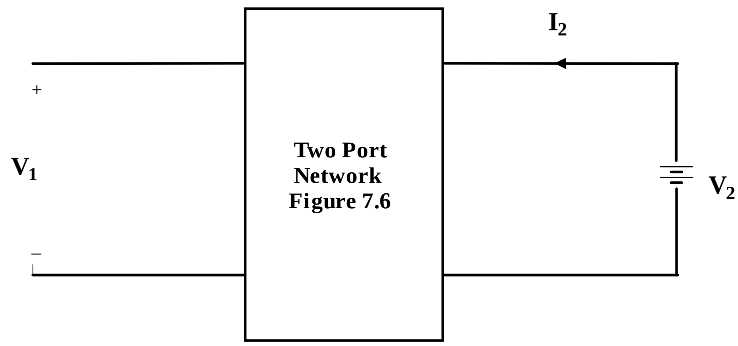


Figure 7.9: Circuit for measuring V1 and I_2 for Part 3LE.

Part 4LE:
Short the input port as shown in Figure 7.10. Measure I_1 and I_2. Use V2 = 10 V.

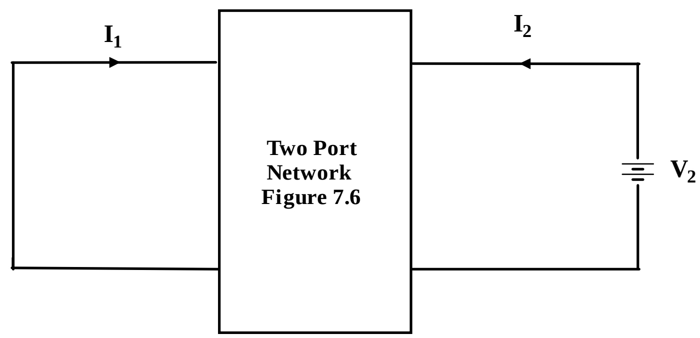


Figure 7.10: Circuit for measuring I_1 and I_2 for Part 4LE.

Before Leaving the Laboratory

Be sure the following is completed before you leave the laboratory.

(a) Check to be sure that you have all the required measured values.

(b) Have the laboratory instructor check your laboratory readings.

(c) Restore your laboratory station (equipment and chairs) to the condition they were in when you arrived. Remove any debris from the work area and floor.

Thank you for your cooperation.