A Robust Multi-resonant PR Regulator for Three-phase Grid-connected VSI Using Direct Pole Placement Design Strategy

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Abstract—An averaged switching model of grid-connected inverter using dual-loop current control with LCL-filter in discrete domain is built under stationary frame. A proportional resonant (PR) regulator is adopted in the current-loop to track the given fundamental sinusoidal current without steady state error. In addition, in order to reduce the low frequency harmonics, the resonance of 5th harmonics is adopted. Because the system is of high order and complex, the theory of root locus is proposed to analyze the influence of parameters in PR regulator, the influence of digital delay, and the LCL-filter. Based on the theoretical analysis, the poles can be properly selected to guarantee the stability of the system and the performance of current-loop during wide grid-fed power. Finally, a 10 kW prototype of grid-connected inverter with LCL-filter is set up to verify the effectiveness, practicality, and robustness of the proposed design method.

I. INTRODUCTION

The environmental pollution caused by consuming fossil fuels makes it vital to develop new technologies of renewable energy, such as wind energy, micro hydro energy and solar energy, whose share in distributed generation (DG) is increasing rapidly [1-7]. An inductor is usually used as an interface between grid-connected inverters and the grid, however LCL-filter is becoming an effective alternative for attenuating the current harmonics caused by pulse width modulation (PWM) recently [8][9]. By offering a bypass for high-frequency current with the additional capacitor branch, higher attenuation can be achieved by a LCL-filter than a pure inductor filter under switching frequency. However, the LCL-filter is a third order system and has resonance peak in the frequency response. The solutions for this problem can be separated into two types: passive damping method (normally, a damping resistance is in series with the capacitance of the LCL filter to achieve the attenuation [10]) and active damping method [11][12] (which is achieved through the improvement of controller to eliminate the resonance). Passive damping methods are simple, reliable, and are not affected by the control systems, however they have lower efficiency resulting from the additional loss of a damping resistor. While the active damping methods mainly include multi-loop control and zero-pole filter compensation.

It is a common and mature concept of active damping to use virtual resistance due to its simplicity and relatively high adaptability. However, its performance could be affected by the resonance parameters, the grid, and the controller as well. Sufficient stability margin should be provided when the damping strategy is adopted for a system with a LCL filter.

In terms of the controllers, there are several methods to enhance the system’s performance of grid-connected inverters [13][14]. Reference [15] analyzes the control of a DG inverter under rotating frame (d-q coordinates) using Park transformation, which converts the three-phase sinusoidal input signals to constant values under d-q coordinates and a proportional integrator (PI) regulators are adopted to eliminate the error. However, the strong coupling of currents under d-q coordinates makes it complicated to design the control strategy. In Reference [16], a control method under stationary frame (α-β coordinates) is proposed. The currents under α-β coordinates do not need to be decoupled, but these sinusoidal values make it impossible for the traditional PI regulators to track accurately without static error. In addition, a series of low frequency harmonics is introduced due to the dead-time of each bridge, however the PI regulator cannot eliminate this kind of harmonics.

In order to solve this problem, the proportional resonant (PR) is introduced [17][18]. The static error of the control signal at specific frequency points can be totally eliminated because the ideal PR regulator can offer infinite gain at these points. Furthermore, if the resonant controller at the 5th and 7th of fundamental frequency is adopted, the harmonics of these frequencies can be reduced or even eliminated, the quality of the current fed to the grid can be greatly improved.

However, it is difficult to design the PR regulators to make the whole system stable. On one hand, the PR regulator is one multi-order system, especially the multi-frequency PR regulator increases the system order; on the other hand, the LCL-filter is of three-order, therefore if the PR regulator is applied in this kind of system, the controller design is complicated. A PR regulator design method based on root-locus analysis theory is proposed [17], in which the proper PR parameters can be selected according to the root-locus trace to assure the system stability and dynamic response.

In this paper, based on the root-locus theory [19], one design strategy that is under stationary frame is proposed. Based on the averaged switching model of the main circuit in discrete domain, the influence of the controller parameters, LCL filter system parameters, and delays is analyzed to insure the system stability and robustness. Based on the above
analysis, the PR current regulator loop design method is proposed by using pole placements. In addition, in order to reduce the low frequency harmonics, the resonance of 5th harmonics is adopted.

One prototype of 10 kW was built to verify the proposed strategy. The parameters which are determined by pole placements in calculation are applied in the experiment. The current total harmonic distortion (THD) of the PR regulator with feedforward [20] is in comparison to that of a PI regulator with feedforward from 1/4 load to full load. Furthermore, the comparison of the 5th harmonics of current between PR with feedforward and PI with feedforward regulator is given. In addition, the dynamic response of grid current and harmonics analysis at full load are given. The THD of the fed current is about 2.2% at full load. The multi-resonant PR regulator can satisfy the THD requirement during wide grid-fed power. The validity and robustness of the proposed multi-resonant PR regulator is verified by the experimental results.

II. AVERAGED MODELING OF SWITCHING PERIOD AND CONTROL STRUCTURE FOR GRID-CONNECTED INVERTER

A. Topology of Three-phase Grid-connected Inverter and Control Strategy

The three-phase inverter system is shown in Fig. 1. An IGBT based three-phase voltage source inverter (VSI) is connected to the grid with an LCL-filter and works in bidirectional power flow mode. The LCL-filter design is discussed in reference [21].

As shown in Fig. 2, the control strategy used in this paper is presented. Instead of the grid side current, the inverter side current is regarded as the control target and fed to the PR regulator, and the currents are controlled under stationary frame. \( U_{\alpha f} \) and \( U_{\beta f} \) are the feedforward value of the grid and \( U_{dc} \) is the DC bus voltage for the controller [20].

The voltage of the DC bus is controlled by a PI regulator, the output of which is the given as the reference current of d coordinate, while the given current of q coordinate is determined manually to control the reactive power that flows into the grid. It is often set to zero to achieve unity power factor. After reversing Park transformation, the given current of stationary frame is achieved. Additionally, a phase-lock-loop (PLL) [22] is applied to get the phase and frequency of the grid requested by the Park transformation.

After the PR regulator, space vector PWM (SVPWM) is adopted to generate the drive signals of the IGBTs. SVPWM is well known for its advantage in DC voltage utilization and current ripple elimination over traditional sinusoidal PWM (SPWM).

B. System Configuration and Average Switching Model

According to Fig. 1, the following equations can be achieved with Kirchhoff's voltage and current laws.

\[
\begin{align*}
     e_k &+ i_{gk} z_{Lg} + i_{sk} z_{Ls} = U_k \\
     (i_{sk} - i_{gk}) z_c &= u_c \quad k = a, b, c \\
     u_c + i_{sk} z_{Ls} &= U_k
\end{align*}
\]

where,

\[
U_k = u_k + U_{DD}
\]

\[
u_k = s_k \frac{U_{dc}}{2}
\]

\[
s_k = \begin{cases} 
1 & \text{conducting of upper switch for bridge } k \\
-1 & \text{conducting of lower switch for bridge } k 
\end{cases}
\]

where, \( z_{Lg}, z_{Ls} \) and \( z_c \) are the impedance of \( L_g, L_s \) and \( R \) respectively, \( k \) is the \( k \)th phase, \( e_k \) is the grid voltage, \( U_k \) is the voltage between the output of the inverter and the ground, \( u_k \) is the voltage between the output of the inverter and neutral point D, and \( U_{DD} \) is the voltage between point D and the ground.

From (1), by eliminating \( i_{gk} \) and \( u_c \), the following equation can be obtained,

\[
e_k + \frac{z_{Lg} z_c + z_{Ls} z_c + z_{Lg} z_{Ls}}{z_c} i_{sk} = U_k \frac{z_c + z_{Lg}}{z_c}
\]

Since PWM control is a nonlinear system, it is difficult to get the system transfer function [23]. However, if the model is averaged over each switching period, the nonlinear switching state can be replaced by a linear value as below:

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Fig. 1. System topology of grid-connected inverter with LCL-filter.

Fig. 2. Control strategy of grid-connected inverter with LCL-filter under stationary frame.
\[ \langle e_i \rangle + \frac{z_1 z_{id} + z_1 z_{iu} + z_1 z_{ud}}{z_e} \langle i_u \rangle = \left( \frac{U_c}{2} \langle s_i \rangle + U_{do} \right) \frac{z_a + z_{sd}}{z_e} \] (6)

where, \( \langle s_i \rangle \) is the averaged value of duty-ratio for each switching period. \( \langle e_i \rangle \) and \( \langle i_u \rangle \) represent respectively the averaged value of the grid voltage and the current of inverter side during each switching period.

Applying Clarke transformation to (4), the equations under stationary frame are as follows,

\[
\begin{align*}
\langle e_i \rangle + \frac{z_1 z_{id} + z_1 z_{iu} + z_1 z_{ud}}{z_e} \langle i_u \rangle &= \frac{z_a + z_{sd}}{z_e} \langle s_i \rangle \\
\langle e_p \rangle + \frac{z_1 z_{id} + z_1 z_{iu} + z_1 z_{ud}}{z_e} \langle i_p \rangle &= \frac{z_a + z_{sd}}{z_e} \langle s_p \rangle 
\end{align*}
\] (7)

The current control diagram of the grid-connected inverter system with LCL-filter under \( \alpha-\beta \) coordinates can then be deduced as Fig. 3. \( G_{pr}(s) \), \( K_{PWM} \) and \( H(s) \) are respectively the PR controller, gain of PWM module, \( H(s) \) is the gain in the current feedback loop. \( i_s \) is the output current of the converter.

III. DESIGN OF PR REGULATOR

An ideal PR regulator has the form as following,

\[ G_{pr}(s) = K_p + \sum_{k=1}^{n} \frac{2K_{hi} \omega_h s}{s^2 + \omega_h^2} \] (8)

which gives an infinite gain at the frequency of \( \omega_h \) and has 180 degree phase shift. There is no phase shift or gain at other frequencies. In order to avoid the infinite gain, which will bring instability problems, the ideal PR regulator is often modified to (9) in practical implementation [24].

\[ G_{pr}(s) = K_p + \sum_{k=1}^{n} \frac{2K_{hi} \omega_h s}{s^2 + 2\omega_h s + \omega_h^2} \] (9)

Fig. 4 shows the frequency response of (9) in dash line. The multi-resonant regulator \( G_{pr} \) is designed to offer resonance at the point of 50 Hz, 150 Hz and 250 Hz. Due to the resonance at \( \omega_h \), where the non-ideal PR regulator provides a large gain, the closed loop of the system can have nearly zero static error at \( \omega_h \).

According to Fig. 3, due to the decoupling of the current in \( \alpha-\beta \) coordinates, the open loop transfer function after adopting PR regulator can be obtained as follows,

\[ \frac{i_{ref}}{s} = \frac{K_{PWM}}{s} \frac{G_{pr}(s)}{H(s)} \frac{z_a + z_{sd}}{z_e} \frac{z_1 z_{id} + z_1 z_{iu} + z_1 z_{ud}}{z_e} \frac{z_a + z_{sd}}{z_e} \]

Table I. LCL-filter Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_g )</td>
<td>1.8mH</td>
</tr>
<tr>
<td>( L_s )</td>
<td>2mH</td>
</tr>
<tr>
<td>( C )</td>
<td>4.7μF</td>
</tr>
<tr>
<td>( R )</td>
<td>2.2Ω</td>
</tr>
</tbody>
</table>
There is no neutral point in the system shown in Fig. 1, therefore there is almost no 3rd harmonics current. In this paper, a PR regulator with two resonant points will be applied in the controller, one is to eliminate the static error of fundamental components, and the other is to eliminate the 5th current harmonics. In order to select the proper controller parameters, namely \( K_p \) and \( K_{hi} \), the parameter related root locus of closed-loop transfer function is adopted in the following analysis. By adjusting one of the parameters each time, the influence of every parameter on system performance is analyzed [17]. The parameters range which assures the system stability can be obtained. According to the pole distribution, the PR parameters which help to achieve the desirable performance can be acquired.

### A. Influence of PR Regulator Parameters

The impact of PR regulator parameters in root locus is shown in Fig. 5. The poles, which are introduced by the digital calculation delay, are obtained by two order approximation. The poles of higher order introduced by the delay, which locate far away from the imaginary axis, have much less effect on the system and can be neglected. As illustrated in the figure, there are four couples of roots in the system. The first couple is introduced by the LCL-filter, the second one is introduced by the PR regulator of the fundamental line frequency, the third one is introduced by the PR regulator of the 5th fundamental line frequency, and the last one is introduced by the sampling-hold part and the LCL-filter.

When \( K_p \) increases, the previous three couples of poles move far away from the imaginary axis and then move close to the imaginary axis, the corresponding damp increases and then decreases. The last couple of poles are on the real axis and moving close to each other when \( K_p \) increases. Both of the poles move slowly; the impact of this couple is much less than the previous discussed couples. \( K_{hi} \) has less impact on the root locus than \( K_p \), making the poles move close to the imaginary axis, the corresponding damping is reduced. \( \omega_{hi} \) has similar impact on root locus of system as \( K_{hi} \), the locus of the system for the variation of these parameters are almost the same. \( K_{hi} \) and \( \omega_{hi} \) have the same effect on system root locus as \( K_{hi} \) and \( \omega_{hi} \), making the poles move close to the imaginary axis, the corresponding damp is reduced. It can be seen in the figure that the system poles enter into the right half plane when the parameters change, thus the system cannot work in stable state. The corresponding parameters’ ranges for each figure are given.

When the grid-connected inverter is controlled by a DSP, the time delay brought by the digital controller needs to be taken into account because it has a significant impact on the stability of the system. Fig. 6 shows the root locus of the closed-loop transfer function when the delay of DSP control is considered. The poles introduced by the LCL-filter and the PR regulator of fundamental frequency move rapidly close to the imaginary axis when the delay time increases, whereas, the poles introduced by PR regulator of 5th line frequency cannot be affected. It can be concluded from the figure that the parameters range which will bring the problem of system instability is enlarged, making the regulator and poles difficult to be designed and placed.

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Fig. 5. Root locus corresponding to the change of parameters of PR regulator
B. Influence of Digital Control Delay

Therefore, the digital control delay has a significant role in the stability characteristic.

C. Influence of LCL-Parameters on the Roots Locus

The system parameters may vary due to the change of temperature when the converter is working, which will affect the system performance significantly. Especially for the LCL-filter, the inductance, the capacitance and the resistance will be changed when the current and temperature increase. Therefore, LCL parameters should be taken into consideration to analyze their influence on system poles and the system robustness.

As shown in Fig. 7, the influence of the LCL-filter parameters on root locus is presented. \( R \) has a huge influence on the roots of the system. With the increasing of \( R \), the roots of the closed-loop system move away from the imaginary axis, which make the system more stable and the damping of the LCL-filter becomes larger. All of the poles except the ones introduced by the PR regulator of 5th line frequency move far away from the imaginary axis so that the damping of the poles are increased correspondingly. Therefore, with the increase of \( R \), the system stability can be improved, and the resonance of poles can be eliminated.

For \( L_s \) and \( L_g \), their impacts on the system poles are relative less. The root locus of the system changes unremarkably. From Fig. 7(b) and Fig. 7(c), with the increasing of \( L_s \), the system poles tend to move away from the imaginary axis, which improves the system stability and increases the damping so that the resonance can be suppressed. However, the increasing of \( L_g \) tends to move the poles close to the imaginary axis, which decreases the system stability. It is the same as the parameter \( R \) that \( L_s \) and \( L_g \) have less effect on the poles introduced by PR regulator of 5th line frequency.

D. Pole Placement and Selection of Parameters

In this paper, the LCL-filter is designed based on the introduction of reference [21]. The parameters of LCL-filter are shown in Table I.

First, based on the principle of stability, the system roots should be placed on the left half plane. Next, the margin of the system parameters should be kept enough to assure the stability even when LCL-filter parameters change. Finally, in order to suppress the resonance of the LCL-filter and improve the current quality, enough damping of the poles introduced by the LCL-filter should be kept to attenuate the resonance, meanwhile, making them move far from the imaginary axis and reduce the impact on the system. Therefore the proper parameters should be the one which helps the first couple of roots achieve the transition point (as shown in Fig. 5(a), where \( K_p \) is selected.). In order to increase the system gain at \( \omega_1 \) and \( \omega_5 \), \( K_{i1} \) and \( K_{i5} \) should be as large as possible to achieve the resonant peak as long as the system has enough stability margin.

In the system, \( T_d \) is determined to be 33.33us. Based on the principle discussed above, the parameters of the multi-PR regulator are determined as following:

\[
K_{i1}=50, \ \omega_{c1}=2, \ \omega_{c5}=10, \ K_p=0.045.
\]
Correspondingly, the roots of the closed-loop transfer function are chosen to be:

\[ P_{1,2} = -1536 \pm 2047j \quad P_{3,4} = -1763 \pm 6879j \]
\[ P_{5,6} = -22 \pm 1503j \quad P_7 = -9399 \quad P_8 = -22 \]

When \( L_g \) and \( L_s \) change between +/-25% of their respective static value, \( R \) changes between +/-30% of its static value; the system poles are placed as shown in Fig. 8. The system stability is achieved even when the parameters of LCL-filter varies within the above considered scope.

The controller proposed above is implemented in DSP board. In order to reduce the grid disturbance and the burden of the PR regulator, the voltage feedforward is adopted in the controller, which is analyzed in [20].

### IV. EXPERIMENTAL VERIFICATION

A 10 kW grid-connected inverter with LCL-filter prototype is built according to the design strategy proposed in this paper, as shown in Fig. 1 and Fig. 2. The parameters of the LCL-filter are listed in Table I and the PR regulator is designed as analyzed in section III. All the control algorithms, including the PR regulator and DC voltage control are implemented in TMS320F28335, which supports floating-point calculation. The carrier frequency of SVPWM is 15 kHz. The sampling and calculation frequency is 30 kHz, therefore the calculation delay time is about 33.33 us. The AC output is directly connected to the grid, and the input voltage is set to be 650 V.

The grid voltage and current waveforms of the half and full load are shown in Fig. 9(a) and Fig. 9(b) respectively. The current THD and the 5th component analysis at different load by adopting the multi-resonant PR regulator with feedforward are in comparison to those by adopting traditional PI regulator with feedforward, as shown in Fig. 10(a) and Fig. 10(b). The current THD at each load point by adopting PR regulator with feedforward is lower than that by adopting PI regulator with feedforward, as shown in Fig. 10(a). It can be seen in Fig. 10(b) that the 5th current harmonics component is significantly reduced. The dynamic response of the grid current is shown in Fig. 11. The harmonics analysis of full load is given in Fig. 12. It can be concluded that the proposed multi-resonant digitized PR regulator can improve the quality of the grid current. During wide range grid-fed power, the robustness of the controller is verified by the experimental results.
In this paper, based on the averaged switching model of the main circuit in discrete domain, the influence of the controller parameters, LCL filter system parameters and delays is analyzed to insure the system stability and robustness. Based on the above analysis, the PR current regulator loop design method is proposed by using pole placements. Furthermore, the resonance of 5th harmonics is adopted to reduce the low frequency harmonics.

One prototype of 10 kW was built to verify the proposed strategy. The parameters which are determined by pole placements in simulation and theoretical analysis are applied in the experiment. The current THD of PR regulator with feedforward is in comparison to that of PI regulator with feedforward from 1/4 load to full load. The comparison of 5th harmonics between PR with feedforward and PI with feedforward regulator is also given. In addition, the dynamic response of the grid current and the harmonics analysis at full load are given. The THD of the fed current is about 2.2% at full load. The multi-resonant PR regulator can satisfy the THD requirement during wide grid-fed power. The validity and robustness of proposed multi-resonant PR regulator is verified by the experimental results.

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