DEFINITIONS FOR NON-PERIODIC CURRENT COMPENSATION

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Abstract – This paper presents a new definition of non-active current, which is valid for single-phase and polyphase systems as well as for periodic and non-periodic waveforms. The definition is applied to a shunt compensation system, and different cases of non-periodic current compensation are studied. A variety of compensation characteristics of non-periodic currents and the rating requirements for the compensator are illustrated by simulation.

1. Introduction

The widespread use of non-linear loads and power electronics converters has increased the generation of non-sinusoidal and non-periodic currents and voltages in power systems. Generally, power electronics converters generate harmonic components whose frequencies are integer multiples of the system fundamental frequency. However, in some cases, such as cycloconverters and line-commutated three-phase thyristor-based rectifiers, the line currents may contain both sub-harmonics (frequency lower than fundamental frequency) and super-harmonics (frequency higher than fundamental frequency but not an integer multiple of it). These waveforms are considered as non-periodic, although mathematically the currents may still have a periodic waveform, but in any event, the period of the currents is not equal to the period of the fundamental voltage [1], [2].

An arc furnace is an example of a non-linear load that may draw rapidly changing non-sinusoidal currents from the source, that is, the current wave shape is constantly changing. Problems such as voltage flicker and harmonic penetration associated with arc furnace loads have been reported in several papers [3]-[5]. A transient disturbance may also be considered as one kind of non-periodic current from the compensation point of view, e.g., the sudden addition of a large load to the system such as starting a motor or a fault.

Several papers have dealt with the definition, characterization, and compensation of non-sinusoidal and non-periodic current/power [6]-[11]. However, most of the previous efforts have focused on the compensation of *periodic*, non-sinusoidal currents instead of *non-periodic* currents. The diversity of the features of non-periodic currents makes their compensation quite difficult, and theoretically, their compensation is very different from that of periodic distorted currents. However, in practice, these two cases may be quite similar to each other [11]. Generally speaking, after compensation, a sinusoidal source current with a constant rms magnitude is preferred for both cases.

If the conventional shunt active filter is used as the compensator, then for both cases it must inject all current components that are the difference between the desired source current and the demanded load current. The work here defines active and non-active currents from a compensation point of view and, in particular, examines the effect that different compensation objectives for various non-periodic currents have on the compensator's ratings requirements [12].

2. Definition of Non-Active Current

Instantaneous active power is defined as the time rate of energy generation, transfer, or utilization. It is a physical quantity and satisfies the principle of conservation of energy. For a single-phase circuit, it is defined as the instantaneous product of voltage and current:

$$p(t) = v(t)i(t) \tag{1}$$

For a polyphase circuit with M phases, each phase's instantaneous active power is still expressed as (1), and instantaneous total active power is the sum of the active powers of each the individual phases:

$$p(t) = \sum_{i=1}^{M} p_i(t) = \sum_{i=1}^{M} v_i(t) i_i(t) \tag{2}$$

Non-active power can be thought of as the useless power that causes increased line current and losses as well as greater generation requirements for utilities. For a single-phase circuit with inductors, capacitors, and/or nonlinear elements, non-active power is the power that circulates back and forth between the source and loads and yields zero average active power over one period of the wave $p(t)$. Therefore, the non-active power for single-phase circuits is based on average or rms values. For a polyphase circuit, non-active power is the power that circulates among the phases as well as the power that circulates back and forth between the source and load.

Some non-active power theories are based on average values and restricted to the frequency domain, while others are formulated in the time domain on an instantaneous base. No matter what mathematical means are used, the goal of these theories is to improve the power factor and to minimize power losses and disturbances by identifying, measuring, and eliminating the useless (non-active) power.

For a single or polyphase power system, a shunt compensator to minimize the nonactive power/current required of the source can be configured as shown in Fig. 1. The shunt compensator is assumed to consist only of passive components (inductor/capacitor) and/or switching devices and no external power source. As a consequence, (neglecting the compensator's power loss), conservation of energy requires the active power of the compensator average zero. In more detail, let $p_S(t)$, $p_L(t)$, and $p_C(t)$ denote the instantaneous power of the source, load and compensator, respectively, and their average values over a time interval T_C be given by

$$P_X(t) = \frac{1}{T_C}\int_{t-T_C}^{t} p_X(\tau)d\tau \text{ where } X = S, L, \text{ or } C. \tag{3}$$

Then

$$P_S(t) + P_C(t) = P_L(t) \tag{4}$$

and

$$P_S(t) = P_L(t),\ P_C(t) = 0 \text{ as } t \to \infty. \tag{5}$$

In (3), T_C is the averaging interval which may be zero, one-half of the voltage's fundamental cycle, one full fundamental cycle, or multiple cycles, depending on the compensation objectives and the passive components' energy storage capacity. The subscripts "S", "L", and "C" denote the source, load, and compensator quantities, respectively, as shown in Fig. 1. Equations (3), (4) and (5) must hold true regardless of whether the system is single-phase or polyphase and regardless of whether the compensation is by passive or active means. Based on these physical and practical limitations, non-active power/current can be defined and formulated.

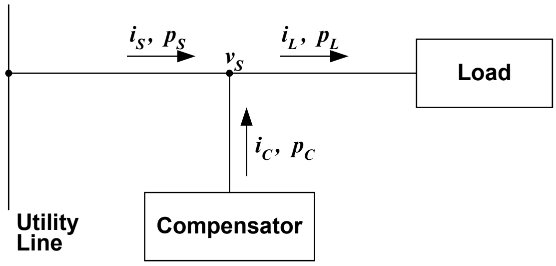


Fig. 1: A shunt compensator configuration.

The definition in this paper is an extension of Fryze's idea of non-active current/power [9]. The instantaneous active current $i_p(t)$ and non-active current $i_q(t)$ are:

$$i_p(t) \equiv \frac{P_L(t)}{V_P^2(t)} v_P(t), \qquad i_q(t) = i_L(t) - i_p(t), \tag{6}$$

where $P_L(t)$ is the average load active power over the interval $[t-T_C, t]$, and $V_P(t)$ is the rms value of the reference voltage $v_p(t)$ over the interval $[t-T_C, t]$:

$$P_L(t) = \frac{1}{T_C}\int_{t-T_C}^{t} p_L(\tau)d\tau = \frac{1}{T_C}\int_{t-T_C}^{t} v(\tau)i(\tau)d\tau = \frac{1}{T_C}\int_{t-T_C}^{t} v(\tau)i_p(\tau)d\tau \tag{7}$$

$$V_P(t) = \sqrt{\frac{1}{T_C}\int_{t-T_C}^{t} v_P^2(\tau)d\tau} \tag{8}$$

The technique is to simply have the compensator provide $i_q(t)$ so the source need only provide $i_p(t)$. The average non-active power is defined as:

$$Q(t) = \frac{1}{T_C}\int_{t-T_C}^{t} v(\tau)i_q(\tau)d\tau \tag{9}$$

where $Q(t) = P_C(t) = 0$, as $t \to \infty$.

$v_P(t)$ is the reference voltage whose specification depends on the compensation objectives. For example, this specification can be the terminal voltage $v_s(t)$ itself or it may be the fundamental component of $v_s(t)$ (i.e., $v_P(t) = v_f(t)$ where $v_s(t) = v_f(t) + v_h(t)$ and $v_f(t)$ is the fundamental and $v_h(t)$ is the harmonic component). The definitions (7) and (8) are valid for single- and polyphase circuits. However, in the case of polyphase circuits, the voltages and currents are expressed in vector form, which for a three-phase system is

$v = [v_a, v_b, v_c]^T$,

$i = [i_a, i_b, i_c]^T$, and

$v^2 = [v_a, v_b, v_c] \cdot [v_a, v_b, v_c]^T = v_a^2 + v_b^2 + v_c^2$.

It was shown in [12] that this new definition has the following features: 1) flexibility to meet many different compensation objectives; 2) valid for non-sinusoidal and non-periodic systems; and 3) valid for single phase and polyphase systems. Table I illustrates that by choosing different voltage reference and time averaging intervals, different source currents will result. Because of its flexibility in regards to compensation objectives, this definition is quite suitable for the mitigation of non-periodic currents. This will be shown in the next section.

Table I. Parameters for Different Compensation Objectives

Compensation Objective	v_P	T_C	Resulting Source Current
Single-phase or polyphase reactive current	v	$T/2$ or T	Unity pf and sinusoidal for sinusoidal v_s
Single-phase or polyphase reactive current and harmonic current	v_f	$T/2$ or T	Sinusoidal regardless of v_s distortion
Instantaneous reactive power for polyphase system	v	$T_C \rightarrow 0$	Instantaneously unity pf for polyphase system
Non-periodic disturbance current	v_f	nT	Reduced amplitude and near sine wave with unity pf
Subharmonic current	v_f	nT	Pure sine wave or smoothed sine wave with unity pf
Stochastic non-periodic current	v_f	nT	Smoothed sine wave with near-unity pf

3. Compensation of Periodic Currents

For compensation of single-phase periodic currents with fundamental period T, using a compensation period T_C that is a multiple of T/2 is enough for complete compensation of the reactive component of the load current such that the source current has a unity power factor. This is seen by examining (8) and noting that the average rms value of a periodic quantity does *not* depend on the time averaging interval T_C if it is an integer multiple of $T/2$. Even if the system consists of harmonics and/or unbalanced components in voltages and/or currents, the system can be completely compensated. Using the fundamental positive-sequence components of the phase voltages as reference voltages, the source currents will follow the reference voltages by choosing $T_C = nT/2$, where n is an integer.

As illustrated in Fig. 2, both the system voltage v_S and load current i_L are unbalanced and have harmonics. The source current i_S is sinusoidal (in phase with the reference voltage) and is balanced as indicated by the zero neutral current i_n after compensation. Fig. 2 also shows that the average powers satisfy $P_S(t) = P_L(t)$ and $P_C(t) = 0$. Note that with the compensator, the amplitude and variation of the instantaneous source power p_S is much smaller than that of the load power p_L.

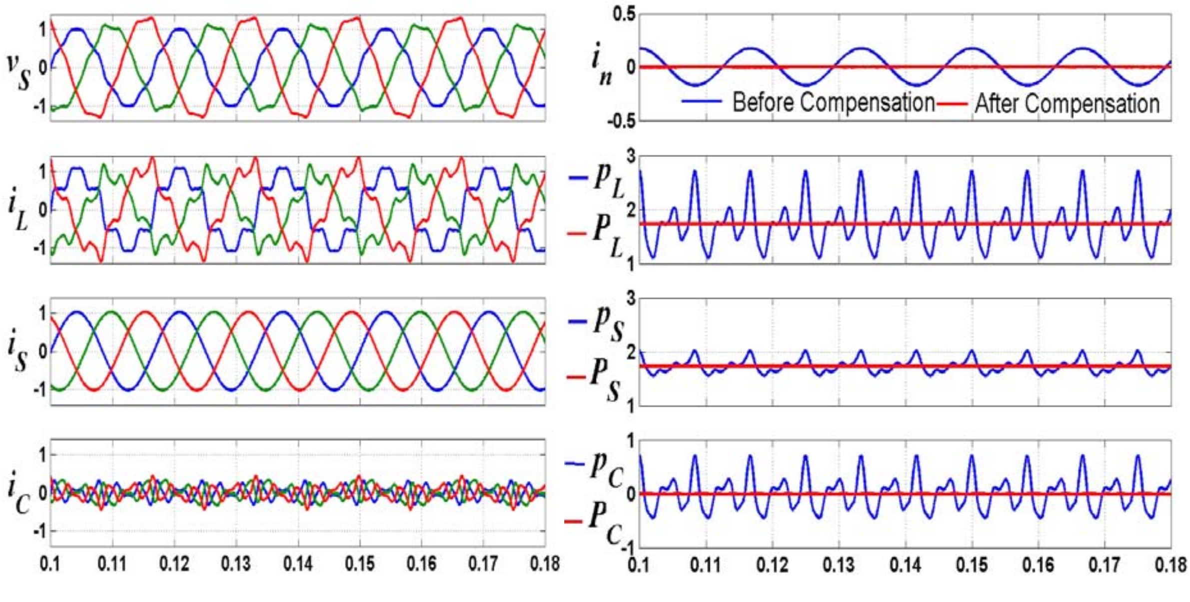


Fig. 2: Three-phase periodic current compensation ($T_C = T/2$).

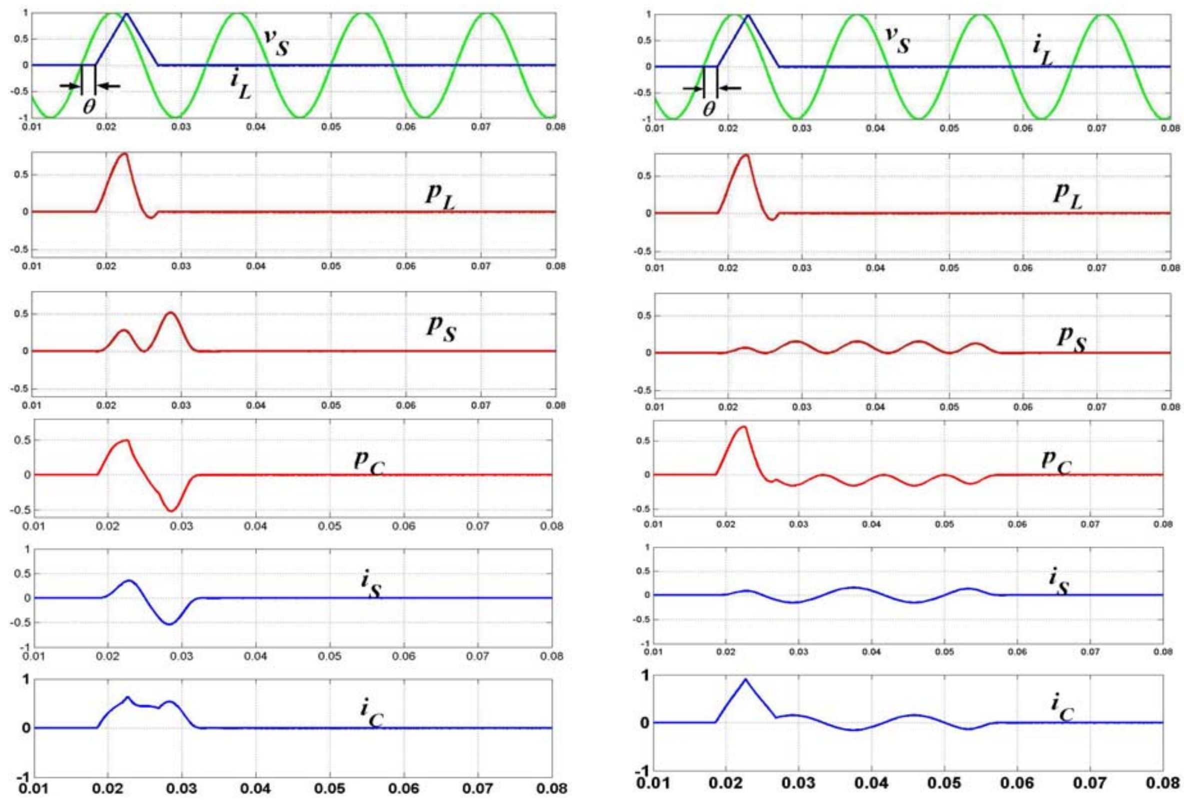
4. Compensation of Non-Periodic Currents

For the purposes of discussion, in this paper non-periodic currents are those currents without a period or those currents whose fundamental period differs from that of the voltage waveform. These currents include load current that has harmonics whose frequency is lower than the voltage fundamental frequency f_f (a subharmonic), is not a multiple of f_f, or it contains some non-periodic components. Unlike in periodic compensation where T_C does not have much influence on the compensation as long as it is a multiple of ½ period of the voltage fundamental frequency period, here the choice of T_C is a critical factor as will be shown in the following sections.

Theoretically, the time averaging interval can be chosen as any arbitrary value in the case of non-periodic currents. However, it is desirable for the interval to be an integer multiple of the line frequency period because of the desire that the source current be sinusoidal and have the same frequency as the source voltage frequency. In general, the period of the line voltage is not the same as the period of a quasi-periodic load current, or there is no period in the case of non-periodic load current. Thus, choosing different T_C will result in quite different source currents and compensator currents. Simulation results of the compensation of three different kinds of non-periodic currents are given in the following subsections.

4.1 Non-periodic Disturbance Currents

The duration of a non-periodic current may be a fraction of the line frequency cycle, or it may be several cycles. Outside of this time period, the current may be zero or a pure sine wave. Fig. 3 shows the simulation result of a single-phase disturbance current (pulse of duration that is ½ of the voltage period) for two different compensation cases: T_C = $T/2$ (Fig. 3a) and T_C = $2T$ (Fig. 3b) where $v_P = v_f$ for both cases. θ is the phase angle between the source voltage and initiation of the load current pulse, and in this case, θ =30° as shown in Fig. 3.



(a) (b)

Fig. 3: Simulation results for disturbance type non-periodic current compensation (a) T_C = $T/2$, (b) T_C = $2T$.

Because the disturbance energy is a fixed value, choosing a different time averaging interval T_C results in different average active power P_L; thus, a different magnitude of source current and compensator current will result. Larger values of T_C result in smaller peak values of $|i_s|$, i.e., the smaller disturbance seen from the source side. However, the compensator current rating will increase accordingly. Compensator design engineers must weigh the tradeoff between minimizing the source current against the cost of additional energy storage devices (capacitance) to accomplish this.

The simulations show that by increasing T_C from $T/2$ to $2T$, the source current decreases significantly (from 60% to 20% of the magnitude of the load current) without a significant increase in the compensator current (from 60% to 90% of the magnitude of the load current). Thus, one can substantially decrease the source current with what may be a cost-effective increase in the compensator energy storage requirements. While these simulations are done for a single-phase case, the same result would be expected in three-phase cases.

4.2 Subharmonic or Quasi-periodic Currents

The main feature of this group of non-periodic currents is that the currents may have a repetitive period. The currents generated by power electronics converters may fall into this group. In the simulation shown in Fig. 4, the quasi-periodic current is composed by adding one subharmonic component (12 Hz) to the fundamental current (60 Hz), and the total harmonic distortion (THD) of load current is 26.7%.

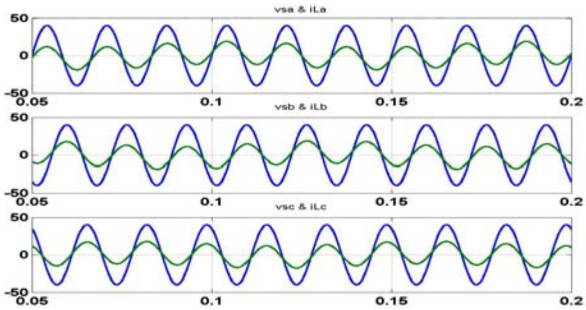
When the fundamental frequency f_f of the source voltage is an odd multiple of the subharmonic frequency f_{sub}, the minimum T_C for complete compensation is ½ of the common period of both f_f and f_{sub}. When f_f is an even multiple of f_{sub}, the minimum T_C for complete compensation is the common period of both f_f and f_{sub}.

Figs. 4b and 4c show the source current with compensation initiating at t = 100 ms. The subharmonic frequency is 12 Hz and the fundamental is 60 Hz, which is 5 times that of the subharmonic frequency. So the subharmonic component can be completely compensated by choosing $T_C = 2.5T$, and the source current is a pure sine wave. Choosing T_C smaller than ½ of the common period will result in a source current that still contains some subharmonic components. When $T_C = T$, the THD of source current is 2.8% (Figure 4b) because not all of the subharmonic component is eliminated; however, the source current THD has been reduced by 89.5% compared to the THD of the load current.

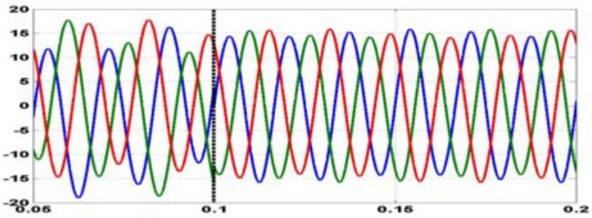
4.3 Stochastic Non-Periodic Currents

The load currents of arc furnaces are typically quite irregular as seen by the simulated arc furnace current shown in Fig. 5a. Thus, it is impossible to choose a specific T_C to make the source current be a sine wave. In the case of non-periodic waveforms, one can mathematically consider the period to be infinite. As can be seen from equations (3) and (4), as T_C goes to infinity, both P_L and V_p become constant and i_p tracks the voltage reference v_p. If v_p is chosen as the fundamental component of the source voltage v_f, then the source current will become sinusoidal. However, choosing the time interval to be infinite is not feasible in a practical application. However, it may still be possible to find some repetitive period in the current waveform that has most of the rms content of the waveform. Choosing that period as T_C may result in an acceptable source current which is quite close to a sine wave.

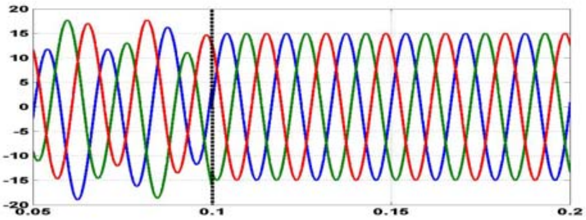
In Fig. 5a, the THD of the load currents i_{La}, i_{Lb}, is i_{Lc} are 75.7%, 47.5%, and 52.7%, respectively. When $T_C = T$ (Figure 6b), the THD of the source current is 6.9%. All of the harmonics with frequencies that are an integral multiple of the fundamental frequency are completely eliminated, and harmonics with frequencies that are not an integral multiple of the fundamental frequency are also mitigated. When T_C increases to $10T$, the source current is closer to a sine wave, with the THD of 0.6%; harmonics are compensated to a low level. In this way, the voltage flicker and harmonic penetration problem [4], [5] associated with this non-periodic current waveform can be mitigated.



(a) 3-phase load current and voltage waveforms.

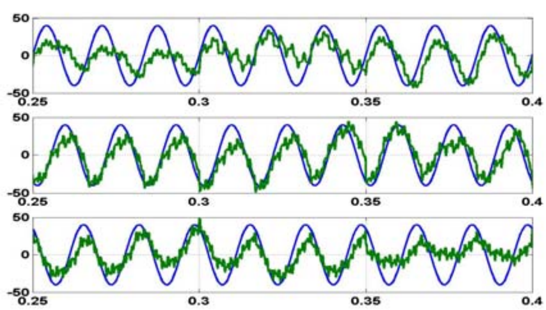


(b) Source currents, compensation at t=100 ms ($T_C = T$)

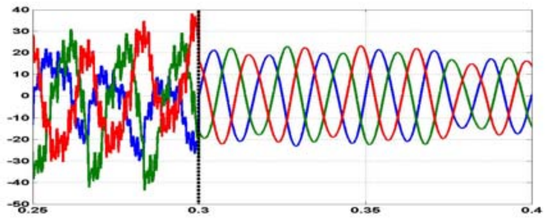


(c) Source currents, compensation at t = 100 ms ($T_C = 2.5T$)

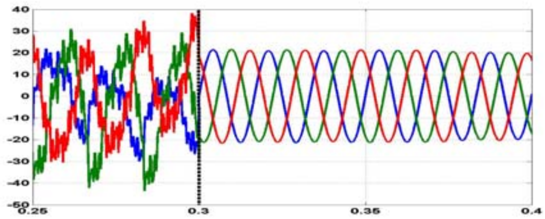
Fig. 4: Subharmonic current compensation.



(a) 3-phase load current and voltage waveforms.



(b) Source currents, compensation at t=300 ms ($T_C = T$)



(c) Source currents, compensation at t = 300 ms ($T_C = 10T$)

Fig. 5: Stochastic current compensation.

5. Compensator Ratings

The energy storage requirement determines how much passive components (inductors and/or capacitors) are needed, whereas the instantaneous power requirement determines voltage and current ratings of the active devices to be used for the compensator. The definition represented as (7) may be interpreted as the time average active power during T_C that is contributed by the active source current i_p where the average active power of the compensator $P_C(t)$ over T_C is assumed to be zero.

$$P_C(t) = \frac{1}{T_C} \int_{t-Tc}^{t} v^T(\tau) i_c \, d\tau = 0 \text{ , with } i_c = i_q \qquad (10)$$

It has been shown in [12] that the compensator average active power defined by (10) is zero. However, during the interval T_C, the instantaneous active power $p_C(t)$ is not necessarily zero. The capacitor is charged or discharged at different time intervals, and therefore a certain amount of energy storage is required. Different requirements will exist for energy storage by the active filter's capacitor to fulfill different compensation tasks. The maximum energy stored in the capacitance occurs at t_{max} when the capacitor changes from charge to discharge mode:

$$\Delta E = \int_0^{t_{\max}} v^T i_q \, dt \qquad (11)$$

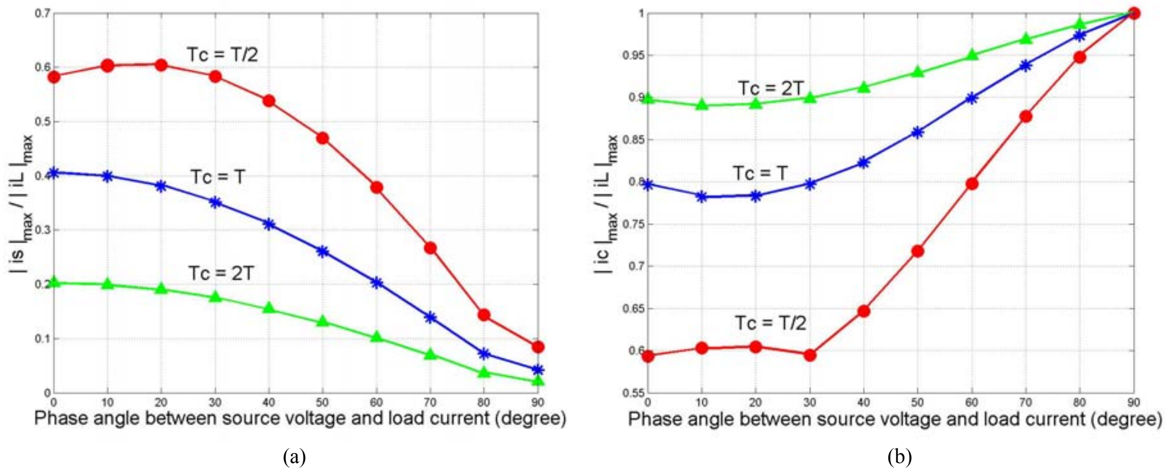


Fig. 6: (a) Peak source current and (b) peak compensator current normalized with respect to load current for different compensation times and load current phase angles in non-periodic disturbance compensation.

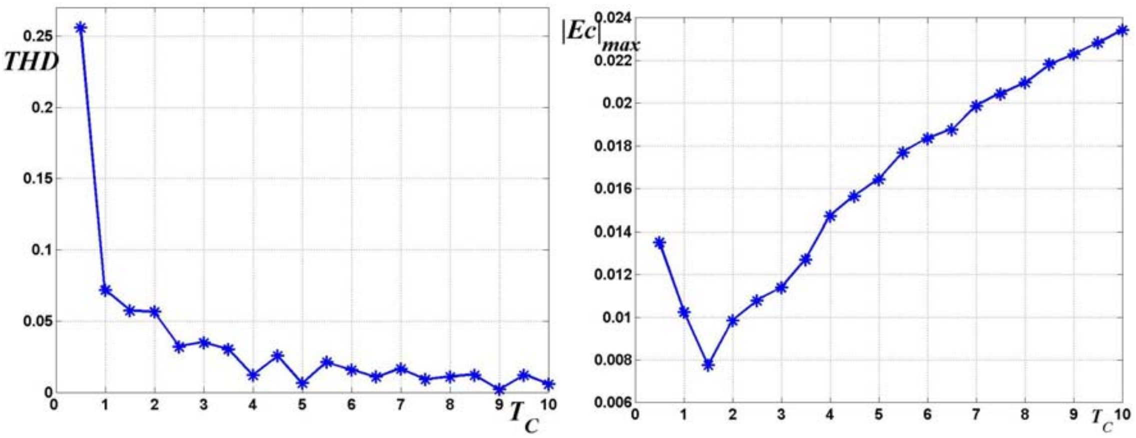
The choice of the time averaging interval T_C is also significant in an energy storage design consideration. Choosing a longer T_C results in a smoother source current with smaller amplitude; however, this requires that the maximum required instantaneous compensator current i_c increase as well as the energy storage requirement of the compensator.

Fig. 6 shows the peak source current normalized with respect to load current for various compensation intervals and phase angle θ between the initiation of the current pulse and source voltage for a triangular pulse waveform such as that shown in Fig. 3. With the averaging interval T_C changing from $T/2$ to $2T$, i_s decreases from 0.6 p.u. to 0.2 p.u. of the load current while i_c increases only modestly from 0.6 to 0.9 p.u. of the load current.

When the phase angle between the source voltage and pulse current is small, which is common for many systems, a small increase in energy storage capacity will result in a much better compensation (i.e., much smaller source current amplitude). In the case of a large phase angle difference, i_c increases approximately 70% (from 0.6 to 1.0 p.u.) from the current required for small phase angle differences for a compensation period of $2T$. However, i_c increases only 10% (from 0.9 to 1.0 p.u. of the load current) for a compensation period of $2T$. This is because if the compensator uses $T_C = 2T$, it must have a larger energy storage capacity (relative to using $T_C = T/2$) that is ready for not only a longer compensation interval, but also a larger instantaneous reactive component. Thus, a compensator with larger energy storage can obtain a better compensation and support a load that has a large reactive component.

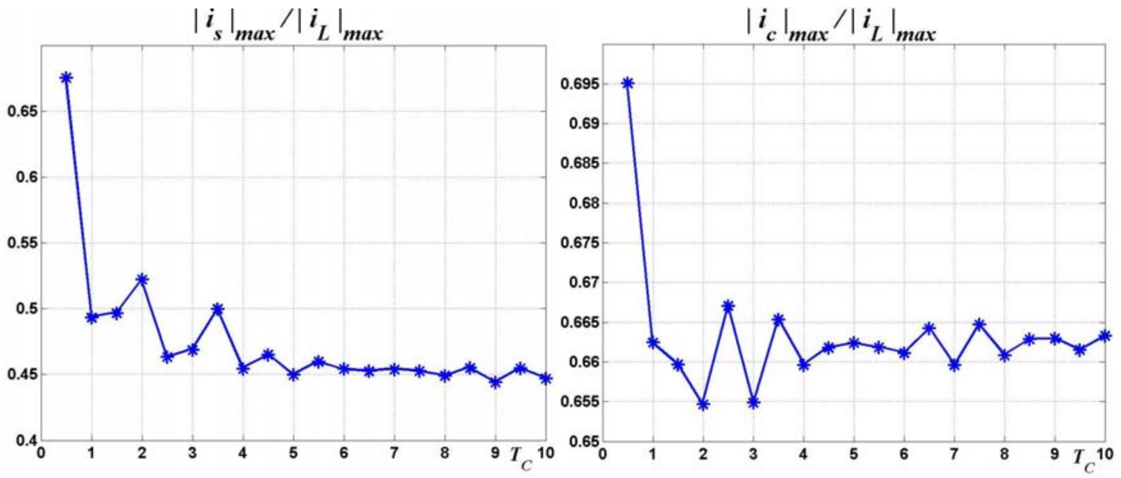
Fig. 7 shows the total harmonic distortion of source current and the compensator's energy storage requirement for different compensation times T_C with stochastic current compensation such as the example shown in Fig. 5. The THD of the load current is 57.3%. With T_C increasing from $0.5T$ to $10T$, the THD of the source current decreases from 25.6% to less than 1% with a rapid decrease for slightly longer compensation periods when T_C is less than $3T$. The THD of the compensated current is lower than 5% when T_C is longer than $2.5T$. Note that in Fig. 7b, the energy requirement of the compensator first decreases as T_C increases from $0.5T$ to $1.5T$, reaches its minimum value at $1.5T$, and then steadily increases for T_C greater than $1.5T$.

Fig. 8 shows the peak source current and peak compensator current as a function of T_C with the stochastic current compensation example shown in Fig. 5. Both i_S and i_C decrease rapidly for longer compensation periods when T_C is less than $2T$. For T_C greater than $4T$, i_S and i_C vary only slightly from average values of 0.45 p.u. and 0.66 p.u. of the load current, respectively.



(a) (b)

Fig. 7: (a) THD of source current and (b) compensator's energy storage requirement for different compensation time T_C for stochastic current compensation



(a) (b)

Fig. 8: (a) Peak source current and (b) peak compensator current plotted as a function of compensation time T_C for stochastic current compensation

Ideally, minimum THD and smooth i_S are desired, but considering the energy storage requirement of the compensator, a tradeoff between better compensation and small energy storage requirements must be made. From Figs. 7 and 8, we can see that when $T_C = 4T$, the THD of the source current is 1.2%; additionally, the peak source current and compensator current are close to their minimum values. If T_C is increased to more than $4T$, the energy storage increases almost linearly, while the THD of source current shows little improvement. Considering the overall requirements of both technology and economics, an optimal T_C can be found.

6. Conclusions

By combining a new non-active current definition and the conventional shunt active power filter, the application presented in this paper accomplishes the compensation of a variety of non-periodic currents in power systems. Simulation results give credibility to the applicability of the definition for a diversity of load currents. According to different compensation cases and the goals to be achieved, different averaging time intervals for the compensator are chosen, which will determine the compensator's energy storage requirement and the extent of residual distortion in the source current.

References

[1]. E. H. Watanabe, M. Aredes. Compensation of non-periodic currents using the instantaneous power theory. IEEE Power Engineering Society Summer Meeting, Seattle, USA, 2000, pp. 994-999.

[2]. L. S. Czarnecki. Non-periodic currents: their properties, identification and compensation fundamentals. IEEE Power Engineering Society Summer Meeting, Seattle, USA, 2000, pp. 971–976.

[3]. G. Carpinelli, M. DiManno, P. Verde, E. Tironi, D. Zaninelli. Ac and dc arc furnaces: a comparison on some power quality aspects. IEEE Power Engineering Society Summer Meeting, Edmonton, Canada, 1999, pp. 499-506.

[4]. S. R. Mendis, M. T. Bishop, J. F. Witte. Investigations of voltage flicker in electric arc furnace power systems. IEEE Industry Applications Magazine, vol. 2, no. 1, Jan./Feb. 1996, pp. 28-34.

[5]. M. M. Morcos, J. C. Gomez. Flicker sources and mitigation. IEEE Power Engineering Review, vol. 22, no. 11, Nov. 2002, pp. 5-10.

[6]. H. Akagi, Y. Kanazawa, A. Nabae. Instantaneous reactive power compensators comprising switching devices without energy storage components. IEEE Transactions on Industry Applications, vol. 20, no. 3, May/June 1984, pp. 625-630.

[7]. L. Rossetto, P. Tenti. Evaluation of instantaneous power terms in multi-phase systems: techniques and application to power conditioning equipment. ETEP, vol. 4, no. 6, Nov./Dec. 1994.

[8]. F. Z. Peng, J. S. Lai. Generalized instantaneous reactive power theory for three-phase power systems. IEEE Transactions on Instrumentation and Measurement, vol. 45, no. 1, Feb. 1996, pp. 293-297.

[9]. L. M. Tolbert, T. G. Habetler. Survey of active and non-active power definitions. IEEE International Power Electronics Congress, Acapulco, Mexico, 2000, pp. 73-79.

[10]. H. Kim, F. Blaabjerg, B. Bak-Jensen, J. Choi. Instantaneous Power Compensation in Three-Phase Systems by Using p-q-r Theory. IEEE Transactions on Power Electronics, vol. 17, no. 5, Sep. 2002, pp. 701-710.

[11]. H. Akagi. Active filters and energy storage systems operated under non-periodic conditions. IEEE Power Engineering Society Summer Meeting, Seattle, USA, 2000, pp. 965-970.

[12]. F. Z. Peng, L. M. Tolbert, Z. Qian. Definitions and compensation of non-active current in power systems. IEEE Power Electronics Specialists Conf., Cairns, Australia, 2002, pp. 1779-1784.