Critical Evaluation of FBD, PQ and Generalized Non-Active Power Theories

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Abstract

Due to the widespread use of non-sinusoidal, non-periodic loads and the existence of distorted voltages, many definitions of non-active power for non-sinusoidal and non-periodic waveforms have been formulated. This paper investigates the major similarities and discrepancies of three non-active power theories which are widely used in control algorithms for shunt compensation systems. The three approaches are FBD, PQ and generalized non-active power theories. The evaluation and comparison of these three theories as the non-active power compensation strategies for single-phase system, three-phase system with unbalanced load, sub-harmonic load, and distorted system voltage will be included in this paper. The conclusions are based on both the simulation and the experimental results of different compensation objectives.

I. Introduction

One of the main points in the development of alternating current (ac) transmission and distribution power systems at the end of the 19th century was based on sinusoidal voltage at constant-frequency generation. Hence, the conventional power theory, based on active, non-active and apparent power definitions, was sufficient for design and analysis of power systems. However, with the widespread use of nonlinear loads and electronic power converters, non-sinusoidal and non-periodic loads are becoming more common in today's electrical systems. These require the improvement and adaptation of non-active compensator technology and metering techniques, which are based on the new non-active power definitions and current decomposition methods, under such conditions.

Two important approaches to power definitions under non-sinusoidal conditions were first formulated by Budeanu, in 1927, and Fryze, in 1932. Fryze defined power in time domain, whereas Budeanu did it in frequency domain [1]. Other non-active power theories, which satisfy the major requirement of non-active compensator technology and metering techniques under such conditions, were derived from them in a few decades. In general, power definitions in the time domain offer a more robust basis for the use in a controller for power electronic devices, because they are also valid during transients [2].

Depenbrock extended Fryze's definition of non-active power and current decomposition method for a single phase to a multiphase system, known as FBD theory [3]. In 1983, Akagi, Kanazawa and Nabae introduced a novel concept, the PQ theory, by applying Park transformation, for the control of active filters connected to three-phase three-wire systems including some notes for four-wire systems [4-6]. Both FBD theory and PQ theory are widely used in the compensation technologies nowadays. More recently, a generalized non-active power definition was proposed [7-9]. Several papers have dealt with the comparison of different theories. However, some of them focus on theoretical analysis [10-12]; some do not include the generalized non-active power definition [13-15]. This paper investigates the major similarities and discrepancies of FBD, PQ, and generalized non-active power theories used for non-active power compensation in different cases, by simulation and experimental results.

This paper starts with the brief introduction of FBD, PQ, and generalized non-active power theories in Section II. The analysis and comparison of these theories in a single-phase system, a three-phase system with unbalanced load, sub-harmonic load, and distorted system voltage, based on simulation results, are given in Section III. The experimental results are provided in Section IV. The final conclusions are discussed in the last section.

II. Non-active power theories

A. FBD theory

Depenbrock extended Fryze's definition of non-active power and current decomposition method for a single phase to a multiphase system with *m* phases. He incorporated some of the apparent power theory published by Buchholz [16]. Multidimensional voltage and current vectors are $\vec{v}$, $\vec{i}$. Buchholz defined the instantaneous collective value of voltages (v_Σ) and currents (i_Σ) in a system with *m* phases as

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}, \ \vec{i} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_m \end{bmatrix} \tag{1}$$

$$v_\Sigma = \sqrt{\sum_{\mu=1}^{m} v_\mu^2}\ , \ i_\Sigma = \sqrt{\sum_{\mu=1}^{m} i_\mu^2} \tag{2}$$

Depenbrock defined collective instantaneous power p_Σ and instantaneous collective power current $i_{\Sigma p}$ as

$$p_\Sigma = \sum_{\mu=1}^{m} v_\mu i_\mu = \sum_{\mu=1}^{m} p_\mu\ , \ i_{\Sigma p} = \frac{p_\Sigma}{v_\Sigma^2} v_\Sigma \tag{3}$$

He then defined instantaneous phase power current $i_{\mu p}$ and phase zero power current $i_{\mu z}$ as

$$i_{\mu p} = \frac{p_\Sigma}{v_\Sigma^2} v_\mu\, , \ i_{\mu z} = i_\mu - i_{\mu p} \qquad (\mu = 1, 2, \cdots, m) \tag{4}$$

Depenbrock also decomposed the power current $i_{\mu p}$ into active current $i_{\mu a}$ and variation current $i_{\mu v}$, which depend on the mean values of p_Σ and v_Σ over one system voltage period, $\overline{p_\Sigma}$ and $\overline{v_\Sigma}$.

$$i_{\mu a} = \frac{\overline{p_\Sigma}}{\overline{v_\Sigma^2}} v_\mu\, , \ i_{\mu v} = i_{\mu p} - i_{\mu a} \tag{5}$$

Then the non-active current $i_{\mu n}$ could be written as

$$i_{\mu n} = i_\mu - i_{\mu a} = i_{\mu z} + i_{\mu v} \tag{6}$$

So the phase current i_μ could be decomposed as

$$i_\mu = i_{\mu p} + i_{\mu z} = i_{\mu a} + i_{\mu v} + i_{\mu z} = i_{\mu a} + i_{\mu n} \tag{7}$$

B. PQ theory

The PQ theory is based on the Park transformation of voltages and currents in three-phase systems (*a*, *b*, *c*) into ($\alpha, \beta, 0$) orthogonal coordinates. The phase voltages in the (*a*, *b*, *c*) coordinates have the form

$$\begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix} = C_1 \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}, \ \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = C_1 \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{8}$$

where $C_1 = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$.

Considering four-wire circuits, three instantaneous power components are defined as

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} v_0 & 0 & 0 \\ 0 & v_\alpha & v_\beta \\ 0 & v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \tag{9}$$

In three-phase three-wire systems, there are no zero-sequence components. In three-phase four-wire systems, the sum of p and p_0 results in the traditional instantaneous power of three-phase systems. The instantaneous currents can be calculated by means of inverse Park transformation.

$$\begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} = C_2 \begin{bmatrix} 0 \\ i_{\alpha p} \\ i_{\beta p} \end{bmatrix}, \begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} = C_2 \begin{bmatrix} 0 \\ i_{\alpha q} \\ i_{\beta q} \end{bmatrix}, \begin{bmatrix} i_{a0} \\ i_{b0} \\ i_{c0} \end{bmatrix} = C_2 \begin{bmatrix} i_0 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

where $C_2 = C_1^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$.

The orthogonal current can be decomposed into instantaneous active ($i_{\alpha p}, i_{\beta p}$) and reactive ($i_{\alpha q}, i_{\beta q}$) currents as

$$i_{\alpha p} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} p,\ i_{\beta p} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} p,\ i_{\alpha q} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} q,\ i_{\beta q} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} q \tag{12}$$

The powers could be decomposed into two components, the average ($\bar{x}$) and oscillating ($\tilde{x}$) component as

$$p = \bar{p} + \tilde{p},\ q = \bar{q} + \tilde{q} \tag{11}$$

The instantaneous active current can also be decomposed into average and oscillating components as

$$i_{\alpha p} = i_{\alpha \bar{p}} + i_{\alpha \tilde{p}} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} \bar{p} + \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} \tilde{p}$$

$$i_{\beta p} = i_{\beta \bar{p}} + i_{\beta \tilde{p}} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} \bar{p} + \frac{v_\beta}{v_\alpha^2 + v_\beta^2} \tilde{p} \tag{13}$$

Finally, the instantaneous phase current could be decomposed as

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} + \begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} + \begin{bmatrix} i_{a0} \\ i_{b0} \\ i_{c0} \end{bmatrix} = \begin{bmatrix} i_{a\bar{p}} \\ i_{b\bar{p}} \\ i_{c\bar{p}} \end{bmatrix} + \begin{bmatrix} i_{a\tilde{p}} \\ i_{b\tilde{p}} \\ i_{c\tilde{p}} \end{bmatrix} + \begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} + \begin{bmatrix} i_{a0} \\ i_{b0} \\ i_{c0} \end{bmatrix} \tag{14}$$

C. Generalized instantaneous non-active power theory

In generalized instantaneous non-active power theory, which is also extended from Fryze's theory, current $\vec{i}$ is decomposed into active current $\vec{i_p}$ and non-active current $\vec{i_q}$.

$$\vec{i}(t) = \vec{i_p}(t) + \vec{i_q}(t) \tag{15}$$

The active current is defined as

$$\vec{i_p}(t) = \frac{P_L(t)}{V_p^2(t)} \vec{v_p}(t) \tag{16}$$

where $V_p(t) = \sqrt{\frac{1}{T_C} \int_{t-T_C}^{t} \vec{v}_p^{\,T}(\tau)\vec{v}_p(\tau)d\tau}$, $P_L(t) = \frac{1}{T_C} \int_{t-T_C}^{t} \vec{v}^{\,T}(\tau)\vec{i}(\tau)d\tau$ (17)

The $\vec{v}_p(t)$ is the reference voltage vector, and depending on the compensation objectives, can be chosen as system voltage, or as the fundamental component of it, or as something else. T_C is the averaging interval. T_C is chosen to average the active power of the load and does not cause any time delay in the system response. $P_L(t)$ is the average load active power over the interval [t-T_C, t], and $V_P(t)$ is the rms value of the reference voltage $\vec{v}_p(t)$ over the interval [t-T_C, t].

It is shown in [8-9], that this definition is flexible to meet many different compensation objectives; it is valid for non-sinusoidal and non-periodic systems; it is valid for single-phase and multiphase systems. More details about the choice of $\vec{v}_p(t)$ and T_C will be given in section III.

III. Simulation results

In this section, the compensation results based on FBD, PQ and generalized non-active power theories for different cases will be shown and compared. For a single or a multiphase system, a shunt compensator can be configured as in Fig. 1. i_s is the source current, i_L is the load current, and i_C is the current that the compensator provides. The simulations are accomplished under ideal conditions: the compensator has no losses.

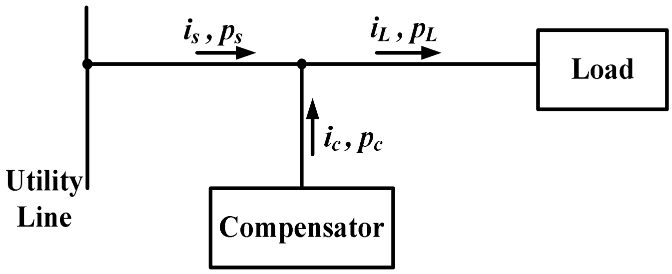


Fig. 1. Shunt compensator configuration.

A. Single-phase system with pulse load

According to the definitions, both FBD and generalized non-active power theories can be used in multiphase systems, including a single phase system. The original PQ theory, proposed by Akagi *et al.*, is based on the Park transformation, which transfer current and voltage in three-phase systems (a, b, c) into ($\alpha, \beta, 0$) orthogonal coordinates. So it cannot be used in single-phase systems. Although a method derived from PQ theory can be used in a single-phase system [17], the simulation results in this subsection are only based on FBD and generalized non-active power theories.

Some loads draw a large magnitude current for a short period, from half of a cycle to a few cycles, which may cause voltage sag in a weak power system. A pulse load current i_L is represented by a triangle waveform in this subsection, as Fig. 2a shows.

Fig. 2b shows the source current i_s compensated by FBD method. It is clear that the non-active current of the pulse load is not compensated completely. That is because in (5), the active current $i_{\mu a}$ is defined based on the mean values of p_Σ and v_Σ, which are over one system voltage period T. The pulse load, however, is a non-periodic load whose period can be thought as infinite. As a result, the FBD method can be applied to the multiphase, steady state, periodical system for compensation purposes, but is not suitable to compensate the non-periodical load currents.

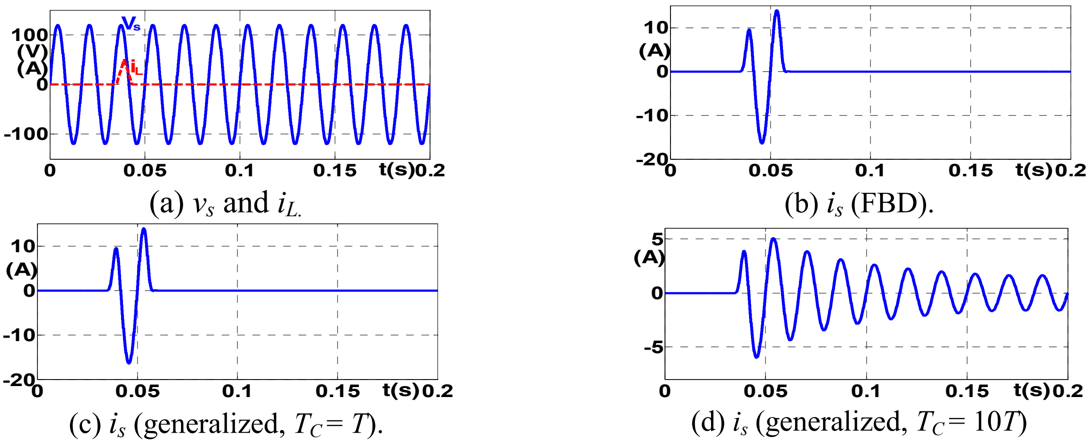
From (16) and (17), in a non-periodic system, the instantaneous current varies with different averaging interval T_C, which is different from the periodic cases. Since the period of the non-periodical load can be thought as infinite, choosing $T_C \to \infty$ in generalized non-active power theory, (17) will be

$$V_p(t) = \lim_{t\to\infty}\sqrt{\frac{1}{t}\int_0^t \vec{v}_p^{\,T}(\tau)\vec{v}_p(\tau)d\tau} = V_p\,,\ P_L(t) = \lim_{t\to\infty}\frac{1}{t}\int_0^t \vec{v}^{\,T}(\tau)\vec{i}(\tau)d\tau = P_L \tag{18}$$

and the active current in (16) will be

$$\vec{i}_p(t) = \frac{P_L(t)}{V_p^2(t)}\vec{v}_p(t) \to \frac{P_L}{V_p^2}\vec{v}_p(t) \tag{19}$$

So the generalized non-active power theory can only compensate non-periodical load current completely when choosing average interval $T_C \to \infty$. However, it is not practical to choose $T_C \to \infty$ in a power system, because T_C must be a finite value. Since a non-periodic current is usually a sum of the fundamental component and non-periodic component, T_C is usually chosen as a few multiples of the fundamental period. Fig. 2c and Fig. 2d show i_s after compensation by generalized theory, choosing $T_C = T$ and $T_C = 10T$, respectively. It is clear that larger T_C will make more components of non-active current be compensated. Less non-active current existing in i_s will reduce its peak value and mitigate its distortion. In addition, Fig. 2b and Fig. 2c are the same, which demonstrates that the FBD method can be deducted from the generalized theory with $T_C = T$ in this case.



(a) v_s and i_L.

(b) i_s (FBD).

(c) i_s (generalized, $T_C = T$).

(d) i_s (generalized, $T_C = 10T$)

Fig. 2. Single-phase systems with pulse load.

B. Three-phase system with unbalanced RL load

In this case, the system voltage v_s is pure sine wave and balanced. The unbalanced load currents have both unequal amplitudes and unbalanced phase-angles. For the periodic system, choosing the reference voltage in generalized non-active power theory as

$$\vec{v}_p(t) = \vec{v}_s(t) = \sqrt{2}V\cdot[\cos(\omega t+\alpha), \cos(\omega t - 2\pi/3+\alpha), \cos(\omega t + 2\pi/3+\alpha)]^T \tag{20}$$

unbalanced load current is

$$\vec{i}(t) = \sqrt{2}\cdot[I_1\cos(\omega t+\beta_1), I_2\cos(\omega t - 2\pi/3+\beta_2), I_3\cos(\omega t + 2\pi/3+\beta_3)]^T \tag{21}$$

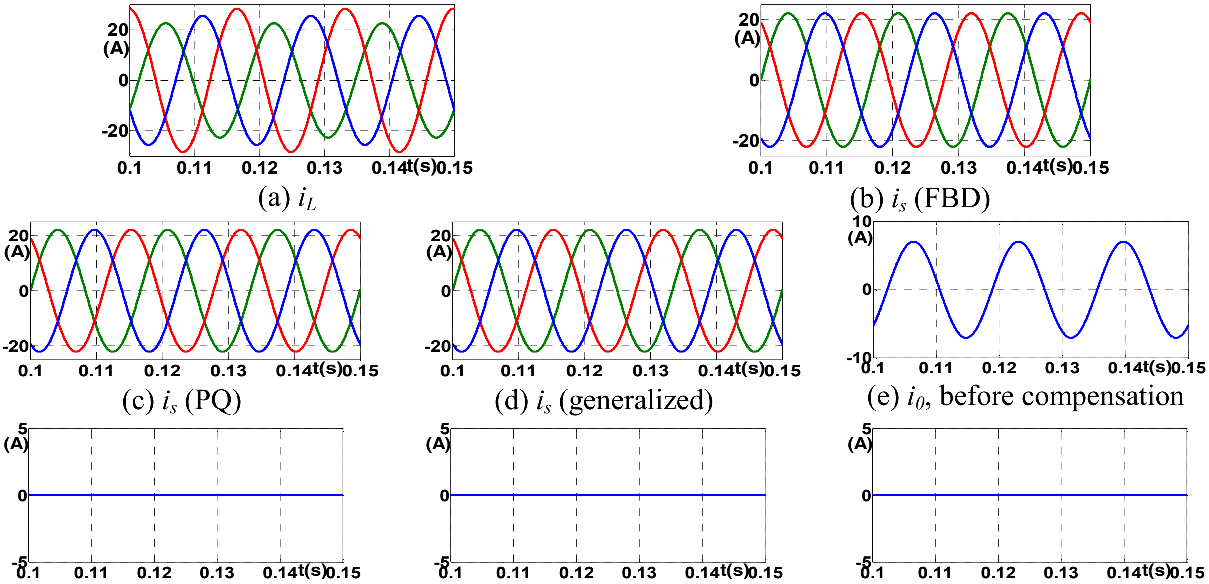
and choosing the averaging interval $T_C = kT/2$, where T is the system line period, $T = 2\pi/\omega$, then (17) will be

$$V_p(t) = V_s(t) = \sqrt{\frac{2}{kT}\int_0^{\frac{kT}{2}} \vec{v}_s^{\,T}(\tau)\vec{v}_s(\tau)d\tau} = V$$

$$P_L(t) = \frac{2}{kT}\int_0^{\frac{kT}{2}} \vec{v}_L^{\,T}(\tau)\vec{i}_L(\tau)d\tau = P_L \tag{22}$$

Equation (22) shows $V_p(t)$ and $P_L(t)$ will be constant values in periodic systems when choosing $T_C = kT/2$. So the non-active current of load $\vec{i}_q(t)$ can be easily compensated completely by the compensator. Since active current $i_{\mu a}$ from FBD method depends on the mean values of power and voltage over T, it will be the same with $k=2$ in (22). For complete compensation, PQ theory requires the dc component of the instantaneous values, which requires at least half of the fundamental cycle to determine it. Therefore, it has the same results as generalized theory when $T_C = T/2$, as (22) shows.

Fig. 3a shows the unbalanced load current. Fig. 3b to Fig. 3d show the compensation results by the three theories in three-phase three-wire systems. Balanced system current i_s can be achieved by all these theories. In three-phase four-wire systems, unbalanced load current will cause large neutral current i_0, as Fig. 3e shows. Fig. 3f to Fig. 3h show i_0 after compensation by these three theories. i_0 is zero after the compensation which also demonstrates the complete compensation for an unbalanced load.



(a) i_L (b) i_s (FBD) (c) i_s (PQ) (d) i_s (generalized) (e) i_0, before compensation (f) i_0 (FBD) (g) i_0 (PQ) (h) i_0 (generalized)

Fig. 3. Three-phase systems with unbalanced load current.

C. Three-phase load with sub-harmonics

Sub-harmonics are the components in a waveform whose frequencies are not an integral multiple of the fundamental frequency. Using a single-phase system to make a simple analysis, the source voltage $v_s(t)$ is

$$v_s(t) = \sqrt{2}V_1 \cos(\omega_1 t + \alpha) \tag{23}$$

and load current is

$$i_L(t) = \sqrt{2}I_1 \cos(\omega_1 t + \beta_1) + \sqrt{2}I_s \cos(\omega_s t + \beta_s) \tag{24}$$

where $\omega_1 = 2\pi f_1$ is the fundamental frequency of system, and $\omega_s = 2\pi f_s$ is the sub-harmonic frequency.

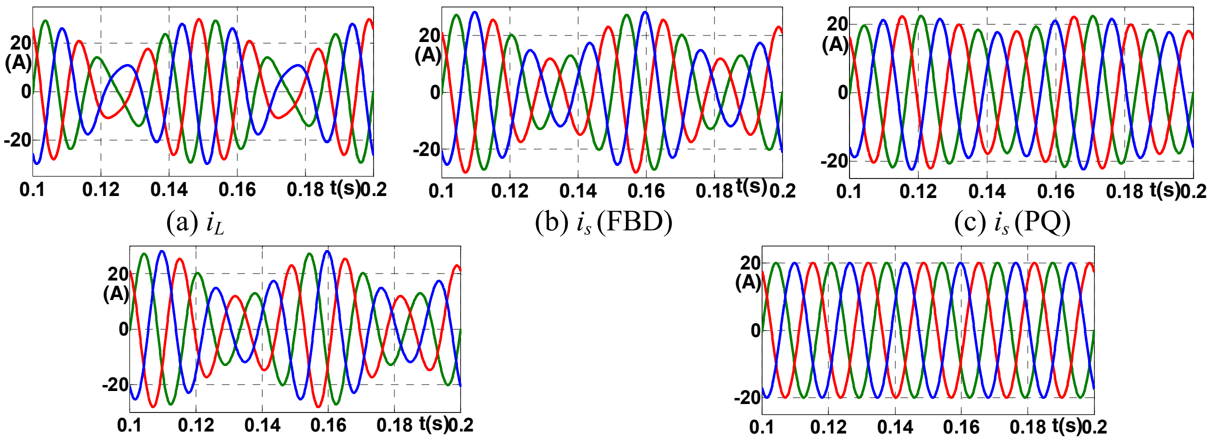
The instantaneous power $p_L(t)$ is

$$\begin{aligned} p_L(t) = v_s(t)i_L(t) &= V_1 I_1 \cos(\alpha_1 - \beta_1) + V_1 I_1 \cos(2\omega_1 t + \alpha_1 + \beta_1) \\ &+ V_1 I_s \cos[(\omega_1 - \omega_s)t + \alpha_1 - \beta_s] + V_1 I_s \cos[(\omega_1 + \omega_s)t + \alpha_1 + \beta_s] \end{aligned} \tag{25}$$

The instantaneous power contains frequencies of $2f_1$, f_1-f_s, f_1+f_s, due to the multiplication between the fundamental and the sub-harmonics.

In generalized non-active power theory, if choosing the reference voltage $v_p(t) = v_s(t)$, then the rms value $V_p(t) = V_1$ is constant. If T_C is an integral multiple of the periods of all the frequencies in $p_L(t)$, the average value $P_L(t)$ will be constant too. From (16), the active current $i_p(t)$ will be pure fundamental sine wave and in phase with $v_s(t)$. For FBD method, it is the same with choosing $T_C = T$ in generalized theory. So the active current obtained from it still contains sub-harmonic components. Since the average values are still used in PQ theory ($\bar{p}$, $\bar{q}$, $\tilde{p}$ and $\tilde{q}$), and data from at least half of a cycle of a fundamental period are used to calculate the averages values, the complete compensation cannot be achieved for sub-harmonic currents whose periods are not integral multiple of half of the fundamental period.

In the simulation of a three-phase sub-harmonic system, the system voltage v_s is 60 Hz pure sine wave, and the load current i_L has 80 Hz sub-harmonic component, as Fig. 4a shows. Using FBD and PQ methods, the non-active current cannot be compensated completely, as shown in Fig. 4b and Fig. 4c. According to the generalized theory, if the averaging interval $T_C = T$, the compensation result is the same with FBD's, as Fig. 4d shows. If choosing $T_C = 3T$, which is the integral multiple of the periods of all the frequencies in $p_L(t)$, complete compensation can be achieved, as shown in Fig. 4e.

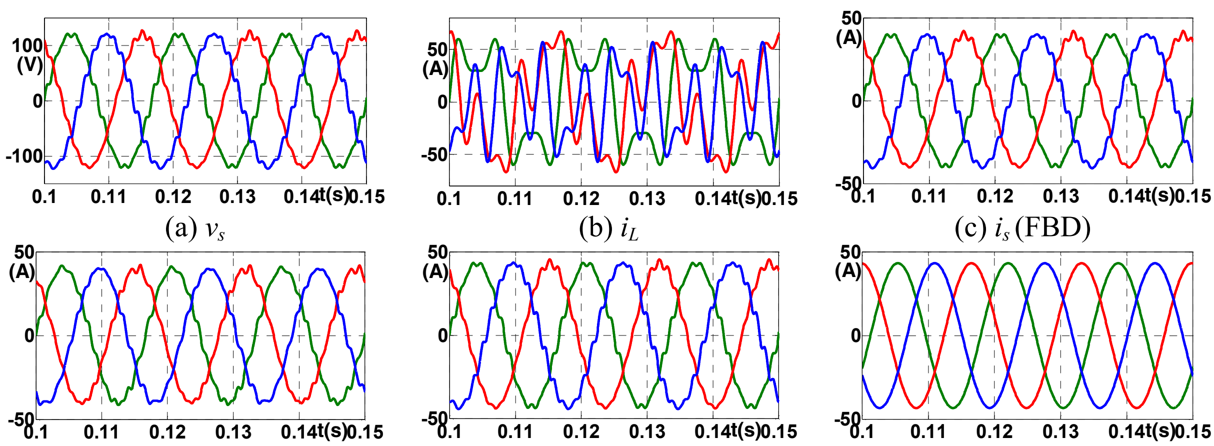


(a) i_L (b) i_s (FBD) (c) i_s (PQ)
(d) i_s (generalized, $T_C = T$) (e) i_s (generalized, $T_C = 3T$)
Fig. 4. Three-phase load with sub-harmonics.

D. Distorted system voltage

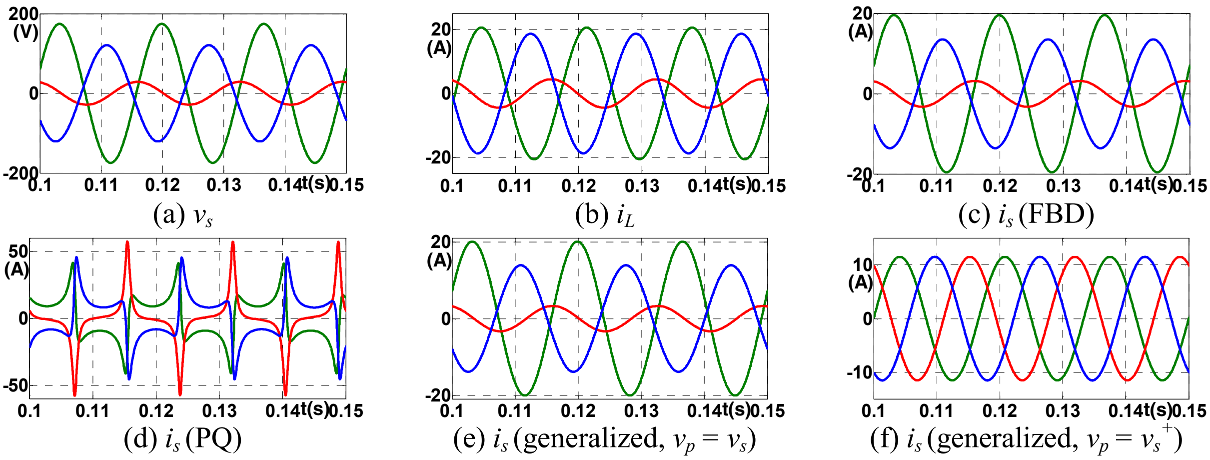
The distorted system voltage includes the system voltage with harmonics, the unbalanced system voltage, and asymmetrical voltage sources.

In the simulation, the system voltage v_s contains harmonics, as Fig. 5a shows, and the load current i_L also contains large 3rd and 5th harmonics, shown in Fig. 5b. According to (5), the active phase current $i_{\mu a}$ obtained from FBD method will contain the same harmonic components as the phase voltage v_μ. Fig. 5c shows the source current i_s compensated by FBD method, and the shape of it is the same with v_s. From (8) - (13), the active power current calculated by PQ theory does not eliminate the harmonics in v_s, so PQ theory does not allow complete compensation in this case either, as Fig. 5d shows. For generalized theory, if choosing reference voltage $v_p(t) = v_s(t)$, from (16) and (17), the harmonic components in v_s will still be in the active current. Fig. 5e shows the result. However, if choosing the reference voltage as the fundamental component of source voltage, $v_p(t) = v_f(t)$, the active current $i_p(t)$ will be pure sine waveform and no harmonics exist, as Fig. 5f shows.



(a) v_s (b) i_L (c) i_s (FBD)
(d) i_s (PQ) (e) i_s (generalized, $v_p = v_s$) (f) i_s (generalized, $v_p = v_f$)
Fig. 5. Harmonic system voltage with harmonic load current.

Fig. 6a shows the unbalanced source voltage. In the simulation, the load is a balanced RL load, but the load current is still unbalanced due to the unbalanced source voltage, shown in Fig. 6b. The unbalanced voltages of three phases can be decomposed into three sequences: a positive sequence $v^+(t)$, a negative sequence $v^-(t)$, and a zero sequence $v^0(t)$. For FBD method, (5) uses phase voltage which contains all of these three sequences to calculate active current, and this leads to current unbalance after compensation. Fig. 6c shows the result. The assumption of PQ theory is that the voltage is a pure sine wave and the three phases are balanced. For balanced, symmetrical voltage sources, the non-active power in (9) is equal to the three-phase non-active power, but for distorted voltage sources, it is not equal to the three-phase non-active power. Zero sequence components in PQ theory are completely attributed to active power without considering its contribution to non-active power. PQ theory will introduce errors if it is applied to a non-active power compensation system for distorted source voltage systems. Fig. 6d shows the source current compensated by PQ theory in an unbalanced source voltage system. It is clear that the source current after compensation is distorted seriously. For generalized non-active power theory, if the reference voltage $v_p(t)$ equals to the unbalanced source voltage $v_s(t)$, the active current calculated by (16) will still be unbalanced, as Fig. 6e shows. So in this case, when generalized theory is used, choosing $v_p(t) = v_s^+(t)$, the source current will not contain negative and zero sequences and it will be balanced, as shown in Fig. 6f. The averaging interval is chosen $T_C = T$ in the simulation.



(a) v_s (b) i_L (c) i_s (FBD)

(d) i_s (PQ) (e) i_s (generalized, $v_p = v_s$) (f) i_s (generalized, $v_p = v_s^+$)

Fig. 6. Unbalanced system voltage.

IV. Experimental results

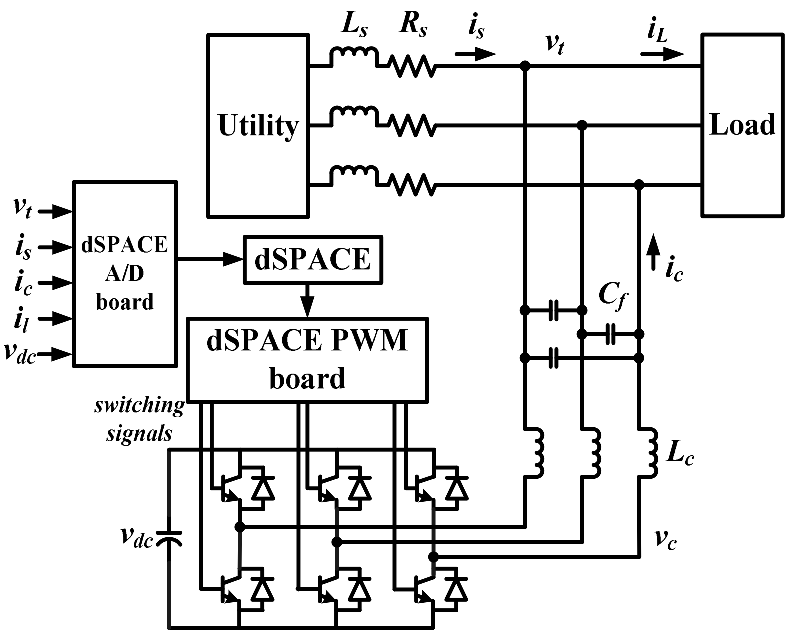


Fig. 7. Experimental configuration.

The experiments are performed in a three-phase system with the voltage rating 208 V (line-to-line rms value). An IGBT based three-phase inverter was used as the inverter for the active filter (DC link voltage is 450V). Fig. 7 shows the configuration. The generalized non-active power theory is used as the compensation algorithm.

System voltage v_s is sinusoidal and balanced, as Fig. 8a shows. Fig. 8b shows the unbalanced RL load current i_L with the phase *a* voltage. The source current i_s after compensation is shown in Fig. 8c. i_s is balanced and in phase with the voltage. When an RL load is connected between phase *a* and phase *b*, the three-phase load currents i_L are shown in Fig. 8d. Fig. 8e shows i_s after compensation. The magnitudes of i_s of phase *a* and phase *b* are reduced, and there is a current in phase *c*. When the load is a diode rectifier, there are harmonics in i_L, as shown in Fig. 8f. After compensation the source current i_s is nearly sinusoidal, as Fig. 8g shows. The spikes in i_s are due to the high *di/dt* in the load current.

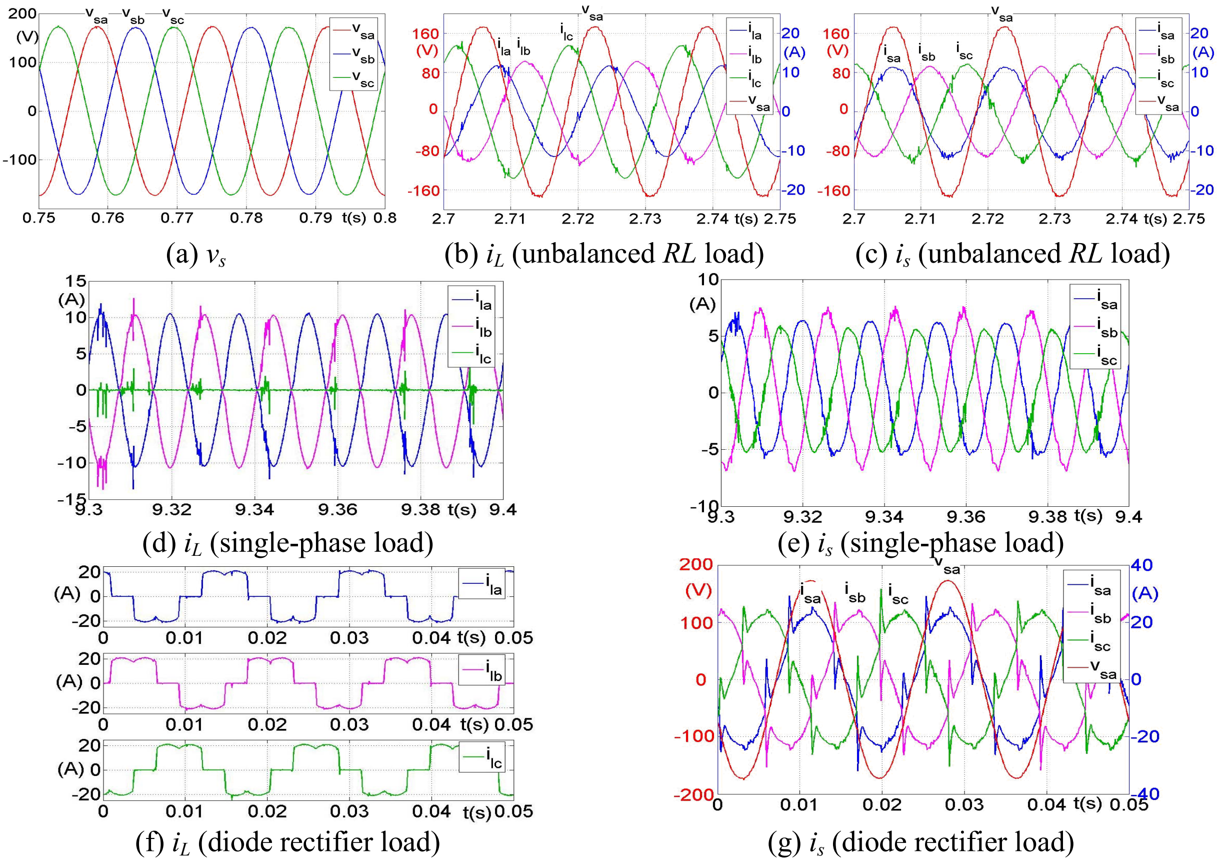


Fig. 8. Experimental results of three different types of loads compensation by generalized theory.

V. Conclusions

This paper compares the compensation results based on FBD, PQ and generalized non-active power theories for different cases. Table I summarizes the applicability of these three theories from the simulation and experimental results presented above. The table shows if each theory is applicable to single-, three-, or multiple-phase systems, and whether the theory can compensate non-active power completely or whether it can partially compensate non-active power.

The generalized non-active power theory allows complete compensation in many cases by specifying suitable v_p and T_C. Practically, the generalized method cannot compensate non-periodical system completely, because T_C cannot be infinite. However, by choosing T_C large enough, the compensation results will be improved significantly. The FBD theory can be deducted from generalized theory, by choosing $T_C = T$ and $v_p = v_s$. PQ theory can achieve complete compensation in three-phase system with pure sine wave source voltage. But it cannot be used in single-phase systems, and it will introduce errors in distorted system voltage cases, because it is based on the Park transformation and assumes the source voltage is pure sine waveform.

Table I. Comparison of three theories' compensation results

Theory	Phase number			Load current			Source voltage distortion
	Single	Three (3&4-wire)	Multiple	Non-periodic	Unbalance	Sub-harmonic	
FBD	Yes	Yes	Yes	Partially	Completely	Partially	Partially
PQ	No	Yes	No	Partially	Completely	Partially	Partially
Generalized	Yes	Yes	Yes	Completely	Completely	Completely	Completely

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