The Use of Power Sums to Solve the Harmonic Elimination Equations for Multilevel Converters

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Abstract

A method is presented to compute the switching angles in a multilevel converter so as to produce the required fundamental voltage while at the same time not generate higher order harmonics. Previous work has shown that the transcendental equations characterizing the harmonic content can be converted to polynomial equations which are then solved using the method of resultants from elimination theory. However, when there are several DC sources, the degree of the polynomials are quite large making the computational burden of their resultant polynomials via elimination theory quite high. Here, it is shown that by reformulating the problem in terms of power sums, the degree of the polynomial equations that must be solved are reduced significantly which in turn reduces the computational burden. In contrast to numerical techniques, the approach here produces all possible solutions.

Introduction

A multilevel inverter is a power electronic device built to synthesize a desired ac voltage from several levels of dc voltages. For example, the output of solar cells are dc voltages, and if this energy is to be fed to into an ac power grid, a power electronic interface is required. A multilevel inverter is ideal for connecting such distributed dc energy sources (solar cells, fuel cells, the rectified output of wind turbines) to an existing ac power grid.

Transformerless multilevel inverters are uniquely suited for utility applications because of the high VA ratings possible with these inverters [1]. The devices in a multilevel inverter have a much lower d*V*/d*t* per switching, and they operate at high efficiencies because they can switch at a much lower frequency than PWM-controlled inverters. Three, four, and five level rectifier-inverter drive systems that have used some form of multilevel PWM as a means to control the switching of the rectifier and inverter sections have been investigated in the literature [2, 3, 4, 5, 6]. Here a fundamental frequency switching scheme (rather than PWM) is considered because, as just mentioned, this results in significantly lower switching losses.

A key issue in the fundamental switching scheme is to determine the switching angles (times) so as to produce the fundamental voltage and not generate specific higher order harmonics. The recent book [7] summarizes the current state of the art in harmonic elimination. Often iterative techniques are used to solve for the switching angles [8], though such an approach does not guarantee finding all the possible solutions. In [9], a genetic algorithm approach is used to solve for the switching angles. In Kato [10], a homotopy technique is used to solve the harmonic elimination equations for a single DC source inverter.

Previous work in [11, 12, 13] has shown that the transcendental equations characterizing the harmonic content can be converted into polynomial equations which are then solved using the method of resultants from elimination theory [14, 15]. However, if there are several dc sources, the degrees of the polynomials in these equations are large. As a result, one reaches the limitations of the capability of contemporary computer algebra software tools (e.g., Mathematica or Maple) to solve the system of polynomial equations using elimination theory (by computing the resultant polynomial of the system).

A major distinction between the work in [11, 12, 13] and the work presented here is that here the theory of power sums [16] is exploited to reduce the degree of the polynomial equations that must be solved so that they are well within the capability of existing computer algebra software tools. As in [13], the approach presented in this work produces all possible solutions in contrast to numerical techniques. Experimental verification that the low order harmonics are indeed eliminated is also presented by driving a three-phase induction motor from an 11-level inverter. A preliminary version of this work was presented in [17].

Cascaded H-bridges

A cascade multilevel inverter consists of a series of H-bridge (single-phase full-bridge) inverter units. The inverter synthesizes a desired voltage from several separate dc sources (SDCSs), which may be obtained from solar cells, fuel cells, batteries, ultracapacitors, etc. The left side of Fig. 1 shows a single-phase structure of a cascade inverter with SDCSs [1]. Each SDCS is connected to a single-phase full-bridge inverter. Each inverter level can generate three different voltage outputs, $+V_{dc}$, 0 and $-V_{dc}$ by connecting the DC source to the AC output side by different combinations of the four switches, S_1, S_2, S_3 and S_4. The ac output of each level's full-bridge inverter is connected in series such that the synthesized voltage waveform is the sum of all of the individual inverter outputs. The number of output phase voltage levels in a cascade multilevel inverter is then $2s + 1$,where s is the number of dc sources. An example phase voltage waveform for an 11-level cascaded multilevel inverter with five SDSCs ($s = 5$) and five full bridges is shown on the right side of Fig. 1.

The output phase voltage is given by $v_n = v_1 + v_2 + v_3 + v_4 + v_5$. With enough levels and an appropriate switching algorithm, the multilevel inverter results in an output voltage that is almost sinusoidal. For the 11-level example shown in Fig. 1, the waveform has less than 5 % total harmonic distortion (THD) with each the H-bridges' active devices switching only at the fundamental frequency. As seen in Fig. 1, the active power drawn from each source is not equal. However, the voltage levels of the sources can be kept balanced by using a pattern swapping scheme such as described in [18-19]. In that scheme, every half-cycle the θ_i are simply rotated through the various levels so that after five half-

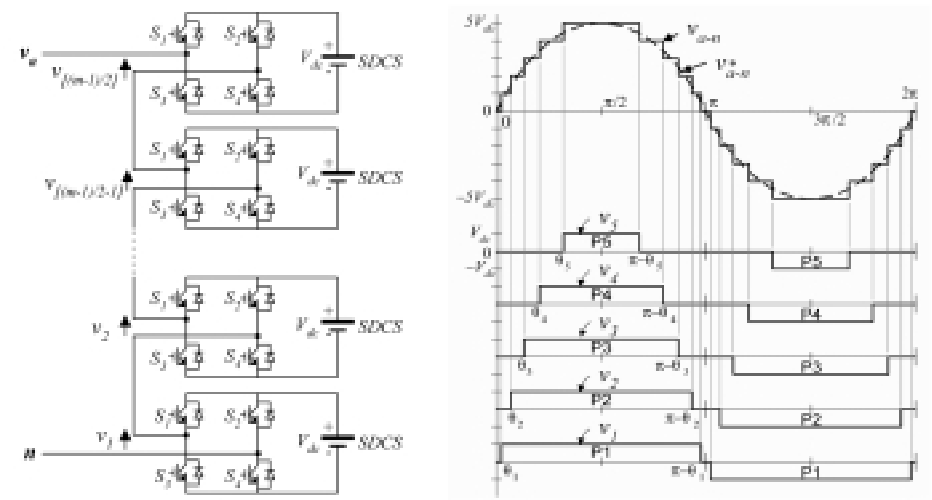

Fig. 1: Left: Single-phase structure of a multilevel cascaded H-bridges inverter. Right: Output waveform of an 11-level cascade multilevel inverter

cycles they are back to their original configuration (see [18-19] for details). For dealing with non equal DC sources, see [20].

Mathematical model of switching

Following the development in [13] (see also [18]), the Fourier series expansion of the (staircase) output voltage waveform of the multilevel inverter as shown in Fig. 1 is

$$V(\omega t)=\frac{4V_{dc}}{\pi}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n}\begin{pmatrix}\cos(n\theta_1)+\cos(n\theta_2)\\ +\cdots+\cos(n\theta_s)\end{pmatrix}\sin(n\omega t) \tag{1}$$

where s is the number of dc sources. Ideally, given a desired fundamental voltage V_1, one wants to determine the switching angles $\theta_1, \ldots, \theta_s$ so that (1) becomes $V(\omega t) = V_1 \sin(\omega t)$. In practice, one is left with trying to do this approximately. The goal here is to choose the switching angles $0 \le \theta_1 < \theta_2 < \cdots < \theta_s \le \pi/2$ so as to make the first harmonic equal to the desired fundamental voltage V_1 and specific higher harmonics of $V(\omega t)$ equal to zero. For three-phase systems, the triplen harmonics in each phase need not be canceled as they automatically cancel in the line-to-line voltages. Specifically, in the case of $s = 5$ dc sources, the desire is to cancel the 5th, 7th, 11th, 13th order harmonics as they dominate the total harmonic distortion. The mathematical statement of these conditions is then

$$\begin{aligned}
&\frac{4V_{dc}}{\pi}(\cos(\theta_1)+\cos(\theta_2)+\cdots\cos(\theta_5)=V_1\\
&\cos(5\theta_1)+\cos(5\theta_2)+\cdots+\cos(5\theta_5)=0\\
&\cos(7\theta_1)+\cos(7\theta_2)+\cdots+\cos(7\theta_5)=0\\
&\cos(11\theta_1)+\cos(11\theta_2)+\cdots+\cos(11\theta_5)=0\\
&\cos(13\theta_1)+\cos(13\theta_2)+\cdots+\cos(13\theta_5)=0
\end{aligned} \tag{2}$$

This is a system of five transcendental equations in the five unknowns θ_1, θ_2, θ_3, θ_4, θ_5. The question here is "When does the set of equations (2) have a solution?". The correct solution to the conditions (2) would mean that the output voltage of the 11–level inverter would not contain the 5th, 7th, 11th and 13th order harmonic components. One approach to solving this set of nonlinear transcendental equations (2) is to use an iterative method such as the Newton-Raphson method [18-21]. In contrast to iterative methods, the following presents a new approach that obtains all possible solutions and requires significantly less computational effort than the approach in [13]. To proceed with the new methodology, first let $s = 5$, and define $x_i = \cos(\theta_i)$ for $i = 1, ..., 5$. Using the trigonometric identities

$$\begin{aligned}
\cos(5\theta)&=5\cos(\theta)-20\cos^3(\theta)+16\cos^5(\theta)\\
\cos(7\theta)&=-7\cos(\theta)+56\cos^3(\theta)-112\cos^5(\theta)+64\cos^7(\theta)\\
\cos(11\theta)&=-11\cos(\theta)+220\cos^3(\theta)-1232\cos^5(\theta)\\
&\quad+2816\cos^7(\theta)-2816\cos^9(\theta)+1024\cos^{11}(\theta)\\
\cos(13\theta)&=13\cos(\theta)-364\cos^3(\theta)+2912\cos^5(\theta)-9984\cos^7(\theta)\\
&\quad+16640\cos^9(\theta)-13312\cos^{11}(\theta)+4096\cos^{13}(\theta),
\end{aligned}$$

the conditions (2) become

$$\begin{aligned}
p_1(x)&\triangleq x_1+x_2+x_3+x_4+x_5-m=0\\
p_5(x)&\triangleq\sum_{i=1}^{5}\left(5x_i-20x_i^3+16x_i^5\right)=0\\
p_7(x)&\triangleq\sum_{i=1}^{5}\left(-7x_i+56x_i^3-112x_i^5+64x_i^7\right)=0\\
p_{11}(x)&\triangleq\sum_{i=1}^{5}\begin{pmatrix}-11x_i+220x_i^3-1232x_i^5+2816x_i^7\\-2816x_i^9+1024x_i^{11}\end{pmatrix}=0\\
p_{13}(x)&\triangleq\sum_{i=1}^{5}\begin{pmatrix}13x_i-364x_i^3+2912x_i^5-9984x_i^7+16640x_i^9\\-13312x_i^{11}+4096x_i^{13}\end{pmatrix}=0
\end{aligned} \tag{3}$$

where $x = (x_1, x_2, x_3, x_4, x_5)$ and $m \triangleq V_1/(4\,V_{dc}/\pi)$. The modulation index is $m_a = m/s = V_1/(s4V_{dc}/\pi)$. (Each inverter has a DC source of V_{dc} so that the maximum output voltage of the multilevel inverter is sV_{dc}. A square wave of amplitude sV_{dc} results in the maximum fundamental output possible of $V_{1max} = 4sV_{dc}/\pi$ so $m_a \triangleq V_1/V_{1max} = V_1/(s4V_{dc}/\pi) = m/s$.)

This is a set of five equations in the five unknowns x_1, x_2, x_3, x_4, x_5. Further, the solutions must satisfy $0 \le x_5 \le \ldots \le x_2 \le x_1 \le 1$. This development has resulted in a set of polynomial equations rather than trigonometric equations. The degrees of the polynomials are large which in turn requires the symbolic computation of the determinant of large square matrices. Contemporary computer algebra software tools cannot solve these equations on a personal computer for inverters with more than four dc sources. Here (cf. [13]) a new approach to solving the system (3) is presented which greatly reduces the computational burden. This is done by taking into account the symmetry of the polynomials making up system (3). Specifically, the theory of power sums [14, 22] is exploited to obtain a new set of relatively low degree polynomials whose resultants can easily be computed using existing computer algebra software tools. Further, in contrast to iterative numerical techniques, the approach here produces all possible solutions.

Solving polynomial equations

For the purpose of exposition, the three source (7 level) multilevel inverter will be used to illustrate the approach. The conditions are then

$$
\begin{aligned}
&p_1(x) \triangleq x_1 + x_2 + x_3 - m = 0, \quad m \triangleq \frac{V_1}{4V_{dc}9/\pi}\\
&p_5(x) \triangleq \sum_{i=1}^{3}\left(5x_i - 20x_i^3 + 16x_i^5\right) = 0\\
&p_7(x) \triangleq \sum_{i=1}^{3}\left(-7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7\right) = 0
\end{aligned}
\tag{4}
$$

Eliminating x_3 by substituting $x_3 = m - (x_1 + x_2)$ into p_5, p_7 gives

$$
\begin{aligned}
p_5(x_1,x_2) &= 5x_1 - 20x_1^3 + 16x_1^5 + 5x_1 - 20x_2^2 + 16x_2^5\\
&\quad + 5(m - x_1 - x_2) - 20(m - x_1 - x_2)^3 + 16(m - x_1 - x_2)^5\\
p_7(x_1,x_2) &= -7x_1 + 56x_1^3 - 112x_1^5 + 64x_1^7 - 7x_2 + 56x_2^3\\
&\quad - 112x_2^5 + 64x_2^7 - 7(m - x_1 - x_2) + 56(m - x_1 - x_2)^3\\
&\quad - 112(m - x_1 - x_2)^5 + 64(m - x_1 - x_2)^7
\end{aligned}
\tag{5}
$$

where

$$
\begin{aligned}
&\deg_{x1}\{p_5(x_1,x_2)\} = 4, \quad \deg_{x2}\{p_5(x_1,x_2)\} = 4\\
&\deg_{x1}\{p_7(x_1,x_2)\} = 6, \quad \deg_{x2}\{p_7(x_1,x_2)\} = 6
\end{aligned}
\tag{6}
$$

Elimination using resultants

In order to explain the computational issues with finding the zero sets of polynomial systems, a brief discussion of the procedure to solve such systems is now given. The question at hand is "Given two polynomial equations $a(x_1, x_2) = 0$ and $b(x_1, x_2) = 0$, how does one solve them simultaneously to eliminate (say) x_2?". A systematic procedure to do this is known as elimination theory and uses the notion of resultants [14-15]. Briefly, one considers $a(x_1, x_2)$ and $b(x_1, x_2)$ as polynomials in x_2 whose coeffcients are polynomials in x_1. Then, for example, letting $a(x_1, x_2)$ and $b(x_1, x_2)$ have degrees 3 and 2, respectively in x_2, they may be written in the form

$$
\begin{aligned}
a(x_1,x_2) &= a_3(x_1)x_2^3 + a_2(x_1)x_2^2 + a_1(x_1)x_2 + a_0(x_1)\\
b(x_1,x_2) &= b_2(x_1)x_2^2 + b_1(x_1)x_2 + b_0(x_1).
\end{aligned}
$$

The $n \times n$ *Sylvester* matrix, where

$$n = \deg_{x2}\{a(x_1,x_2)\} + \deg_{x2}\{b(x_1,x_2)\} = 3 + 2 = 5$$

is defined by

$$
S_{a,b}(x_1) = \begin{bmatrix}
a_0(x_1) & 0 & b_0(x_1) & 0 & 0\\
a_1(x_1) & a_0(x_1) & b_1(x_1) & b_0(x_1) & 0\\
a_2(x_1) & a_1(x_1) & b_2(x_1) & b_1(x_1) & b_0(x_1)\\
a_3(x_1) & a_2(x_1) & 0 & b_2(x_1) & b_1(x_1)\\
0 & a_3(x_1) & 0 & 0 & b_2(x_1)
\end{bmatrix}
$$

The resultant polynomial is then defined by

$$r(x_1) = \mathrm{Res}\big(a(x_1,x_2), b(x_1,x_2), x_2\big) \triangleq \det S_{a,b}(x_1) \tag{7}$$

and is the result of solving $a(x_1, x_2) = 0$ and $b(x_1, x_2) = 0$ simultaneously for x_1, i.e., elimi-nating x_2 (See [14-15] for a discussion of elimination theory and resultants). The point here is that as the degrees of the polynomials increase, the size of the corresponding Sylvester matrix increases, and therefore the symbolic computation of its determinant becomes much more computationally intensive.

Power sums

Consider once again the system of polynomial equations (5). In [13] (see also [11-12]) the authors computed the resultant polynomial of the pair $\{p_5(x_1, x_2), p_7(x_1, x_2)\}$ to obtain the solutions to (4). This involved setting up a 10×10 Sylvester matrix ($10 = \deg_{x2}\{p_5(x_1, x_2)\} + \deg_{x2}\{p_7(x_1, x_2)\}$) and then computing its determinant to obtain the resultant polynomial $r(x_1)$ whose degree turned out to be 22. However, as one adds more dc sources to the multilevel inverter, the degrees of the polynomials go up rapidly. For example, in the case of four dc sources, the final step of the method requires computing (symbolically) the determinant of a 27×27 Sylvester matrix to obtain a resultant polynomial of degree 221. In the case of five sources, using this method, the authors were only able to get the system of five polynomial equations in five unknowns to reduce to three equations in three unknowns. The computation to get it down to two equations in two unknowns requires the symbolic computation of the determinant of a 33×33 Sylvester matrix. To get around this diffculty, a new approach is developed here which exploits the fact that the polynomials in (3) are symmetric.

The polynomials $p_1(x)$, $p_2(x)$, $p_3(x)$ in (4) can be written in terms of power sums, that is, define the power sums (polynomials) t_1, t_2, t_3 as

$$
\begin{aligned}
t_1 &\triangleq x_1 + x_2 + x_3\\
t_2 &\triangleq x_1^2 + x_2^2 + x_3^2\\
t_3 &\triangleq x_1^3 + x_2^3 + x_3^3\\
&\vdots
\end{aligned}
\tag{8}
$$

Using the power sums, the polynomials (4) become

$$\begin{aligned} p_1(t) &= t_1 - m \\ p_5(t) &= 5t_1 - 20t_3 + 16t_5 \\ p_7(t) &= -7t_1 + 56t_3 - 112t_5 + 64t_7 \end{aligned} \tag{9}$$

This is now a set of three equations in the four unknowns t_1, t_3, t_5, t_7. However the polynomials of (4) are symmetric in the x_i, i.e., for example, if one interchanges x_1 and x_3, the polynomials remain the same. (This also is seen from the fact that the system (4) has been rewritten in (9) in terms of the power sums which are symmetric in the x_i.) As a result, the theory of power sums says that any set of symmetric polynomials in the variables x_1, x_2, ..., x_n can be rewritten in terms of the power sums t_1, t_2, ..., t_n (see [14] page 317). In the case of (9), it turns out that

$$\begin{aligned} t_5 &= \left(t_1^5 - 5t_1^3 t_2 + 5t_1^2 t_3 + 5t_2 t_3\right)/6 \\ t_7 &= \left(t_7 - 21t_1^3 t_2^2 + 7t_1^4 t_3 + 21t_2^2 t_3 + 28t_1 t_3^2\right)/36 \end{aligned}$$

These are then substituted into (9) to obtain ($t \triangleq (t_1, t_2, t_3)$)

$$\begin{aligned} p_1(t) &= t_1 - m \\ p_5(t) &= \left(15t_1 + 8t_1^5 - 40t_1^3 + t_2 - 60t_3 + 40t_1^2 t_3 + 40t_2 t_3\right)/3 \\ p_7(t) &= (-63t_1 - 168t_1^5 + 16t_1^7 + 840t_1^3 t_2 - 336t_1^3 t_2^2 + 504t_3 \\ &\quad - 840t_1^2 t_3 + 112t_1^4 t_3 - 840t_2 t_3 + 336t_2^2 t_3 + 448t_1 t_3^2)/9 \end{aligned}$$

which is now a system of three polynomials in three unknowns. One uses $p_1(t) = t_1 - m = 0$ to eliminate t_1 so that

$$q_5(t_2,t_3) \triangleq p_5(m,t_2,t_3) = (1/3)\begin{pmatrix} 15m + 8m^5 - 40m^3 + t_2 - 60t_3 \\ +\ 40m^2 t_3 + 40t_2 t_3 \end{pmatrix}$$

$$q_7(t_2,t_3) \triangleq p_7(m,t_2,t_3) = (1/9)\begin{pmatrix} -63m - 168m^5 + 16m^7 \\ +\ 840m^3 t_2 - 336m^3 t_2^2 + 504t_3 \\ -\ 840m^2 t_3 + 112m^4 t_3 - 840t_2 t_3 \\ +336t_2^2 t_3 + 448m t_3^2 \end{pmatrix}$$

where

$$\begin{aligned} &\deg_{t2}\{q_5(t_2,t_3)\} = 1, \quad \deg_{t3}\{q_5(t_2,t_3)\} = 1, \\ &\deg_{t2}\{q_7(t_2,t_3)\} = 2, \quad \deg_{t3}\{q_7(t_2,t_3)\} = 2, \end{aligned}$$

The key point here is that the degrees of these polynomials in t_2, t_3 are much less than the degrees of $p_5(x_1, x_2)$, $p_7(x_1, x_2)$ in x_1, x_2 (see equation (6)). In particular, the Sylvester matrix of the pair $\{q_5(t_2, t_3), q_7(t_2, t_3)\}$ is 3×3 rather than being 10×10 in the case of $\{p_5(x_1, x_2), p_7(x_1, x_2)\}$ in (5). Eliminating t_2, the resultant polynomial $\mathrm{Res}(q_5(t_2, t_3), q_7(t_2, t_3), t_2)$ is given by

$$\begin{aligned} \mathrm{Res}(q_5, q_7, t_2) &= \frac{16}{81} m(m^3 - t_3)(-4725m + 25200m^3 - 5040m^5 \\ &\quad + 256m^9 + 12600t_3 - 100800m^2 t_3 + 20160m^4 t_3 \\ &\quad - 3840m^6 t_3 + 100800m t_3^3 - 44800t_3^3 \end{aligned}$$

which factors into a polynomial of degree 1 in t_3 and of degree 3 in t_3. For each m, one solves $\mathrm{Res}(q_5, q_7, t_2) = 0$ for the roots $\{t_{3i}\}_{i = 1, 2, 3}$. These roots are then used to solve $q_5(t_2, t_{3i}) = 0$ for the root t_{2i} resulting in the set of 3-tuples $\{(t_1, t_2, t_3) \in \mathbb{C}^3 \mid (t_1, t_2, t_3) = (m, t_{2i}, t_{3i})$ for $i = 1, 2, 3\}$ as the only possible solutions to (9).

Solving the Power Sums

For each solution triple (t_1, t_2, t_3), the corresponding values of (x_1, x_2, x_3) are required to obtain the switching angles. To do so, one simply uses the resultant method to solve the system of polynomials

$$\begin{aligned} f_1(x_1,x_2,x_3) &= t_1 - (x_1 + x_2 + x_3) = 0 \\ f_2(x_1,x_2,x_3) &= t_2 - (x_1^2 + x_2^2 + x_3^2) = 0 \\ f_3(x_1,x_2,x_3) &= t_3 - (x_1^3 + x_2^3 + x_3^3) = 0 \end{aligned}$$

for (x_1, x_2, x_3). That is, one computes

$$\begin{aligned} r_1(x_2,x_3) &\triangleq \mathrm{Res}\big(f_1(x_1,x_2,x_3),\ \ f_2(x_1,x_2,x_3),\ \ x_1\big) \\ &= t_1^2 - t_2 - 2t_1 x_2 + 2x_2^2 - 2t_1 x_3 + 2x_2 x_3 + 2x_3^2 \end{aligned}$$

$$\begin{aligned} r_2(x_2,x_3) &\triangleq \mathrm{Res}\big(f_1(x_1,x_2,x_3),\ \ f_3(x_1,x_2,x_3),\ \ x_1\big) \\ &= t_1^3 - t_3 - 3t_1^3 x_2 + 3t_1 x_2^2 - 3t_1^2 x_3 + 6t_1 x_2 x_3 \\ &\quad - 3x_2^2 x_3 + 3t_1 x_3^2 - 3x_2 x_3^2 \end{aligned}$$

and finally

$$\begin{aligned} r(x_3) &\triangleq \mathrm{Res}\big(r_1(x_2,x_3), r_2(x_2,x_3), x_2\big) \\ &= \left(t_1^3 - 3t_1 t_2 + 2t_3 - 3t_1^2 x_3 + 3t_2 x_3 + 6t_1 x_3^2 - 6x_3^3\right)^2. \end{aligned} \tag{10}$$

The procedure is to substitute the solutions of (9) into (10) and solve for the roots $\{x_{3i}\}$. For each x_{3i}, one then solves $r_1(x_2, x_{3i})$ for the roots x_{2j}. Finally, one solves $f_1(x_1, x_{2j}, x_{3i}) = 0$ for x_{1j} to get the triples $\{(x_1, x_2, x_3) = (x_{1j}, x_{2j}, x_{3i}),\ i = 1, 2, 3,\ j = 1, 2\}$ as the only possible solutions to (4). This finite set of possible solutions can then be checked as to which are solutions of (4) satisfying $0 \le x_3 \le x_2 \le x_1 \le 1$.

The five DC source case

In this section, the five DC source case is summarized. The polynomials $p_1(x)$, $p_2(x)$, $p_3(x)$, $p_4(x)$, $p_5(x)$ in (3) are symmetric polynomials and therefore may be rewritten in terms of the power sums

$$\begin{aligned} t_1 &\triangleq x_1 + x_2 + x_3 + x_4 + x_5 \\ t_2 &\triangleq x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 \\ t_3 &\triangleq x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 \\ t_4 &\triangleq x_1^4 + x_2^4 + x_3^4 + x_4^4 + x_5^4 \\ t_5 &\triangleq x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5. \end{aligned}$$

Rewriting the polynomials $p_i(x)$ in terms of the power sums polynomials gives $p_i(t)$ for $i = 1, ..., 5$ where $t = (t_1, t_2, t_3, t_4, t_5)$. Now, $p_1(t) = t_1 - m$, so that t_1 is eliminated immediately by substituting $t_1 = m$. This leaves four polynomials in the four unknowns t_2, t_3, t_4, t_5.

$$q_5(t_2,t_3,t_4,t_5) \triangleq p_5(m,t_2,t_3,t_4,t_5)$$

$$q_7(t_2,t_3,t_4,t_5) \triangleq p_7(m,t_2,t_3,t_4,t_5)$$

$$q_{11}(t_2,t_3,t_4,t_5) \triangleq p_{11}(m,t_2,t_3,t_4,t_5)$$

$$q_{13}(t_2,t_3,t_4,t_5) \triangleq p_{13}(m,t_2,t_3,t_4,t_5)$$

The explicit expressions for the polynomials are rather long and are not needed to show the significance of reformulating the polynomials in terms of the power sums. Rather, the key point here is that the maximum degrees of each of these polynomials in t_2, t_3, t_4, t_5 are much less than the maximum degrees of $p_1(x)$, $p_5(x)$, $p_7(x)$, $p_{11}(x)$, $p_{13}(x)$ in x_1, x_2, x_3, x_4, x_5 as seen by comparing their values given in the two tables below.

	deg in t_2	deg in t_3	deg in t_4	deg in t_5
$q_5(t)$	0	1	0	1
$q_7(t)$	2	1	1	1
$q_{11}(t)$	4	3	2	2
$q_{13}(t)$	4	3	2	2

	degrees in x_1, x_2, x_3, x_4, x_5
$p_5(x_1, x_2, x_3, x_4, x_5)$	5
$p_7(x_1, x_2, x_3, x_4, x_5)$	7
$p_{11}(x_1, x_2, x_3, x_4, x_5)$	11
$p_{13}(x_1, x_2, x_3, x_4, x_5)$	13

Consequently, the computational burden of finding the resultant polynomials (i.e., the determinants of the Sylvester matrices) is greatly reduced. Proceeding, the indeterminate t_5 is eliminated first by computing

$$r_{q5,q7}(t_2,t_3,t_4) = \mathrm{Res}\big(q_5(t_2,t_3,t_4,t_5),\ q_7(t_2,t_3,t_4,t_5),\ t_5\big)$$
$$r_{q5,q11}(t_2,t_3,t_4) = \mathrm{Res}\big(q_5(t_2,t_3,t_4,t_5),\ q_{11}(t_2,t_3,t_4,t_5),\ t_5\big)$$
$$r_{q5,q13}(t_2,t_3,t_4) = \mathrm{Res}\big(q_5(t_2,t_3,t_4,t_5),\ q_{13}(t_2,t_3,t_4,t_5),\ t_5\big)$$

where

	deg in t_2	deg in t_3	deg in t_4
$r_{q5,q7}(t_2, t_3, t_4)$	2	1	1
$r_{q5,q11}(t_2, t_3, t_4)$	4	2	2
$r_{q5,q13}(t_2, t_3, t_4)$	4	3	2

Eliminating t_4 from these three polynomials gives the two polynomials

$$r_1(t_2,t_3) \triangleq \mathrm{Res}\big(r_{q5,q7}(t_2,t_3,t_4),\ r_{q5,q11}(t_2,t_3,t_4),\ t_4\big)$$

$$r_2(t_2,t_3) \triangleq \mathrm{Res}\big(r_{q5,q7}(t_2,t_3,t_4),\ r_{q5,q13}(t_2,t_3,t_4),\ t_4\big)$$

where

	deg in t_2	deg in t_3
$r_1(t_2, t_3)$	2	4
$r_2(t_2, t_3)$	3	4

Both of these polynomials have a lower degree in t_2 than in t_3, so the resultant computation is less intensive if t_2 is eliminated in the next step rather than t_3. Proceeding, t_2 is eliminated from $r_1(t_2, t_3)$ and $r_2(t_2, t_3)$ to get

$$\begin{aligned} r_1(t_3) &\triangleq \mathrm{Res}\big(r_1(t_2,t_3),\ r_2(t_2,t_3),\ t_2\big) \\ &= Cm^5(m^3 - t_3)^5 g_1^2(m,t_3) g_2(m,t_3) \end{aligned}$$

where C is a constant, $g_1(m, t_3)$ is a polynomial of degree 2 in t_3, and $g_2(m, t_3)$ is a polynomial of degree 9 in t_3 and degree 39 in $m = t_1$. It turns out that neither $g_1(m, t_3) = 0$ nor $t_3 = m_3$ lead to solutions for the switching angles as the corresponding x_i's do not satisfy $0 \le x_5 \le \cdots \le x_2 \le x_1 \le 1$. Consequently, only the 9 roots of $g_2(m, t_3)$ need be computed for each value of m.

One then back solves these equations for the five tuples $(t_1, t_2, t_3, t_4, t_5)$ that are solutions to the system of polynomial equations $p_i(t)$ for $i = 1, ..., 5$. To obtain the corresponding values of $(x_1, x_2, x_3, x_4, x_5)$ for each of the solutions $(t_1, t_2, t_3, t_4, t_5)$, elimination theory is again used to solve the system of polynomial equations

$$f_1(x) = t_1 - (x_1 + x_2 + x_3 + x_4 + x_5) = 0$$

$$f_2(x) = t_2 - (x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2) = 0$$

$$f_3(x) = t_3 - (x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3) = 0$$

$$f_4(x) = t_4 - (x_1^4 + x_2^4 + x_3^4 + x_4^4 + x_5^4) = 0$$

$$f_5(x) = t_5 - (x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5) = 0$$

As in the case of three dc sources, this is straightforwardly accomplished (See above).

Computational results

Using the fundamental switching scheme of Fig. 1, the solutions of (2) were computed using the method described above. These solutions are plotted in Fig. 2 versus the parameter m. As the plots show, for m in the intervals [2.21, 3.66] and [3.74, 4.23] as well as $m = 1.88, 1.89$, the output waveform can have the desired fundamental with the 5^{th},7^{th}, 11^{th}, 13^{th} harmonics absent. Further, in the subinterval [2.53, 2.9] two sets of solutions exist, while in the subinterval [3.05, 3.29], there are three sets of solutions. In the case of multiple solution sets, one would typically choose the set that gives the lowest total harmonic distortion (THD). In those intervals for which no solutions exist, one must use a different switching scheme (see [23] for a discussion on such possibilities). The corresponding total harmonic distortion (THD) in percent was computed out to the 31^{st} according to

$$THD = 100 \times \sqrt{\frac{V_5^2 + V_7^2 + V_{11}^2 + V_{13}^2 + \cdots + V_{31}^2}{V_1^2}}$$

and is plotted versus m in Fig. 3 for each of the solution sets shown in Fig. 2. As this figure shows, one can choose a particular solution for the switching angles such that the THD is 6.5 % or less for $2.25 \le m \le 4.23$ $(0.45 \le m_a \le 0.846)$.

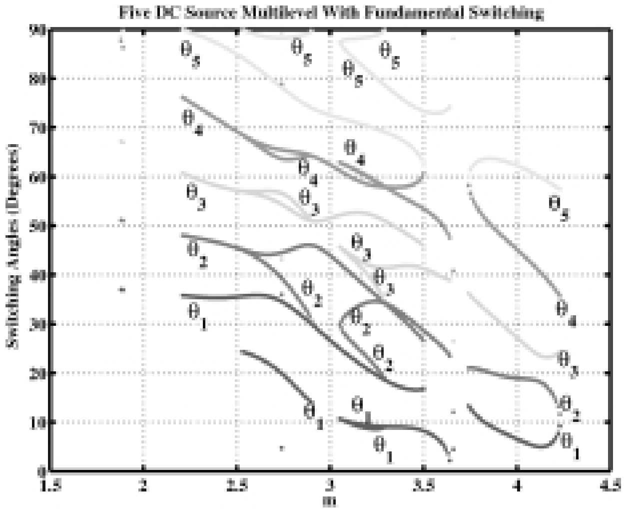

Fig. 2: Switching angles vs m for the 5 dc source multilevel converter ($m_a = m/s$ with $s = 5$)

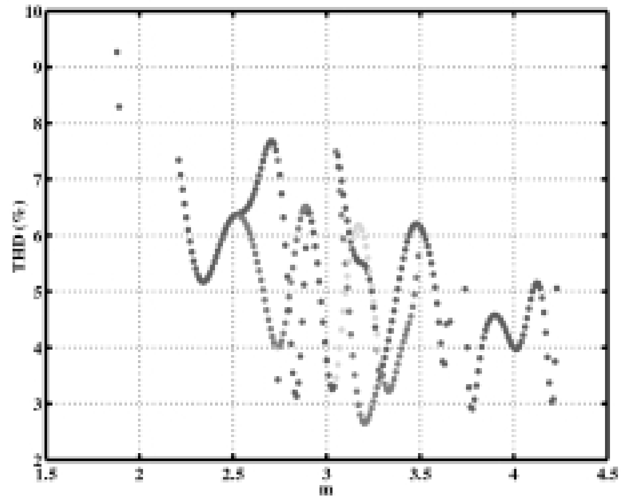

Fig. 3: The total harmonic distortion versus m for each solution set ($m_a = m/s$ with $s = 5$)

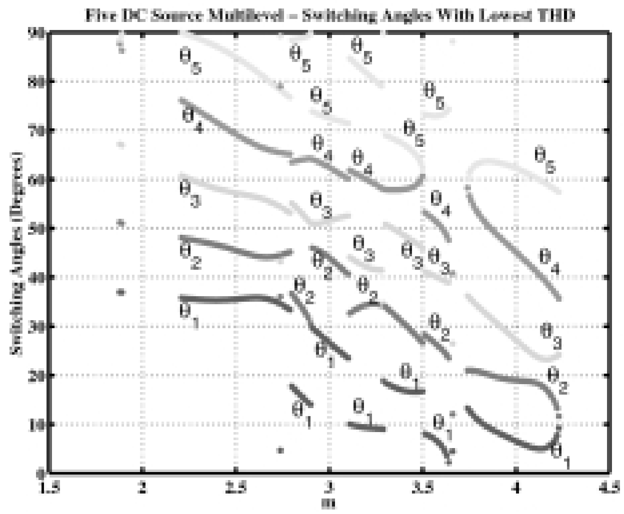

Fig. 4: The switching angles vs m which give the lowest THD for the 5 dc source multilevel converter ($m_a = m/s$ with $s = 5$)

For those values of m for which multiple solution sets exist, an appropriate choice is the one that results in the lowest THD. This was done and is shown in Fig. 4. Fig. 2 shows that there is a solution set for m in the interval [2.21, 3.66] that is continuous as a function of m_o. However, Fig. 4 shows that in the subintervals [2.8, 2.9] and [3.11, 3.29], one chooses a different solution set to obtain a smaller THD. A look at Fig. 3 shows that this difference in THD can be as much as 3.5 %, which is significant.

If one had used an iterative method such as Newton-Raphson, then in all likelihood only one solution set would be found and would most certainly not be the solution set that results in the lowest THD for m in the subintervals [2.8, 2.9] and [3.11, 3.29]. The reason the Newton-Raphson method would not have found this solution set is simply due to the way it is implemented. One starts with an initial guess for the angles at $m = 2.21$ (It would take some guessing to even know what value of m to start with!). Then the solution set for this value of m would be used as the initial guess for the solution when m is incremented by Δm to its next value and so on. At $m = 2.21$, there is only one possible solution as Fig. 2 shows. Then, as m is incremented, the Newton-Raphson algorithm would give the solution set in Fig. 2 that is continuous as a function of m, which is not always the solution set with the smallest THD. In contrast, the method proposed here finds the complete solution set and allows one to be sure that the solution with the lowest THD is used.

Experimental results

The same experimental setup described in [13] was used for this work. It is a three-phase 11-level (5 dc sources) wye-connected cascaded inverter using 100 V, 70 A MOSFETs as the switching devices [19]. The gate driver boards and MOSFETs are shown in Fig. 5. A battery bank of 15 SDCSs of 36 V (not shown) each feed the inverter (5 SDCSs per phase). The ribbon cable shown in the figure provides the communication link between the gate driver board and the real-time processor.

In this work, the RT-LAB real-time computing platform from Opal-RT-Technologies Inc. [24] was used to interface the computer (which generates the logic signals) to this cable. This system allows one to implement the switching algorithm as a lookup table in Simulink which is then converted to *C* code using RTW (real-time workshop) from Mathworks. The RT-LAB software provides icons to interface the Simulink model to the digital I/O board and converts the *C* code into executables.

The step size for the realtime implementation was 32 microseconds. This small step was used to obtain an accurate resolution for implementing the switching times. Using the XHP (extreme high performance) option in RT-LAB as well as the multiprocessor option to spread the computation between two processors, an execution time of 32 microseconds can be achieved.

Note that while the computation of the lookup table of Figs. 2 and 4 requires some off-line computational effort, the real-time implementation is accomplished by putting the data (i.e., Figs. 2 and 4) in a lookup table and therefore does not require high computational power for implementation. The multilevel converter was attached to a three phase induction motor with the following name plate data: rated hp =1 /3 hp, rated current = 1.5 A, rated speed = 1725 rpm and rated voltage = 208 V (RMS line-to-line at 60 Hz).

In this experiment, $m = 3.2$ was chosen to produce a fundamental voltage of $V_1 = m\,(4V_{dc}/\pi) = 3.2(4 \times 36/\pi) = 146.7$ along with $f = 60$Hz. As can be seen in Fig. 3, there are three different solution sets for $m = 3.2$. The solution set that gave the smallest THD (= 2.65 % see Fig. 3) was used. Fig. 6 shows the output voltages for the three phases.

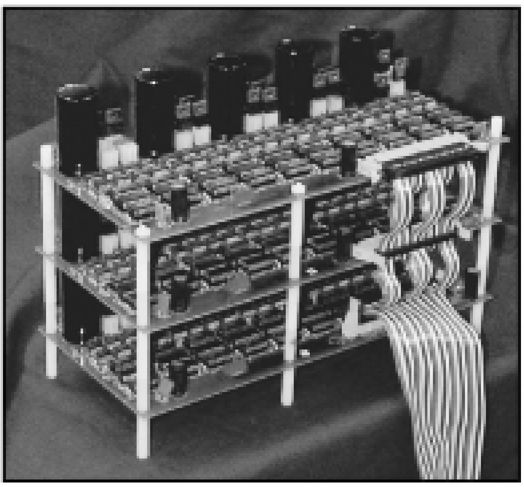

Fig. 5: Gate driver boards and MOSFETs for the mulitlevel inverter

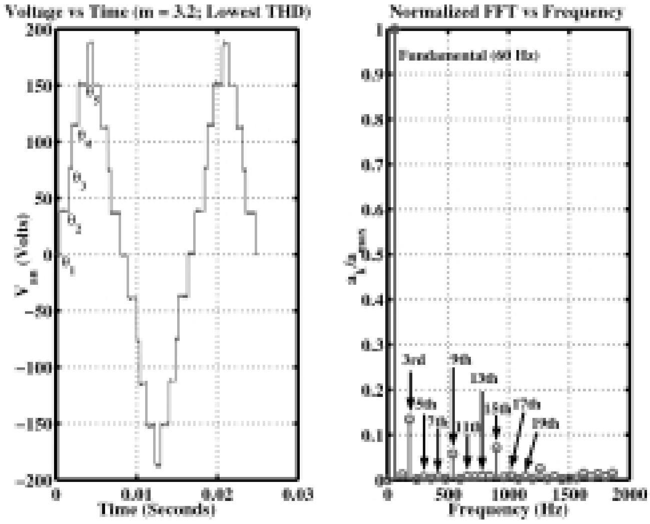

Fig. 7: Phase *a* output voltage waveform (m = 3.2) using the solutions set with the lowest THD and its normalized FFT.

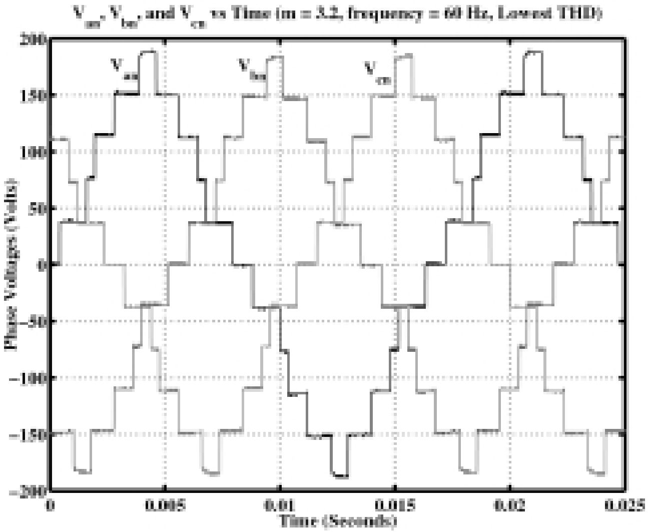

Fig. 6: Three phase output voltages for m = 3.2.

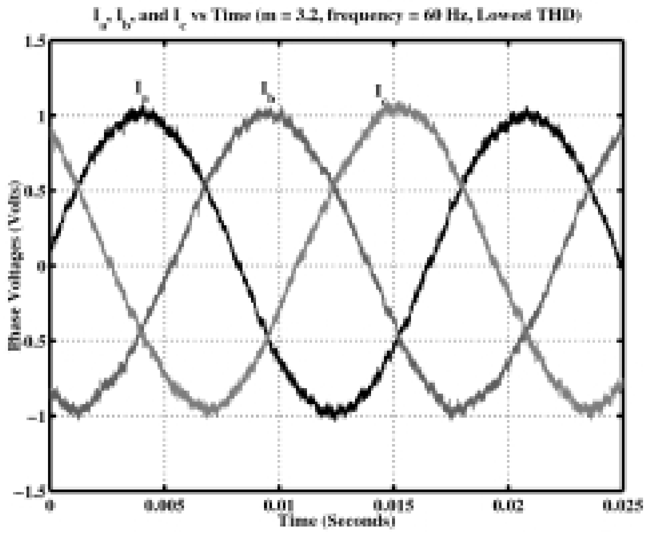

Fig. 8: Three phase currents for m = 3.2.

Fig. 7 shows the phase *a* voltage and its corresponding FFT showing that the 5^{th}, 7^{th}, 11^{th} and 13^{th} are absent from waveform as predicted. The THD of the line-line voltage was computed using the data in Fig. 7 and was found to be 2.8 %, comparing favorably with the value of 2.65 % predicted in Fig. 3.

The motor currents corresponding to the output voltages of Fig. 6 are shown in Fig. 8.

Fig. 9 shows the phase *a* current and its corresponding FFT illustrating that the harmonic content of the current (1.9 % THD) is much less than the voltage due to the filtering by the motor's inductance.

Conclusions

A procedure to eliminate harmonics in a multilevel inverter has been given which exploits the properties of the transcendental equations that define the harmonic content of the converter output. Specifically, it was shown that one can transform the transcendental equations into symmetric polynomials which are then further transformed into another set of polynomials using power sums.

This formulation resulted in a drastic reduction in the degrees of the polynomials that characterize the solution. Consequently, the computation of solutions of this final set of polynomial equations were easily carried out using elimination theory (resultants) as the required symbolic computations were well within the capabilities of contemporary computer algebra software tools. This methodology resulted in the complete characterization of the solutions to the harmonic elimination problem. That is, for each *m*, it produces all possible solutions or it shows that no solution exists. This is in contrast to numerical techniques such as Newton-Raphson, optimization software, etc. (for example, see [8], [25]) where one gets only one solution or no solution and is left to ponder whether a solution exists or not. On the other hand, the approach here is more computationally complex than the Newton Raphson technique. (For example, the equations to be solved in [26] involve both sines and cosines resulting in high degree polynomials for which a computer algebra system can have diffculty in computing the resultants.) Finally, experiments were performed and the data presented corresponded well with the predicted results.

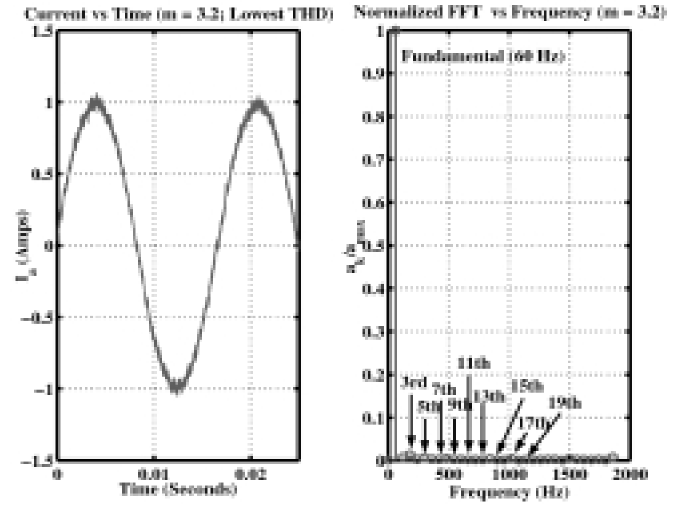

Fig. 9: Phase *a* current corresponding to the voltage in Fig. 7 and its normalized FFT.

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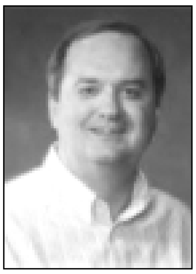
The Authors



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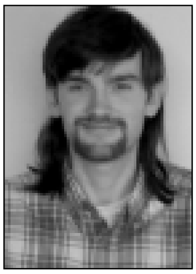
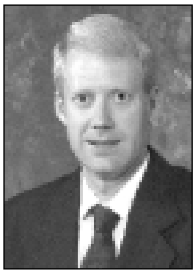


Keith J. McKenzie (S'01) received his B.S. degree in electrical engineering from The University of Tennessee (UT), Knoxville, in December 2001 and is currently pursuing the M.S. degree at the same university.



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Agenda

Technically sponsored conferences

4 – 8 April 2005, Niigata, Toki Messe, Japan
IPEC-Niigata 2005,
Secretariat of IPEC-Niigata 2005
c/o ICS Convention Design., Inc.
Sumitomo Corporation Jinbocho Building
3-24, Kanda-Nishikicho, Chiyoda-ku,
Tokyo, 101-8449, Japan
Phone: +81-3-3219-3541
Fax: +81-3-3292-1811
e-mail: ipec2005@ics-inc.co.jp
http://www.iee.or.jp/ias/ipec05/

1 to 3 June, 2005, Gda_sk, Poland,
4th International Workshop, Compatibility in Power Electronics, CPE 2005
Contact: Gdynia Maritime University
Faculty of Electrical Engineering
CPE2005
81-225 Gdynia, Morska str. 81-87
Poland
Phone: (+48 58) 690 16 51,
fax: (+48 58) 621-99-38
e-mail: cpe2005@am.gdynia.pl
www.am.gdynia.pl/cpe2005

27 – 30 June 2005, St. Petersburg, Russia
POWERTECH 2005
Energy Systems Institute, Irkutsk
130, Lermontov str.
Irkutsk 664033, Russia
Fax: +7 3952 426796
E-mail: conf2005@isem.sei.irk.ru
http://www.sei.irk.ru/PowerTech2005/

11-14 September 2005, Dresden, Germany
EPE 2005, 11th European Conference on Power Electronics and Applications
Contact:
EPE Association
c/o VUB-TW
Pleinlaan 2
B-1050 Brussels
Tel: +32 2 629 28 19
Fax: +32 2 629 36 20
e-mail: epe-association@vub.ac.be
http://www.epe2005.com

28 – 30 September 2005, Santander (Spain)
SAAEI'2005 - XII Annual Seminar on Automatic Control, Industrial Electronics and Instrumentation
Contact: Grupo de Ingeniería Microelectrónica
Departamento TEISA
E.T.S.I.I.T. Universidad de Cantabria
Avda. Los Castros s/n
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http://www.saaei.org

13 & 14 october 2005, Paris/Versailles, France
Civil or Military All-Electric Ship – AES 2005
Contact: AES 2005
SEE
17, rue Hamelin
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France
Tel: +33 1 56 90 37 04
Fax: + 33 1 56 90 37 08
http://www.aes2005.org

17-19 October, 2005, Warsaw, Poland
International Conference on Power Electronics and Intelligent Control for Energy Conservation
Contact: Institute of Control and Industrial Electronics, Warsaw University of Technology
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00-662 Warszawa POLAND
tel.:(+48 22) 628 06 65
fax.: (+48 22) 625 66 33
http:/ /pelincec. isep.pw.edu.pl

12-14 June, 2006, Lund, Sweden
NORPIE 2006
Contact: Lund University, Lund Institute of Technology
Department of Industrial Electrical Engineering and Automation
Lund, Sweden
http://www.iea.lth.se/norpie2006