Low Switching Frequency Active Harmonic Elimination in Multilevel Converters with Unequal DC Voltages

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***Abstract* — This paper presents an active harmonic elimination modulation control method for the cascaded H-bridges multilevel converter when supplied by unequal DC sources. First, the multilevel converter is decoupled into individual unipolar converters, and the low order harmonics (such as the 5th, 7th, 11th, and 13th) are eliminated by using resultant theory while at the same time the minimum specified harmonic distortion (HD) of a combination of unipolar converters for multilevel converter control is found. Next, the switching angle sets corresponding to the minimum harmonic distortion are used as initial guesses with the Newton climbing method to eliminate the specified higher order harmonics. If the solutions are not available for some modulation indices, the unipolar switching scheme is used to eliminate high order harmonics and the active harmonic elimination method is used to eliminate low order harmonics. This method has lower switching frequency than that of the previously proposed active harmonic elimination method.**

***Keywords*—Multilevel converter, harmonic elimination.**

I. INTRODUCTION

The multilevel converter is a promising technology for renewable/distributed energy applications of power electronics because of its easy connection with renewable/distributed energy modules such as fuel cells, solar panels, and wind turbines. Other benefits also include its low electromagnetic interference (EMI) and high efficiency with low switching frequency control methods [1].

There are four kinds of control methods for multilevel converters. They are the selective harmonic elimination method, space vector control method, traditional PWM control method, and space vector PWM method [2-4]. The space vector control method, the space vector PWM method, and the traditional PWM control method with carrier phase shifting require equal DC voltages for the multilevel converters. Control of multilevel converters with unequal DC voltages is much more complicated [5-6].

To address the issue of eliminating higher order harmonics at low modulation indices, the active harmonic elimination method has been proposed [7]. The active harmonic elimination method with unequal DC sources uses a unipolar switching scheme in which the switch angles are determined using elimination theory to remove low-order harmonics. Then, the specifically chosen high-order harmonics (e.g., the odd non triplen harmonics) are eliminated by using an additional switching angle (one for each higher harmonic) to generate the negative of the harmonic to cancel it [8].

The active harmonic elimination in [7-8] has a disadvantage in that it uses a high switching frequency to eliminate higher order harmonics. This paper proposes a modification to the previously proposed harmonic elimination method in [7-8] that is equally effective at removing harmonics but uses a lower switching frequency. First, the multilevel converter is decoupled into individual unipolar converters and the low-order harmonics (such as the 5th, 7th, 11th, and 13th) are eliminated by using resultant theory while at the same time the minimum specified harmonic distortion (HD) of a combination of unipolar converters for multilevel converter control is found. Next, the switching angle sets corresponding to the minimum harmonic distortion are used as initial guesses for the Newton climbing method to eliminate the specified high-order harmonics.

However, for some modulation indices, no solutions exist. Under such situations, the active harmonic elimination method can use the unipolar switching scheme to eliminate high-order harmonics, and use the active harmonic method to eliminate low-order harmonics to decrease the required switching frequency. Compared to the active harmonic elimination method proposed for H-bridge multilevel converters previously [8], this method has lower switching frequency. Therefore, it is referred as to the optimized harmonic elimination method.

A 5-level example and a 7-level example are given in this paper. Matlab simulations and experiments are employed to validate the proposed method. The experimental results show that the method can effectively eliminate the specific harmonics, and the output voltage waveforms have low total harmonic distortion (THD) as expected in theory.

II. UNIPOLAR SWITCHING SCHEME AND ITS SOLUTION

Based on harmonic elimination theory [9-11], the control of the sinusoidal wave generation is to choose a series of switching angles to synthesize a desired sinusoidal voltage waveform. A typical 5-angle unipolar switching output is shown in Fig. 1. The Fourier series expansion of the output voltage waveform shown in Fig. 1 is

$$V(t)=\sum_{n=1,3,5\ldots}^{\infty}\frac{4V_{dc}}{n\pi}[\cos(n\theta_1)-\cos(n\theta_2)+\cos(n\theta_3)-\cos(n\theta_4)+\cos(n\theta_5)]\sin(n\omega t) \quad (1)$$

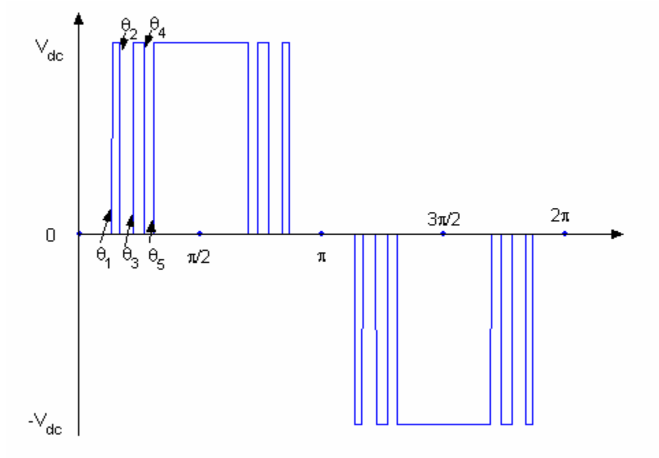


Fig. 1. Five-angle unipolar switching output.

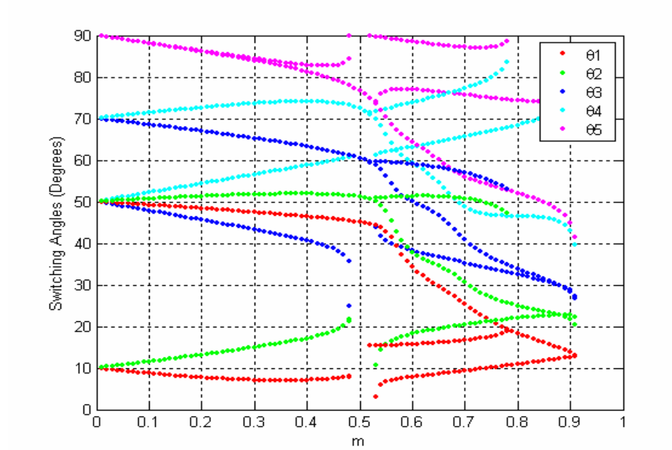


Fig. 2. Switching angle solutions to 5-angle unipolar switching scheme vs. *m*.

Ideally, given a desired fundamental voltage V_1, one wants to determine the switching angles θ_1, θ_2, θ_3, θ_4, and θ_5 so that $V(\omega t)=V_1(\omega t)$, and specific higher harmonics of $V_n(\omega t)$ are equal to zero. For a three-phase application, the triplen harmonics in each phase need not be canceled as they automatically cancel in the line-to-line voltages. Here, the 5th, 7th, 11th, and 13th order harmonics are chosen to be removed.

That is, the switching angles must satisfy the following equations:

$$\begin{aligned}
&\cos(\theta_1)-\cos(\theta_2)+\cos(\theta_3)-\cos(\theta_4)+\cos(\theta_5)=m\\
&\cos(5\theta_1)-\cos(5\theta_2)+\cos(5\theta_3)-\cos(5\theta_4)+\cos(5\theta_5)=0\\
&\cos(7\theta_1)-\cos(7\theta_2)+\cos(7\theta_3)-\cos(7\theta_4)+\cos(7\theta_5)=0\\
&\cos(11\theta_1)-\cos(11\theta_2)+\cos(11\theta_3)-\cos(11\theta_4)+\cos(11\theta_5)=0\\
&\cos(13\theta_1)-\cos(13\theta_2)+\cos(13\theta_3)-\cos(13\theta_4)+\cos(13\theta_5)=0
\end{aligned} \tag{2}$$

Here, m is defined as the modulation index as

$$m=\pi V_1/(4V_{dc}), \tag{3}$$

and the THD is computed as

$$THD=\sqrt{\sum_{i=5,7,11,13,\cdots}^{49} V_i^2}\Big/V_1 \cdot \tag{4}$$

The resultant method described in [12] is used to compute the solutions to (2), and these are shown in Fig. 2. Fig. 3 shows the THD corresponding to these solutions.

From the switching angle solutions shown in Fig. 2, it can be derived that the solutions exist in a range of the modulation indices from 0 to 0.91. Some modulation indices have a single solution, and there are multiple solution sets for other modulation indices.

Fig. 3 shows that different solution sets have different THD values. Another feature is the THD is very high (> 50%) for the low modulation index range (m < 0.5).

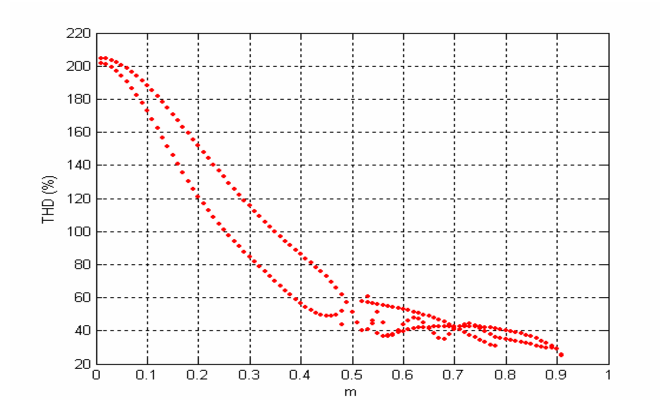


Fig. 3. THD vs. *m* for switching angles shown in Fig. 2.

III. Proposed Optimized Harmonic Elimination Method

Cascaded H-bridge multilevel converters can be viewed as unipolar converters connected in series; the total modulation index for a multilevel converter when decoupled control is applied is

$$m=\sum_{i=1}^{s} c_i k_i m_i \cdot \tag{5}$$

Here, V_{dc} is the nominal DC voltage, V_{dci} is the ith DC voltage, c_i ($c_i \in \{-1, 0, 1\}$) is called the combination coefficient, and $k_i = V_{dci}/V_{dc}$ is called the unbalance coefficient. If $c_i = 1$, level i produces a positive voltage; if $c_i = 0$, the level is bypassed; and if $c_i = -1$, the level produces a negative voltage.

For each 5-angle unipolar converter, the residual higher harmonic contents also can be computed by (1). The normalized harmonic magnitudes are computed as

$$V_n=\frac{4}{n\pi}[\cos(n\theta_1)-\cos(n\theta_2)+\cos(n\theta_3)-\cos(n\theta_4)+\cos(n\theta_5)] \cdot \tag{6}$$

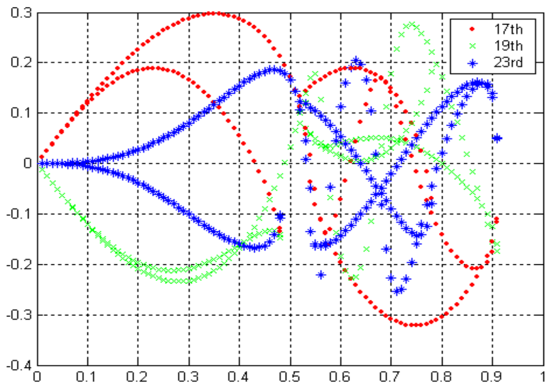


Fig. 4. Normalized 17th, 19th, and 23rd harmonic magnitudes vs. *m*.

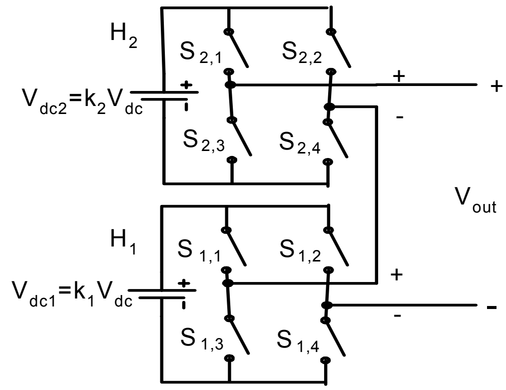


Fig. 5. Five-level multilevel converter topology.

For example, Fig. 4 shows the normalized 17th, 19th, and 23rd harmonic magnitudes vs. *m*. It can be seen that they are positive for some modulation indices and are negative for other modulation indices. Thus, it is possible to obtain a combination of several unipolar converters for each modulation index *m* of the multilevel converter with the lowest harmonic distortion for the specified harmonics.

A. Five-level case

The multilevel circuit topology for the 5-level case is shown in Fig. 5. There are two H-bridges, H_1 and H_2, and two unequal DC sources, $V_{dc1} = k_1V_{dc}$ and $V_{dc2} = k_2V_{dc}$. If we choose to have a 5-angle unipolar output for each H-bridge level, then there are a total of 10 switching angles for the 5-level case. Therefore, one degree of freedom is used for fundamental control and any 9 harmonics could then be eliminated.

The modulation index for this 5-level inverter is

$$m = \sum_{i=1}^{2} c_i k_i m_i \ . \qquad (7)$$

In (7), m and k_i are given so that c_i, m_i are chosen to satisfy (7). Next, the resultant method is then used to solve the system (2) twice; once for bridge H_1 with $m = m_1$ in (2), and then for bridge H_2 with $m = m_2$ in (2). The solution set for $m = m_1$ is denoted as θ_{11}, θ_{12}, θ_{13}, θ_{14}, θ_{15} while the solution set for $m = m_2$ is denoted as θ_{21}, θ_{22}, θ_{23}, θ_{24}, θ_{25}.

Resultant theory has eliminated the lower order harmonics (5^{th}, 7^{th}, 11^{th}, 13^{th}). However, to eliminate harmonics up to the 29th, different angles need to be chosen. The initial switching angle guess is found by checking which combination of the solution sets found with the resultant theory has the lowest harmonic distortion for the remaining harmonics to be eliminated, which in this example are the 17th, 19th, 23rd, 25th, and 29th:

$$HD = V_{17}^2 + V_{19}^2 + V_{23}^2 + V_{25}^2 + V_{29}^2 , \qquad (8)$$

where, V_n is the nth harmonic which can be expressed as

$$V_n = \sum_{i=1}^{2} \frac{4c_i k_i V_{dc}}{n\pi} [\cos(n\theta_{i1}) - \cos(n\theta_{i2}) + \cos(n\theta_{i3}) - \cos(n\theta_{i4}) + \cos(n\theta_{i5})] \ . \qquad (9)$$

The switching angle sets corresponding to the lowest harmonic distortion (HD_{min}) are used as initial guesses for the Newton Climbing technique to solve (10) to control the fundamental and to eliminate the odd, non-triplen harmonics (5th, 7th, 11th, 13th, 17th, 19th, 23rd, 25th, and 29th):

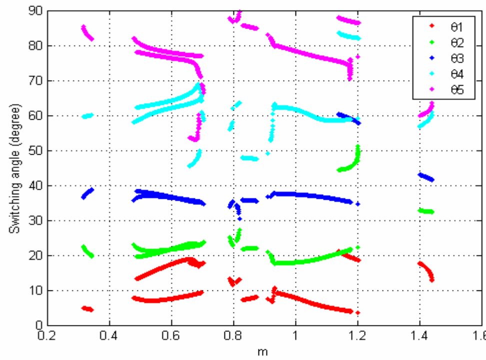
$$\begin{aligned} &\sum_{i=1}^{2} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(\theta_{ij}\right) = m \\ &\sum_{i=1}^{2} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(5\theta_{ij}\right) = 0 \\ &\vdots \\ &\sum_{i=1}^{2} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(13\theta_{ij}\right) = 0 \\ &\vdots \\ &\sum_{i=1}^{2} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(29\theta_{ij}\right) = 0 \end{aligned} \ . \qquad (10)$$

As a numerical example, an unequal case (k_1=0.75, k_2=1) is computed using this method. The switching angles are shown in Fig. 6. The switching angles shown in Fig. 6(a) are for bridge H_1, and the switching angles shown in Fig. 6(b) are for bridge H_2.

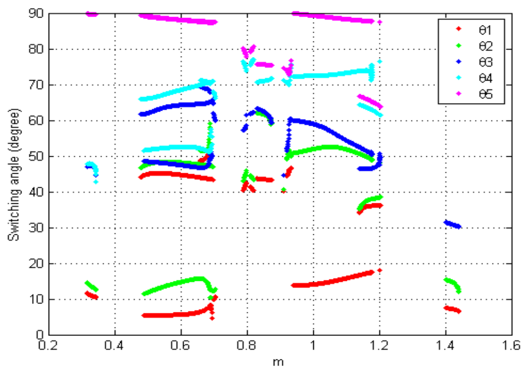
A variation of this method is to eliminate the 31st harmonic (for instance) and not the 7th harmonic (for instance). The 7th harmonic could then be eliminated using the active harmonic elimination method [7-8]. The harmonic distortion is computed up to 31st by

$$HD = V_{17}^2 + V_{19}^2 + V_{23}^2 + V_{25}^2 + V_{29}^2 + V_{31}^2 . \qquad (11)$$

The switching angle sets corresponding to the lowest harmonic distortion (HD_{min}) could be used as initial guesses for the Newton climbing method to solve (12) to eliminate the odd, non-triplen harmonics 5th, 11th, 13th, 17th, 19th, 23rd, 25th, 29th and 31st.

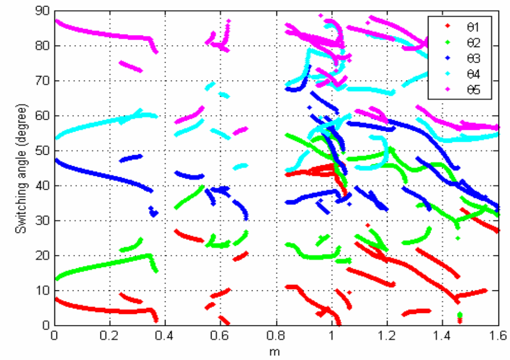


(a)

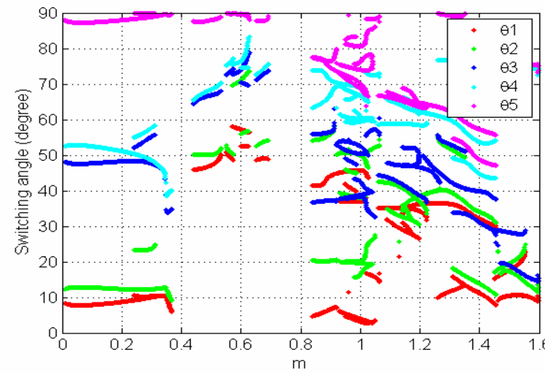


(b)

Fig. 6. Switching angles for 5-level multilevel converter to eliminate harmonics below the 29th ($k_1 = 0.75$, $k_2 = 1.0$), (a) switching angles for bridge H_1; (b) switching angles for bridge H_2.



(a)



(b)

Fig. 7. Switching angles for 5-level multilevel converter to eliminate harmonics below the 31st ($k_1 = 0.75$, $k_2 = 1.0$), (a) switching angles for bridge H_1; (b) switching angles for bridge H_2.

$$\begin{aligned}
&\sum_{i=1}^{2} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(\theta_{ij}\right) = m \\
&\sum_{i=1}^{2} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(5\theta_{ij}\right) = 0 \\
&\sum_{i=1}^{2} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(11\theta_{ij}\right) = 0 \\
&\vdots \\
&\sum_{i=1}^{2} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(31\theta_{ij}\right) = 0
\end{aligned} \tag{12}$$

Note that the 7th is not eliminated in (12). Here, the 7th harmonic, not the 5th harmonic, is chosen to be removed by the active harmonic elimination method because canceling the 5th harmonic will generate a new 25th harmonic. The switching angles are shown in Fig. 7. Similarly, the switching angles shown in Fig. 7(a) are for bridge H_1, and the switching angles shown in Fig. 7(b) are for bridge H_2.

If there are no solutions available for some modulation indices, the unipolar switching scheme can be used to eliminate high-order harmonics and the active harmonic elimination method can be used to eliminate low-order harmonics to decrease the switching frequency [7].

For example, the equations to eliminate high-order harmonics up to the 31st using the 5-angle unipolar switching scheme are:

$$\begin{aligned}
&\cos(\theta_1) - \cos(\theta_2) + \cos(\theta_3) - \cos(\theta_4) + \cos(\theta_5) = m \\
&\cos(19\theta_1) - \cos(19\theta_2) + \cos(19\theta_3) - \cos(19\theta_4) + \cos(19\theta_5) = 0 \\
&\cos(23\theta_1) - \cos(23\theta_2) + \cos(23\theta_3) - \cos(23\theta_4) + \cos(23\theta_5) = 0 \\
&\cos(29\theta_1) - \cos(29\theta_2) + \cos(29\theta_3) - \cos(29\theta_4) + \cos(29\theta_5) = 0 \\
&\cos(31\theta_1) - \cos(31\theta_2) + \cos(31\theta_3) - \cos(31\theta_4) + \cos(31\theta_5) = 0
\end{aligned} \tag{13}$$

These equations are solved by the Newton climbing method [13]. The initial guesses for the Newton climbing method are obtained from the solutions to eliminate the 5th, 7th, 11th, and 13th harmonics using the resultant method. Fig. 8 shows the solutions. The solutions can be used to obtain the lowest THD combination and eliminate the specified harmonics.

Next, active harmonic elimination is used where the magnitudes and phases of the residual lower order harmonics (5th, 7th, 11th, 13th, 17th, and 25th) are computed, generated, and subtracted from the original voltage waveform to eliminate them. In this method, the additional number of switchings required to eliminate the 5th, 7th, 11th, 13th, 17th, and 25th harmonics is bounded by

$$N_{sw} \le \sum_{n \in \{5,7,11,13,17,25\}} n \,. \tag{14}$$

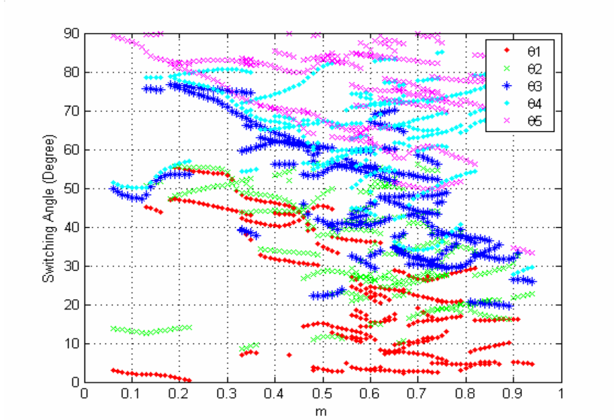


Fig. 8. Switching angles for 5-angle unipolar switching scheme to eliminate the 19th, 23rd, 29th and 31st harmonics vs. *m*.

The upper bound of the additional number of switchings for this example is 78 by (14). It is about one half of the active harmonic elimination method in theory [8]. Thus, this method can also decrease the required number of switchings.

B. Seven-level case

The multilevel circuit topology for the 7-level case is shown in Fig. 9. There are three H-bridges, H_1, H_2, and H_3, and three unequal DC sources, $V_{dc1} = k_1 V_{dc}$, $V_{dc2} = k_2 V_{dc}$, and $V_{dc3} = k_3 V_{dc}$.

If we choose to have a 5-angle unipolar output for each level, then there are a total of 15 switching angles for the 7-level case (3 H-bridges); therefore, any 14 harmonics could be eliminated (one degree of freedom is used for fundamental frequency amplitude control).

The modulation index for this 7-level inverter is

$$m = \sum_{i=1}^{3} c_i k_i m_i \tag{15}$$

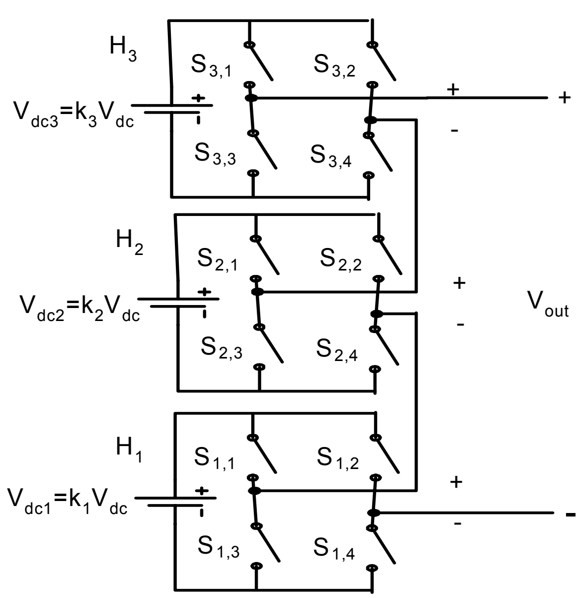
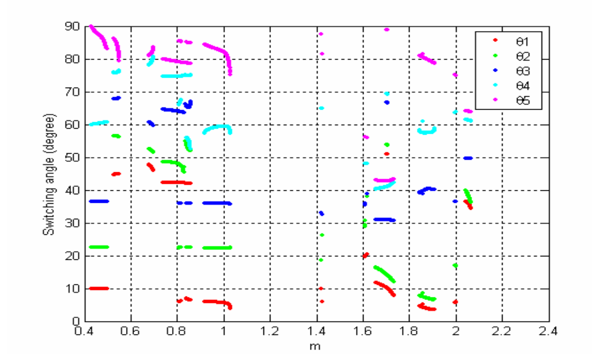
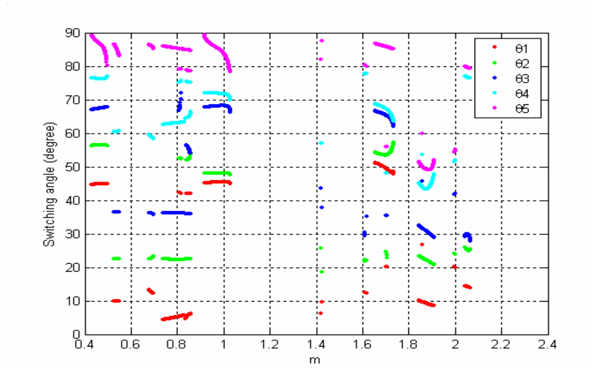


Fig. 9. Seven-level multilevel converter topology.



(a)



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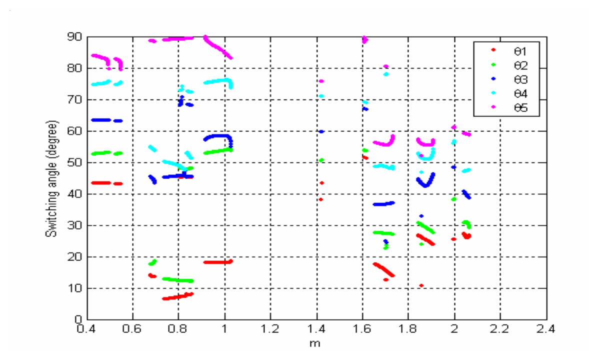


(c)

Fig. 10. Switching angles for 7-level multilevel converter to eliminate harmonics below the 43rd ($k_1 = 0.9$, $k_2 = 0.95$, $k_3 = 1.0$),
(a) switching angles for bridge H_1;
(b) switching angles for bridge H_2;
(c) switching angles for bridge H_3.

Just as in the previous section, resultant theory can be used to eliminate the lower order harmonics (5th, 7th, 11th, 13th). However, to eliminate harmonics up to the 43rd, different angles need to be chosen. The initial switching angle guess is found by checking which combination of the solution sets found with the resultant theory has the lowest harmonic distortion for the remaining harmonics to be eliminated, which in this example are the odd, nontriplen harmonics from the 17th through the 43rd:

$$\begin{aligned} HD = {} & V_{17}^2 + V_{19}^2 + V_{23}^2 + V_{25}^2 + V_{29}^2 \\ & + V_{31}^2 + V_{35}^2 + V_{37}^2 + V_{41}^2 + V_{43}^2 \end{aligned} \tag{16}$$

The switching angle sets corresponding to the lowest harmonic distortion (HD_{min}) are used as initial guesses for the Newton Climbing technique to solve (17) to control the fundamental and to eliminate the odd, non-triplen harmonics (5th, 7th, 11th, 13th, 17th, 19th, 23rd, 25th, 29th, 31st, 35th, 37th, 41st, and 43rd):

$$\begin{aligned}
&\sum_{i=1}^{3} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(\theta_{ij}\right) = m \\
&\sum_{i=1}^{3} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(5\theta_{ij}\right) = 0 \\
&\vdots \\
&\sum_{i=1}^{3} k_i c_i \sum_{j=1}^{5} (-1)^{j+1} \cos\left(43\theta_{ij}\right) = 0
\end{aligned} \tag{17}$$

As an example, an unequal case ($k_1 = 0.9$, $k_2 = 0.95$, $k_3 = 1$) is computed. The switching angles are shown in Fig. 10. The switching angles shown in Fig. 10(a) are for bridge H_1, the switching angles shown in Fig. 10(b) are for bridge H_2, and the switching angles shown in Fig. 10(c) are for bridge H_3.

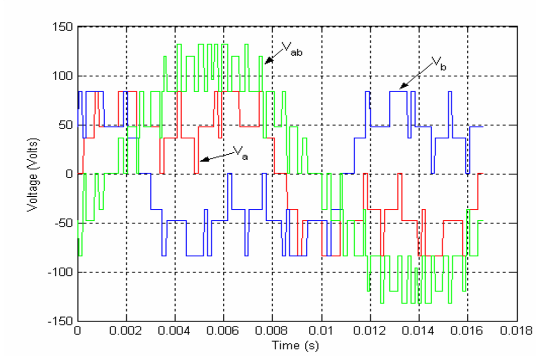
IV. Simulation and Experiment

Simulation has been used to validate the proposed algorithm. Fig. 11(a) shows voltage waveforms of a 5-level simulation case with $k_1 = 0.75$, $k_2 = 1.0$, $V_{dc} = 48$ V, $m = 1.080$, THD = 19.19% to eliminate harmonics up to the 29th. From the normalized FFT analysis shown in Fig. 11(b), it can be derived that all the harmonics up to 29th are zero. This confirmed the computation results.

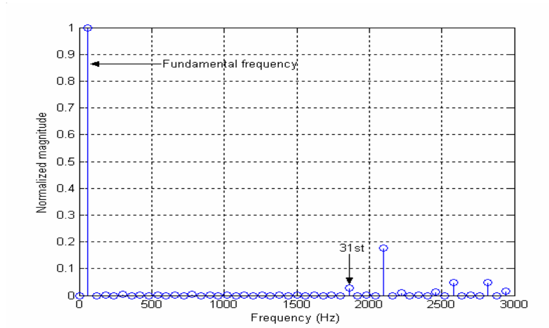
A prototype three-phase 11-level cascaded H-bridge multilevel converter has been built using 60 V, 70 A MOSFETs as the switching devices to implement the algorithm using a field programmable gate array (FPGA) controller with 8 µs control resolution.

The 5-level simulation case is chosen to implement with the multilevel converter. Fig. 12(a) shows the experimental line-line voltage, and Fig. 12(b) shows its corresponding normalized line-line voltage FFT analysis. Fig. 12(b) shows that the harmonics have been eliminated up to 29th. The experimental THD is 17.4%, and it corresponds very well with the theoretical computation of 19.34% and simulation result of 19.19%.

A 7-level case with with $k_1 = 0.9$, $k_2 = 0.95$, $k_3 = 1.0$, $V_{dc} = 38$ V, and $m = 1.66$ is also chosen to implement with the multilevel converter to validate the proposed algorithm. Fig. 13(a) shows the experimental line-line voltage, and Fig. 13(b) shows its corresponding normalized line-line voltage FFT analysis. Fig. 13(a) shows that the harmonics have been eliminated up to 43rd. The experimental THD is 6.42%, and it corresponds very well with the theoretical computation of 6.87% and simulation result of 6.89%.

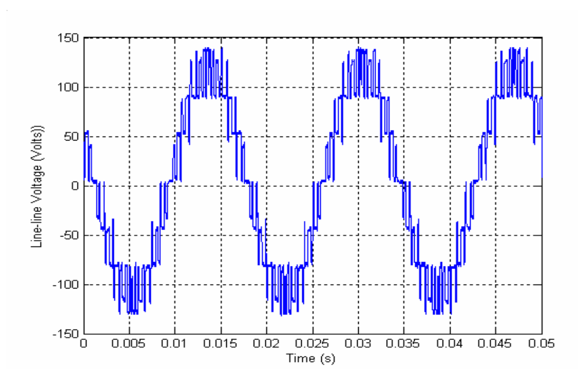


(a)

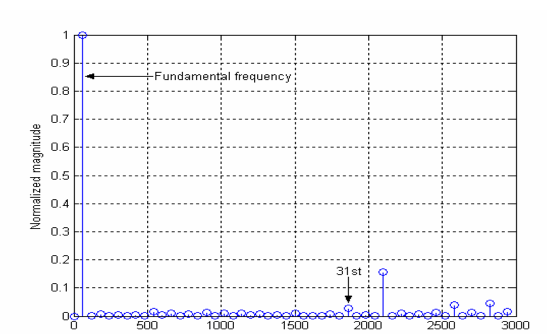


(b)

Fig. 11. Simulation of a 5-level case ($k_1 = 0.75$, $k_2 = 1$, $V_{dc} = 48$ V, $m = 1.080$, THD = 19.19%) to eliminate harmonics up to 29th, (a) voltage waveform; (b) normalized FFT analysis of line-line voltage.

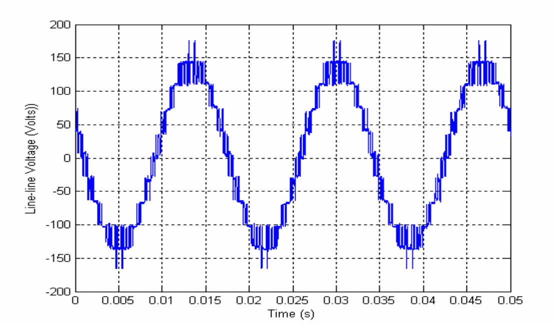


(a)

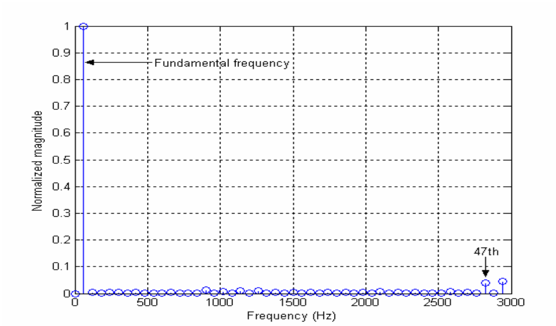


(b)

Fig. 12. Experiment of 5-level case ($k_1 = 0.75$, $k_2 = 1$, $V_{dc} = 48$ V, $m = 1.080$, THD = 17.4%) to eliminate harmonics up to 29th, (a) line-line voltage; (b) normalized FFT analysis of line-line voltage.



(a)



(b)

Fig. 13. Experimental voltages for 7-level case ($k_1 = 0.9$, $k_2 = 0.95$, $k_3 = 1$, $V_{dc} = 38$ V, $m = 1.66$, THD = 6.42%) to eliminate harmonics up to 43rd, (a) line-line voltage; (b) Normalized FFT analysis of line-line voltage

V. CONCLUSIONS

This paper presents an active harmonic elimination modulation control method for a cascaded H-bridges multilevel converter with unequal DC sources. This method has lower switching frequency than the earlier proposed method [8].

First, the multilevel converter is decoupled into individual unipolar converters and the low order harmonics (such as the 5th, 7th, 11th and 13th) are eliminated by using elimination theory while at the same time the minimum specified harmonic distortion (HD) of a combination of unipolar converters for multilevel converter control is found. Next, the switching angle sets corresponding to the minimum harmonic distortion are used as initial guesses with the Newton climbing method to eliminate the specified higher order harmonics.

If the solutions are not available for some modulation indices, the unipolar switching scheme is used to eliminate high order harmonics and the active harmonic elimination method is used to eliminate low order harmonics. The experiments validated that the proposed method can eliminate the specified harmonics as expected.

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REFERENCES

[1] J. S. Lai, F. Z. Peng, "Multilevel converters – A new breed of power converters," *IEEE Transactions on Industry Applications*, vol. 32, no.3, May./June 1996, pp. 509-517.

[2] J. Rodríguez, J. Lai, F. Peng, "Multilevel inverters: a survey of topologies, controls and applications," *IEEE Transactions on Industry Applications*, vol. 49, no. 4, Aug. 2002, pp. 724-738.

[3] G. Carrara, S. Gardella, M. Marchesoni, R. Salutari, G. Sciutto, "A new multilevel PWM method: a theoretical analysis," *IEEE Transactions on Power Electronics*, vol. 7, no. 3, July 1992, pp. 497-505.

[4] J. K. Steinke, "Control strategy for a three phase AC traction drive with a 3-Level GTO PWM inverter," *IEEE PESC*, 1988, pp. 431-438.

[5] L. M. Tolbert, J. N. Chiasson, Z. Du, and K. J. McKenzie, "Elimination of harmonics in a multilevel converter with non equal DC sources," *IEEE Transactions on Industry Applications*, vol. 41, no. 1, Jan./Feb. 2005, pp. 75-82.

[6] L. M. Tolbert, F. Z. Peng, T. Cunnyngham, J. N. Chiasson, "Charge balance control schemes for multilevel converter in hybrid electric vehicles," *IEEE Transactions on Industrial Electronics,* vol. 49, no. 5, October 2002, pp. 1058-1065.

[7] Z. Du, L. M. Tolbert, J. N. Chiasson, "Harmonic elimination for multilevel converter with programmed PWM method," *IEEE Industry Applications Society Annual Meeting,* October 3-7, 2004, Seattle, Washington, pp. 2210-2215.

[8] Z. Du, L. M. Tolbert, J. N. Chiasson, "Active Harmonic Elimination in Multilevel Converters Using FPGA Control," *IEEE COMPEL*, August 15-18, 2004, Urbana-Champaign, Illinois, pp. 127-132.

[9] H. S. Patel, R. G. Hoft, "Generalized harmonic elimination and voltage control in thyristor inverters: Part I –harmonic elimination," *IEEE Transactions on Industry Applications*, vol. 9, May/June 1973. pp. 310-317.

[10] H. S. Patel, R. G. Hoft, "Generalized harmonic elimination and voltage control in thyristor inverters: Part II –voltage control technique," *IEEE Transactions on Industry Applications*, vol. 10, Sept./Oct. 1974, pp. 666-673.

[11] P. N. Enjeti, P. D. Ziogas, J. F. Lindsay, "Programmed PWM techniques to eliminate harmonics: A critical evaluation" *IEEE Transactions on Industry Applications*, vol. 26, no. 2, March/April. 1990. pp. 302 – 316.

[12] J. N. Chiasson, L. M. Tolbert, K. J. McKenzie, Z. Du, "Control of a multilevel converter using resultant theory, " *IEEE Transactions on Control Systems Technology*, vol. 11, no. 3, May 2003, pp. 345-354.

[13] Z. Du, L. M. Tolbert, J. N. Chiasson, "Modulation extension control for multilevel converters using triplen harmonic injection with low switching frequency," *IEEE Applied Power Electronics Conference,* March 6-10, 2004, Austin, Texas, pp. 419-423.

[14] C. K. Duffey, R. P. Stratford, "Update of harmonic standard IEEE-519: IEEE recommended practices and requirements for harmonic control in electric power systems," *IEEE Transactions on Industry Applications*, vol. 25, no. 6, Nov./Dec. 1989, pp. 1025-1034.