Harmonic Elimination in Multilevel Converters

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Abstract— **A method is presented to compute the switching angles in a multilevel converter so as to produce the required fundamental voltage while at the same time not generate higher order harmonics. Using a fundamental switching scheme, previous work has shown that this is possible only for specific ranges of the modulation index. Here it is shown for a three DC source multilevel inverter that, by modifying the switching scheme, one can extend the range of modulation indices for which the switching angles exist to achieve the fundamental while eliminating the 5^{th} and 7^{th} harmonics. In contrast to numerical techniques, the approach here produces all possible solutions.**

Keywords— **Multilevel Converters, Harmonic Elimination, Resultants, Symmetric Polynomials**

I. Introduction

Electric power production in the 21st Century will see dramatic changes in both the physical infrastructure and the control and information infrastructure. A shift will take place from a relatively few large, concentrated generation centers and the transmission of electricity over mostly a high voltage ac grid to a more diverse and dispersed generation infrastructure that also has a higher percentage of dc transmission lines [1].

The general function of the multilevel inverter is to synthesize a desired ac voltage from several levels of dc voltages. For this reason, multilevel inverters are ideal for connecting either in series or in parallel an ac grid with distributed energy resources such as photovoltaics or fuel cells or with energy storage devices such as capacitors or batteries[2]. Additional applications of multilevel converters include such uses as medium voltage adjustable speed motor drives, static var compensation, dynamic voltage restoration, harmonic filtering, or for a high voltage dc back-to-back intertie[3]. Transformerless multilevel inverters are uniquely suited for this application because of the high VA ratings possible with these inverters [4]. The multilevel voltage source inverter's unique structure allows it to reach high voltages with low harmonics without the use of transformers or series-connected, synchronized-switching devices.

A fundamental issue for a multilevel converter is to find the switching angles (times) so that the converter produces the required fundamental voltage and does not generate specific lower order dominant harmonics. In this work, a method is presented to compute the switching angles in a multilevel converter so as to achieve this goal. Using a fundamental switching scheme (see Figure 2), previous work in [5][6] has shown that this is possible only for specific ranges of the modulation index. Here it is shown that, by modifying the switching scheme, one can extend the lower range of modulation indices for which the switching angles exist. Further, in contrast with the PWM technique proposed in [7], the switching schemes proposed here are only slightly above the fundamental frequency. In contrast to numerical techniques such as used in [8], the approach here produces all possible solutions.

II. Cascaded H-bridges

The cascade multilevel inverter consists of a series of H-bridge (single-phase full-bridge) inverter units. The general function of this multilevel inverter is to synthesize a desired voltage from several separate dc sources (SDCSs), which may be obtained from solar cells, fuel cells, or ultracapacitors. Figure 1 shows a single-phase structure of a cascade inverter with SDCSs [4].

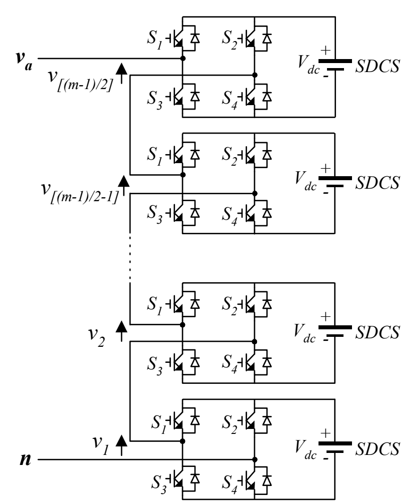

Fig. 1. Single-phase structure of a multilevel cascaded H-bridges inverter.

Each SDCS is connected to a single-phase full-bridge inverter. Each inverter level can generate three different voltage outputs, $+V_{dc}, 0$ and $-V_{dc}$ by connecting the dc source to the ac output side by different combinations of the four switches, S_1, S_2, S_3 and S_4. The ac output of each level's full-bridge inverter is connected in series such that the synthesized voltage waveform is the sum of all of the individual inverter outputs. The number of output phase (line-neutral) voltage levels in a cascade mulitilevel inverter is then $2s+1$, where s is the number of dc sources.

An example phase voltage waveform for a 7-level cascaded multilevel inverter with three SDSCs ($s = 3$) is shown in Figure 2. The output phase voltage is given by $v_{an} = v_{a1} + v_{a2} + v_{a3}$.

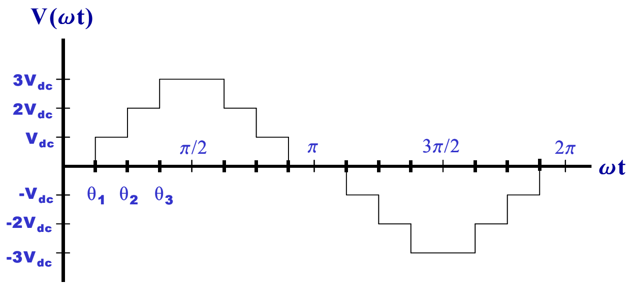

Fig. 2. Output waveform of an 7-level (3 DC source) cascade multilevel inverter.

Each of the active devices of the H-bridges switch only at the fundamental frequency, and consequently this is referred to as the fundamental switching scheme. Also, each H-bridge unit generates a quasi-square waveform by phase-shifting its positive and negative phase legs' switching timings. Each switching device always conducts for 180° (or $\frac{1}{2}$ cycle) regardless of the pulse width of the quasi-square wave so that this switching method results in equalizing the current stress in each active device.

Using the fundamental switching scheme of Figure 2, it has been shown in [6] that achieving the fundamental while eliminating specified lower order harmonics is only achievable for certain ranges of the modulation index. For example, Figure 3 is a plot of the switching solution angles in the case of three DC sources where the fundamental is achieved while the 5^{th} and 7^{th} harmonics are eliminated. Here the parameter m is related to the modulation index by $m = sm_a$ where s is the number of DC sources ($s = 3$ in Figure 3). Note that for m in the interval $[1.15, 2.52]$ there is a solution (with two different solutions in the subinterval $[1.49, 1.85]$). On the other hand, for $m \in [0, 0.8]$ ($m_a \in [0, 0.27]$), $m \in [0.83, 1.14]$ ($m_a \in [0.28, 0.383]$) and $m \in [2.53, 2.76]$ ($m_a \in [0.84, 0.92]$) there are *no* solutions.

The objective is to show how the range of values of the modulation index $m_a = m/s$ can be extended for which the fundamental is still achieved and the 5^{th} and 7^{th} harmonics are also eliminated. This is done by having more switchings per cycle. At very low modulation indices, one would surmise that only one of the DC sources would be used with multiple switchings. This is simply the unipolar programmed PWM switching scheme of Patel and Hoft [9][10] (see Figure 4). At slightly higher modulation indices one would surmise that two DC sources would be used with a scheme such as in [8] (see Figure 5) or a combination of the unipolar scheme and that of 2 DC source multilevel scheme (see Figure 6). In the following, it is shown how the transcendental equations that characterize the harmonic content for each of these switching schemes can be solved to find *all* solutions that eliminate the 5^{th} and 7^{th} harmonics while achieving the fundamental.

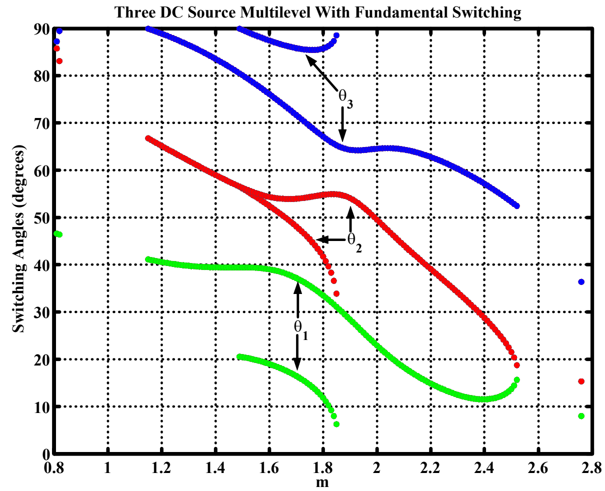

Fig. 3. Scheme 0: Fundamental switching scheme. The switching angles $\theta_1, \theta_2, \theta_3$ in degrees vs m. There are $s = 3$ DC sources and the modulation index is given by $m_a = m/s$

III. Mathematical Model of Switching

The three switching schemes illustrated in Figures 4, 5 and 6 are considered to eliminate the 5^{th} and 7^{th} harmonics at lower modulation indices while still achieving the fundamental voltage. These schemes use 4 switching angles in contrast to the 3 switching angles used by the fundamental switching scheme of Figure 2. Consequently, their switches turn on and off at an overall frequency just above the fundamental frequency. To proceed, note that each of the waveforms of Figures 4, 5 and 6 have a Fourier series expansion of the form

$$V(\omega t) = \frac{4V_{dc}}{\pi}\sin(n\omega t) \times \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n}\Big(\ell_1\cos(n\theta_1) + \ell_2\cos(n\theta_2) + \ell_3\cos(n\theta_3) + \ell_4\cos(n\theta_4)\Big) \quad (1)$$

where $0 \le \theta_1 \le \theta_2 \le \theta_3 \le \theta_4 \le \pi/2$ and $\ell_i = \pm 1$ depending on the switching scheme. Specifically, $\ell = (\ell_1, \ell_2, \ell_3, \ell_4) = (1, -1, 1, -1)$ in the case of scheme 1 (unipolar switching), $\ell = (1, 1, -1, 1)$ in the case of scheme 2 ("virtual stage"), and $\ell = (1, -1, 1, 1)$ for scheme 3. As the Fourier series is summed over only the odd harmonics and $\cos(n(\pi - \theta_i)) = -\cos(n\theta_i)$ for n odd, equation (1) may be rewritten in the form

$$V(\omega t) = \frac{4V_{dc}}{\pi}\sin(n\omega t) \times \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n}\Big(\cos(n\theta'_1) + \cos(n\theta'_2) + \cos(n\theta'_3) + \cos(n\theta'_4)\Big) \quad (2)$$

where $\theta'_i = \theta_i$ if $\ell_i = 1$ and $\theta'_i = \pi - \theta_i$ if $\ell_i = -1$. The conditions $\theta_1 \le \theta_2 \le \theta_3 \le \theta_4$ become

	Inequality Conditions
Scheme 1	$0 \le \theta'_1 \le \pi - \theta'_2 \le \theta'_3 \le \pi - \theta'_4 \le \pi/2$
Scheme 2	$0 \le \theta'_1 \le \theta'_2 \le \pi - \theta'_3 \le \theta'_4 \le \pi/2$
Scheme 3	$0 \le \theta'_1 \le \pi - \theta'_2 \le \theta'_3 \le \theta'_4 \le \pi/2$

. (3)

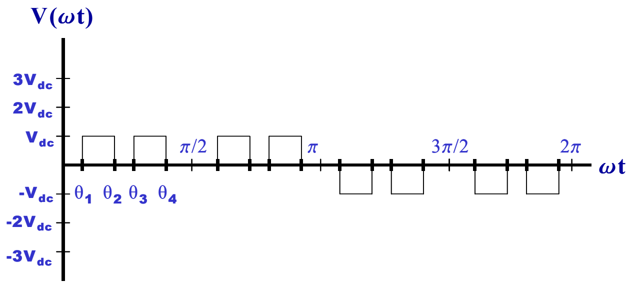

Fig. 4. Scheme 1: Output waveform using a single DC source with a unipolar programmed PWM scheme.

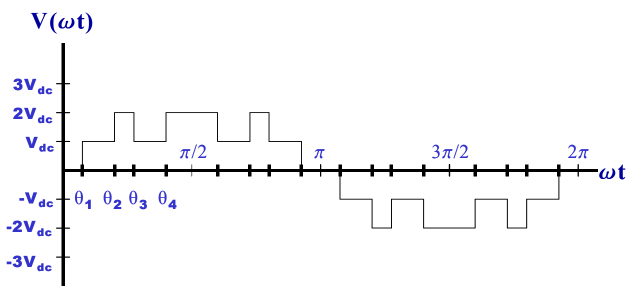

Fig. 5. Scheme 2: Output waveform using 2 DC sources and a "virtual stage" scheme [8].

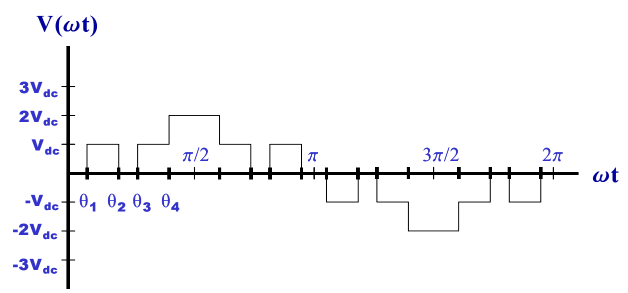

Fig. 6. Scheme 3: Output waveform using 2 DC sources with a combination of a unipolar PWM scheme and a 2 DC sources multilevel scheme.

The desire here is to use these switching schemes to achieve the fundamental voltage and eliminate the 5^{th} and 7^{th} harmonics for those values of the modulation index m_a for which solutions did not exist for the switching scheme of Figure 2 (see Figure 3). That is, choose the switching angles $\theta_1', \theta_2', \theta_3', \theta_4'$ to satisfy

$$\begin{aligned} \cos(\theta_1') + \cos(\theta_2') + \cos(\theta_3') + \cos(\theta_4') &= m \\ \cos(5\theta_1') + \cos(5\theta_2') + \cos(5\theta_3') + \cos(5\theta_4') &= 0 \\ \cos(7\theta_1') + \cos(7\theta_2') + \cos(7\theta_3') + \cos(7\theta_4') &= 0 \end{aligned} \quad (4)$$

and the inequalities (3). Here $m \triangleq V_1/(4V_{dc}/\pi)$ and the modulation index is given by $m_a \triangleq V_1/V_{1\,\max} = V_1/(s4V_{dc}/\pi) = m/s$.

This is a system of 3 transcendental equations in the 4 unknowns $\theta_1', \theta_2', \theta_3', \theta_4'$. In order to get a fourth constraint, consider the possibility of also eliminating the 11^{th} harmonic using this extra switching. That is, append the condition

$$\cos(11\theta_1') + \cos(11\theta_2') + \cos(11\theta_3') + \cos(11\theta_4') = 0 \quad (5)$$

to the conditions (4). The fundamental question is "When does the set of transcendental equations (4), (5) have a solution?" To answer this question, define

$$x_1 = \cos(\theta_1'), x_2 = \cos(\theta_2'), x_3 = \cos(\theta_3'), x_4 = \cos(\theta_4')$$

and use the trigonometric identities

$$\begin{aligned} \cos(5\theta) &= 5\cos(\theta) - 20\cos^3(\theta) + 16\cos^5(\theta) \\ \cos(7\theta) &= -7\cos(\theta) + 56\cos^3(\theta) - 112\cos^5(\theta) + 64\cos^7(\theta) \\ \cos(11\theta) &= -11\cos(\theta) + 220\cos^3(\theta) - 1232\cos^5(\theta) \\ &\quad +2816\cos^7(\theta) - 2816\cos^9(\theta) + 1024\cos^{11}(\theta) \end{aligned}$$

to transform the conditions (4) and (5) to

$$\begin{aligned} p_1(x) &\triangleq x_1 + x_2 + x_3 + x_4 - m = 0 \\ p_5(x) &\triangleq \sum_{i=1}^{4}\left(5x_i - 20x_i^3 + 16x_i^5\right) = 0 \\ p_7(x) &\triangleq \sum_{i=1}^{4}\left(-7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7\right) = 0 \\ p_{11}(x) &\triangleq \sum_{i=1}^{4}\left(-11x_i + 220x_i^3 - 1232x_i^5 + 2816x_i^7\right. \\ &\quad \left. -2816x_i^9 + 1024x_i^{11}\right) = 0 \end{aligned} \quad (6)$$

where $x \triangleq (x_1, x_2, x_3, x_4)$, and the angle conditions become

	Inequality Conditions
Scheme 1	$0 \le -x_4 \le x_3 \le -x_2 \le x_1 \le 1$
Scheme 2	$0 \le x_4 \le -x_3 \le x_2 \le x_1 \le 1$
Scheme 3	$0 \le x_4 \le x_3 \le -x_2 \le x_1 \le 1$

. (7)

This is a set of four polynomial equations in the four unknowns x_1, x_2, x_3, x_4. In the next section, a systematic method is presented to solve these equations for all of their possible solutions. It is interesting to note that in [11] polynomial systems were also considered.

IV. Solving Polynomial Equations

The first equation of (6) can be solved as $x_4 = m - (x_1 + x_2 + x_3)$ to eliminate x_4 from the remaining three equations. However, one is still left with three polynomial equations in the three unknowns (x_1, x_2, x_3). The pertinent question is then, "Given two polynomial equations $a(x_1, x_2, x_3) = 0$ and $b(x_1, x_2, x_3) = 0$, how does one solve them simultaneously to eliminate (say) x_3?". A systematic procedure to do this is known as *elimination theory* and uses the notion of *resultants* [12][13]. Briefly, one considers $a(x_1, x_2, x_3)$ and $b(x_1, x_2, x_3)$ as polynomials in x_3 whose coefficients are polynomials in (x_1, x_2). Then, for example, letting $a(x_1,x_2,x_3)$ and $b(x_1,x_2,x_3)$ have degrees 3 and 2, respectively in x_3, they may be written in the form

$$\begin{aligned} a(x_1,x_2,x_3) &= a_3(x_1,x_2)x_3^3 + a_2(x_1,x_2)x_3^2 \\ &\quad + a_1(x_1,x_2)x_3 + a_0(x_3,x_2) \\ b(x_1,x_2,x_3) &= b_2(x_1,x_2)x_3^2 + b_1(x_1,x_2)x_3 + b_0(x_3,x_2). \end{aligned}$$

The $n \times n$ *Sylvester* matrix $S_{a,b}$, where $n = \deg_{x_3}\{a(x)\} + \deg_{x_3}\{b(x)\} = 3 + 2 = 5$, is defined by $S_{a,b}(x_1,x_2) \triangleq$

$$\begin{bmatrix} a_0(x_1,x_2) & 0 & b_0(x_1,x_2) & 0 & 0 \\ a_1(x_1,x_2) & a_0(x_1,x_2) & b_1(x_1,x_2) & b_0(x_1,x_2) & 0 \\ a_2(x_1,x_2) & a_1(x_1,x_2) & b_2(x_1,x_2) & b_1(x_1,x_2) & b_0(x_1,x_2) \\ a_3(x_1,x_2) & a_2(x_1,x_2) & 0 & b_2(x_1,x_2) & b_1(x_1,x_2) \\ 0 & a_3(x_1,x_2) & 0 & 0 & b_2(x_1,x_2) \end{bmatrix}.$$

The *resultant* polynomial $r(x_1,x_2)$ is defined by

$$\begin{aligned} r(x_1,x_2) &= \mathrm{Res}\Big(a(x_1,x_2,x_3), b(x_1,x_2,x_3), x_3\Big) \\ &\triangleq \det S_{a,b}(x_1,x_2) \end{aligned} \tag{8}$$

and is the result of solving $a(x_1,x_2,x_3) = 0$ and $b(x_1,x_2,x_3) = 0$ simultaneously for (x_1,x_2), i.e., eliminating x_3.

In previous work [6], the method of resultants was used to solve three polynomial equations in three unknowns to obtain the switching angles in Figure 3. However, in the present problem, there are four polynomial equations in four unknowns as given in (6). As the number of equations increases, the degrees of the polynomials increase so that one has to compute *symbolically* the determinant of a large $n \times n$ Sylvester matrix. For example, after $x_4 = m - (x_1 + x_2 + x_3)$ is used in (6) to eliminate x_4, the remaining three polynomials

$$\begin{aligned} q_5(x_1,x_2,x_3) &\triangleq p_5(x_1,x_2,x_3,m-x_1-x_2-x_3) \\ q_7(x_1,x_2,x_3) &\triangleq p_7(x_1,x_2,x_3,m-x_1-x_2-x_3) \\ q_{11}(x_1,x_2,x_3) &\triangleq p_{11}(x_1,x_2,x_3,m-x_1-x_2-x_3) \end{aligned}$$

have degrees $4, 6, 10$, respectively in x_3. In particular, to eliminate x_3 from $q_7(x_1,x_2,x_3) = 0, q_{11}(x_1,x_2,x_3) = 0$ would require the symbolic computation of a $(6+10) \times (6+10) = 16 \times 16$ Sylvester matrix.

This symbolic calculation is carried out using computer algebra software (e.g., the `Resultant` command in Mathematica [14]). However, these computations are time consuming, and one quickly encounters the computational limits of such systems as the size of the Sylvester matrix increases. To get around this, use is made of the fact that the polynomials making up the system (6) are symmetric. The *theory of symmetric polynomials* [12] is then exploited to obtain a new set of relatively low order polynomials whose resultants can easily be computed using existing computer algebra software tools. In contrast to numerical techniques, the approach here produces all possible solutions.

A. Symmetric Polynomials

The polynomials $p_1(x), p_5(x), p_7(x), p_{11}(x)$ in (6) are *symmetric polynomials*, that is, $p_i(x_1,x_2,x_3,x_4) = p_i(x_2,x_1,x_3,x_4) = p_i(x_3,x_2,x_1,x_4)$, etc. Define the *elementary symmetric functions* (polynomials) s_1, s_2, s_3, s_4 as

$$\begin{aligned} s_1 &\triangleq x_1 + x_2 + x_3 + x_4 \\ s_2 &\triangleq x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4 \\ s_3 &\triangleq x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \\ s_4 &\triangleq x_1x_2x_3x_4 \end{aligned} \tag{9}$$

A basic property of symmetric polynomials is that they can be rewritten in terms of the elementary symmetric functions [12] (e.g., using the `SymmetricReduction` command in Mathematica [14]). In the case at hand, it follows that with $s = (s_1, s_2, s_3, s_4)$ and using (9), the polynomials (6) become

$$\begin{aligned} p_1(s) &= s_1 - m \\ p_5(s) &= 5s_1 - 20s_1^3 + 16s_1^5 + 60s_1s_2 - 80s_1^3s_2 + 80s_1s_2^2 \\ &\quad -60s_3 + 80s_1^2s_3 - 80s_2s_3 \\ p_7(s) &= -7s_1 + 56s_1^3 - 112s_1^5 + 64s_1^7 - 168s_1s_2 \\ &\quad +560s_1^3s_2 - 448s_1^5s_2 - 560s_1s_2^2 + 896s_1^3s_2^2 \\ &\quad -448s_1s_2^3 + 168s_3 - 560s_1^2s_3 + 448s_1^4s_3 \\ &\quad +560s_2s_3 - 1344s_1^2s_2s_3 + 448s_2^2s_3 + 448s_1s_3^2 \\ p_{11}(s) &= -11s_1 + 220s_1^3 - 1232s_1^5 + 2816s_1^7 - \cdots \end{aligned} \tag{10}$$

(The complete expression for $p_{11}(s)$ is given in the Appendix). One uses $p_1(s) = s_1 - m = 0$ to eliminate s_1. The table below gives the degrees of the three polynomials $p_5(s), p_7(s), p_{11}(s)$ in the indeterminates s_2, s_3, s_4.

	degree in s_2	degree in s_3	degree in s_4
$p_5(s)$	2	1	1
$p_7(s)$	3	2	1
$p_{11}(s)$	5	3	2

The key point here is that the degrees of these polynomials in s_2, s_3, s_4 are much less than the degrees of $p_5(x), p_7(x), p_{11}(x_1)$ in x_1, x_2, x_3 (see (6)). In particular, the Sylvester matrix of the pair $\{p_7(s_2,s_3,s_4), p_{11}(s_2,s_3,s_4)\}$ is a 3×3 matrix (if the variable s_4 is to be eliminated) rather than the 16×16 Sylvester matrix required to eliminate x_3 in the case of $\{p_7(x_1,x_2,x_3), p_{11}(x_1,x_2,x_3)\}$ in (6).

To proceed, one then eliminates s_4 by computing

$$\begin{aligned} r_{q_5,q_7}(s_2,s_3) &= \mathrm{Res}\Big(q_5(s_2,s_3,s_4), q_7(s_2,s_3,s_4), s_4\Big) \\ r_{q_5,q_{11}}(s_2,s_3) &= \mathrm{Res}\Big(q_5(s_2,s_3,s_4), q_{11}(s_2,s_3,s_4), s_4\Big) \end{aligned}$$

and finally, computing

$$r(s_2) = \mathrm{Res}\Big(r_{q_5,q_7}(s_2,s_3), r_{q_5,q_{11}}(s_2,s_3), s_3\Big)$$

gives a polynomial in the single variable s_2. For each m, one solves $r(s_2) = 0$ for the roots $\{s_{2i}\}$. Each root s_{2i} is then used to solve $r_{q_5,q_7}(s_{2i}, s_3) = 0$ for the roots $\{s_{3j_i}\}$. Each pair (s_{2i}, s_{3j_i}) is used to solve $p_5(m, s_{2i}, s_{3j_i}, s_4) =$

0 to get the roots $s_{4k_{i,j}}$. Then the set of 4-tuples $\{s \mid (s_1, s_2, s_3, s_4) = (m, s_{2i}, s_{3j_i}, s_{4k_{i,j}})$ for some $i, j, k\}$ are the only possible solutions to (10).

For each solution triple (s_1, s_2, s_3, s_4), it is the corresponding values of (x_1, x_2, x_3, x_4) which are required to obtain the switching angles. Consequently, the system of polynomial equations (9) must be solved for the x_i. To do so, one simply uses the resultant method again to solve the system of polynomials

$$\begin{aligned} f_1(x) &= s_1 - (x_1 + x_2 + x_3 + x_4) \\ f_2(x) &= s_2 - (x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4) \\ f_3(x) &= s_2 - (x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4) \\ f_4(x) &= s_4 - x_1x_2x_3x_4. \end{aligned} \tag{11}$$

The Appendix shows how (11) is solved using resultants. The solutions of (11) which satisfy (7) are then straightforwardly used to compute the switching angles.

V. Results

Using the above techniques, the switching angles for the three schemes vs the parameter m (modulation index is $m_a = m/s$) were computed. The switching angles for scheme 1 (unipolar programmed PWM) of Figure 4 were computed first. Figure 7 shows these results where only the switching angle set for each value of m that gave the smallest THD is plotted (see Figure 11). It turns out that for $0 \le m \le 0.49$ there are actually three different solution sets and for $m \in [0.5, 0.55]$, $m \in [0.68, 0.87]$ there are two different solutions sets. This shows that for low modulation indices where only one DC source is used, this scheme can achieve the fundamental without generating the $5^{th}, 7^{th}$ or 11^{th} harmonics.

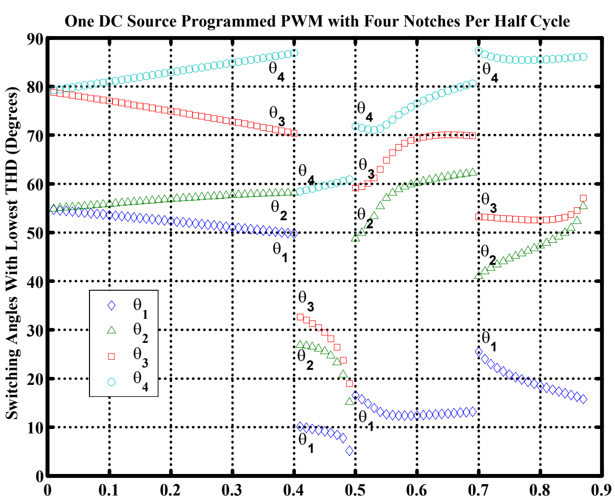

Fig. 7. Switching angles for scheme 1 of Figure 4

The switching angles for scheme 2 are shown in Figure 8. As the fundamental switching scheme does not achieve the desired result for $m < 1.15$, Figure 8 shows that scheme 2 can be used for $0.97 < m < 1.15$ (where scheme 1 will also not work) to achieve the fundamental without generating the $5^{th}, 7^{th}$ or 11^{th} harmonics.

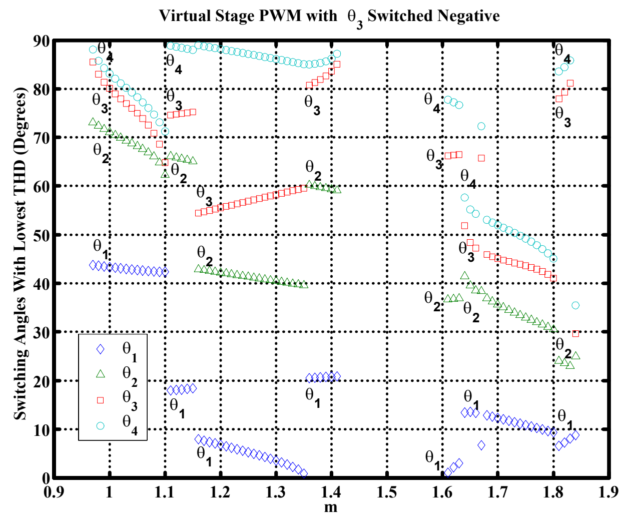

Fig. 8. Switching angles for Scheme 2 of Figure 5.

Finally, the switching angles for scheme 3 are shown in Figure 9. Figure 9 shows that scheme 3 can be used for $0.56 < m < 0.69$ as neither scheme 2 nor the fundamental switching scheme (scheme 0) work in that range of m. Of course, this scheme will also not generate the 11^{th} harmonic.

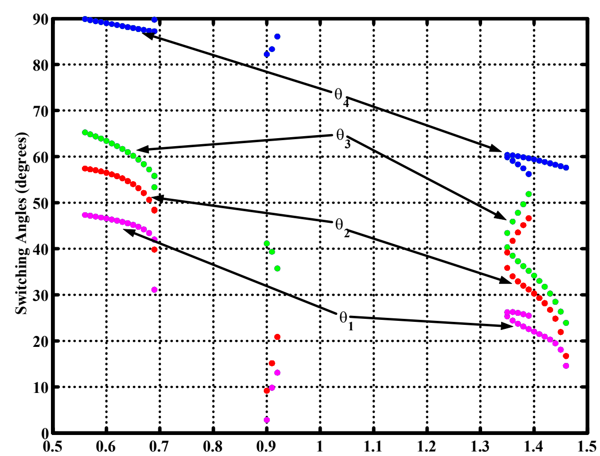

Fig. 9. Switching angles for scheme 3 of Figure 6.

None of the above schemes is able to achieve the desired result for $0.87 < m < 0.97$ (or $0.29 < m_a < 0.323$). In the ranges of m for which more than one scheme will work, a natural choice is the one which generates the smallest distortion due to higher order harmonics. Figures 10, 11, 12 and 13 are plots of the harmonic distortion vs m due to the $11^{th}, 13^{th}, 17^{th}$ and 19^{th} harmonics for each scheme. These figures show for low modulation indices ($m < 0.87$ or $m_a < 0.29$), the unipolar PWM (scheme 1) should be used except for $0.55 < m < 0.7$ ($0.55/3 < m_a < 0.7/3$) where scheme 3 (see Figure 13) will produce the lowest harmonics. As pointed out above, Scheme 2 can be used for $0.97 < m < 1.15$ (where no other scheme works) and Figure 12 shows that the harmonic distortion will be between 10% and 18% in this range.

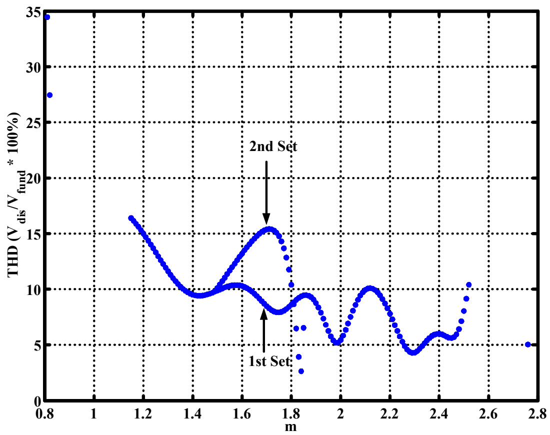

Fig. 10. Scheme 0 (See Figure 2) THD due to the $11^{th}, 13^{th}, 17^{th}$ and 19^{th} vs m.

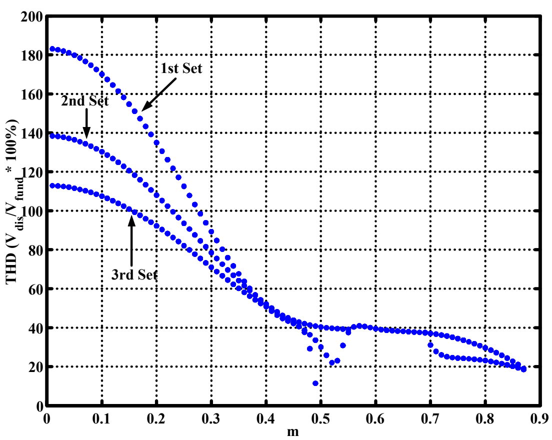

Fig. 11. Scheme 1 (See Figure 4) THD due to the $11^{th}, 13^{th}, 17^{th}$ and 19^{th} vs m.

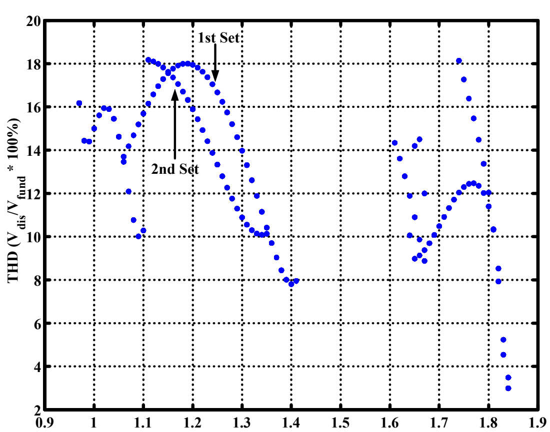

Fig. 12. Scheme 2 (See Figure 5) THD due to the $11^{th}, 13^{th}, 17^{th}$ and 19^{th} vs m.

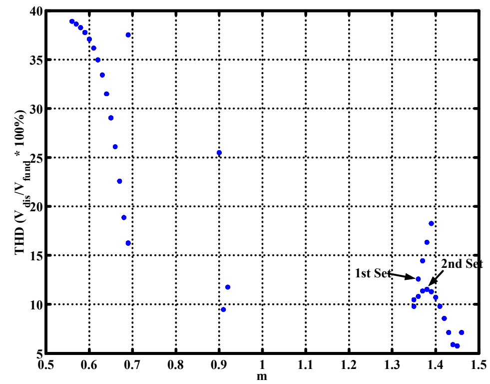

Fig. 13. Scheme 3 (See Figure 6) THD due to the $11^{th}, 13^{th}, 17^{th}$ and 19^{th} vs m.

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