A generalised instantaneous non-active power theory for STATCOM

Y. Xu, L.M. Tolbert, J.N. Chiasson, J.B. Campbell and F.Z. Peng

Abstract: A generalised instantaneous non-active power theory is presented. Comprehensive definitions of instantaneous active and non-active currents, as well as instantaneous, average and apparent powers, are proposed. These definitions have flexible forms that are applicable to different power systems, such as single-phase or multi-phase, periodic or non-periodic and balanced or unbalanced systems. By changing the averaging interval and the reference voltage, various non-active power theories can be derived from this theory. The definitions of instantaneous active and non-active currents provide an algorithm for a STATCOM to calculate the non-active current in the load current. The theory is implemented by the STATCOM, and four cases (three-phase balanced *RL* load, three-phase unbalanced *RL* load, diode rectifier load and single-phase load) are tested. The experimental results show that the STATCOM can perform instantaneous non-active power compensation, and both the fundamental non-active component and the harmonics are eliminated from the utility so that nearly unity power factor can be achieved. The STATCOM also has a fast dynamic response for transients.

1 Introduction

As early as in the 1920s and 1930s, Fryze [1] and Budeanu [2] proposed instantaneous non-active power theories for periodic (but non-sinusoidal) waveforms in the time and frequency domains, respectively. Different theories have been formulated to describe the increasingly complex phenomena in power systems. Owing to the existence of nonlinear loads, currents and voltages are distorted so that they are no longer pure sinusoids and sometimes not even periodic. They can be divided into the following categories: single-phase or multi-phase, periodic or non-periodic and balanced or unbalanced.

Various non-active power theories in the time domain have been discussed [3], and most of them can be divided into two categories. The first category is Fryze's theory [1] and its extensions, and the second is p–q theory [4, 5] and its extensions. In the frequency domain, the periodic voltage and current waveforms are decomposed to Fourier series, and the non-active power/current is defined on the basis of voltage and current [2, 6]. These theories proposed instantaneous or average definitions for non-active current and/or non-active power in different cases mentioned above.

Parallel non-active compensation provides the non-active component in the load current so that the current from the utility is a fundamental sinusoid in phase with the voltage (unity power factor). A STATCOM is a parallel non-active power compensator that uses switches with both turn-on and turn-off capability so that the STATCOM has full control capability and is independent of the system voltage. Different control strategies and practical issues such as the application of non-active power theories to non-active power compensation are discussed in [7–10].

A generalised instantaneous non-active power theory is presented in this article. It defines the instantaneous non-active current component in a load current, which is the current component that the STATCOM should provide. In addition, this theory proposes comprehensive definitions of instantaneous currents (instantaneous active and instantaneous non-active currents), instantaneous powers (instantaneous power, instantaneous active power and instantaneous non-active power), average powers (average power, average active power and average non-active power) and apparent powers (apparent power, apparent active power and apparent non-active power). These definitions are not only consistent with the standard definitions in a system with fundamental sinusoidal waveforms [11], but also a more general theory from which other theories mentioned above can be derived.

2 Generalised non-active power theory

The generalised non-active power theory [12] is based on Fryze's idea of non-active power/current [1] and is an extension of the theory proposed in [13]. Let a voltage vector $\boldsymbol{v}(t)$ and a current vector $\boldsymbol{i}(t)$ in an m-phase system be given by

$$\boldsymbol{v}(t) = [v_1(t), v_2(t), \ldots, v_m(t)]^{\mathrm{T}} \quad (1)$$

$$\boldsymbol{i}(t) = [i_1(t), i_2(t), \ldots, i_m(t)]^{\mathrm{T}} \quad (2)$$

The instantaneous power $p(t)$ is defined by

$$p(t) = \boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{i}(t) = \sum_{k=1}^{m} v_k(t) i_k(t) \quad (3)$$

doi:10.1049/iet-epa:20060290

Paper first received 23rd July and in revised form 5th October 2006

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The average power $P(t)$ is defined as the average value of the instantaneous power $p(t)$ over the averaging interval $[t - T_c, t]$, that is

$$P(t) = \frac{1}{T_c}\int_{t-T_c}^{t} p(\tau)\,d\tau \tag{4}$$

The instantaneous active current $\boldsymbol{i}_a(t) = [i_{a1}(t), i_{a2}(t), \ldots, i_{am}(t)]^T$ and instantaneous non-active current $\boldsymbol{i}_n(t) = [i_{n1}(t), i_{n2}(t), \ldots, i_{nm}(t)]^T$ are

$$\boldsymbol{i}_a(t) = \frac{P(t)}{V_p^2(t)}\boldsymbol{v}_p(t) \tag{5}$$

$$\boldsymbol{i}_n(t) = \boldsymbol{i}(t) - \boldsymbol{i}_a(t) \tag{6}$$

In (5), voltage $\boldsymbol{v}_p(t)$ is the reference voltage, which is chosen on the basis of the characteristics of the system and the desired compensation results. $\boldsymbol{v}_p(t)$ is often chosen to be the fundamental of the source voltage, and it will be discussed in detail in the next section. $V_p(t)$ is the corresponding rms value of the reference voltage $\boldsymbol{v}_p(t)$, that is

$$V_p(t) = \sqrt{\frac{1}{T_c}\int_{t-T_c}^{t} \boldsymbol{v}_p^T(\tau)\boldsymbol{v}_p(\tau)\,d\tau} \tag{7}$$

Based on the above definitions for $\boldsymbol{i}_a(t)$ and $\boldsymbol{i}_n(t)$, the instantaneous active power $p_a(t)$ and instantaneous non-active power $p_n(t)$ are defined as

$$p_a(t) = \boldsymbol{v}^T(t)\boldsymbol{i}_a(t) = \sum_{k=1}^{m} v_k(t)\boldsymbol{i}_{ak}(t) \tag{8}$$

$$p_n(t) = \boldsymbol{v}^T(t)\boldsymbol{i}_n(t) = \sum_{k=1}^{m} v_k(t)\boldsymbol{i}_{nk}(t) \tag{9}$$

The average active power $P_a(t)$ is defined as the average value of the instantaneous active power $p_a(t)$ over the averaging interval $[t - T_c, t]$, that is

$$P_a(t) = \frac{1}{T_c}\int_{t-T_c}^{t} p_a(\tau)\,d\tau \tag{10}$$

The average non-active power $P_n(t)$ is defined as the average value of the instantaneous non-active power $p_n(t)$ over the averaging interval $[t - T_c, t]$, that is

$$P_n(t) = \frac{1}{T_c}\int_{t-T_c}^{t} p_n(\tau)\,d\tau \tag{11}$$

The rms value of the system voltage is

$$V(t) = \sqrt{\frac{1}{T_c}\int_{t-T_c}^{t} \boldsymbol{v}^T(\tau)\boldsymbol{v}(\tau)\,d\tau} \tag{12}$$

The rms values of the active current $\boldsymbol{i}_a(t)$, non-active current $\boldsymbol{i}_n(t)$ and current $\boldsymbol{i}(t)$ are

$$I_a(t) = \sqrt{\frac{1}{T_c}\int_{t-T_c}^{t} \boldsymbol{i}_a^T(\tau)\boldsymbol{i}_a(\tau)\,d\tau} \tag{13}$$

$$I_n(t) = \sqrt{\frac{1}{T_c}\int_{t-T_c}^{t} \boldsymbol{i}_n^T(\tau)\boldsymbol{i}_n(\tau)\,d\tau} \tag{14}$$

$$I(t) = \sqrt{\frac{1}{T_c}\int_{t-T_c}^{t} \boldsymbol{i}^T(\tau)\boldsymbol{i}(\tau)\,d\tau} \tag{15}$$

Based on the rms values defined earlier, the apparent power $S(t)$ is defined by

$$S(t) = V(t)I(t) \tag{16}$$

The apparent active power $P_p(t)$ is defined by

$$P_p(t) = V(t)I_a(t) \tag{17}$$

The apparent non-active power $Q(t)$ is defined by

$$Q(t) = V(t)I_n(t) \tag{18}$$

These definitions provide instantaneous values (defined for each time t), whereas the standard definitions of average powers and rms voltage/current for a fundamental sinusoidal system do not provide instantaneous values.

3 Characteristics of the theory

The averaging interval T_c and the reference voltage $\boldsymbol{v}_p(t)$ are two important factors in the generalised non-active power theory. They are discussed in this section, and the characteristics of the generalised non-active power theory are presented as well.

3.1 Averaging interval, T_c

The averaging time interval T_c can be chosen arbitrarily from zero to infinity, and for different T_c's, the resulting active current and non-active current will have different characteristics. The flexibility of choosing different T_c's as well as the reference voltage results in this theory being applicable for defining non-active power for a larger class of systems than in the current literature. For each case, a specific value of T_c can be chosen to fit the application or to achieve an optimal result. The choice of T_c is as follows.

1. $T_c = 0$: The definitions of average powers are the same as the instantaneous powers, and the rms definitions have different forms

$$V(t) = \boldsymbol{v}^T(t)\boldsymbol{v}(t) \tag{19}$$

$$I(t) = \boldsymbol{i}^T(t)\boldsymbol{i}(t) \tag{20}$$

$$V_p(t) = \boldsymbol{v}_p^T(t)\boldsymbol{v}_p(t) \tag{21}$$

If $\boldsymbol{v}_p(t) = \boldsymbol{v}(t)$, the instantaneous active power $p_a(t)$ is equal to the instantaneous power $p(t)$, and the instantaneous non-active power $p_n(t)$ is identically zero, that is

$$p_a(t) = \boldsymbol{v}^T(t)\boldsymbol{i}_a(t) \equiv p(t) \tag{22}$$

$$p_n(t) = \boldsymbol{v}^T(t)\boldsymbol{i}_n(t) \equiv 0 \tag{23}$$

More specifically, in a single-phase system, the instantaneous active current $\boldsymbol{i}_a(t)$ is always equal to the current $\boldsymbol{i}(t)$, and the instantaneous non-active current $\boldsymbol{i}_n(t)$ is always zero; therefore $T_c = 0$ is not suitable for non-active power/current definitions in single-phase systems.

2. T_c is a finite value: For most applications, T_c will be chosen as a finite value. For a periodic system with a fundamental period T, T_c is usually chosen as $T_c = T/2$. If $\boldsymbol{v}_p$(t) is chosen as a periodic waveform with period T, then the average power $P(t)$ and the rms value $V_p(t)$ are both constant numbers, that is, $P(t) = P$ and $V_p(t) = V_p$

$$\boldsymbol{i}_a(t) = \frac{P}{V_p^2}\boldsymbol{v}_p(t) \tag{24}$$

The instantaneous active current is proportional to (and thus the same shape as) the reference voltage. By choosing different reference voltages, the instantaneous active current can have different waveforms.

The generalised non-active power theory does not specify the characteristics of the voltage $\boldsymbol{v}(t)$ and current $\boldsymbol{i}(t)$, that is, they can theoretically be any waveforms. However, in a power system, the voltage is usually sinusoidal with/without harmonic distortion, and the distortion of the voltage is usually lower than that of the current [the total harmonic distortion (THD) of the voltage is usually <5%]. Therefore the voltage is assumed to be periodic for all cases. In applications such as non-active power compensation, T_c is usually chosen to be one to ten times that of the fundamental period based on the system specifications and the desired compensation results.

3. $T_c \to \infty$: This is a theoretical analysis of a non-periodic system. A non-periodic system is referred to as a system with a periodic voltage and a non-periodic current. In a non-periodic system, the instantaneous active current varies with different averaging interval T_c, which is different from the periodic cases [14]. For a non-periodic system, choosing $T_c = t$, the average power $P(t)$, the rms value of the reference voltage $\boldsymbol{v}_p(t)$ and the active current $\boldsymbol{i}_a(t)$ are defined as

$$P(t) = \frac{1}{T_c}\int_{t-T_c}^{t} \boldsymbol{v}^T(\tau)\boldsymbol{i}(\tau)\,d\tau = \frac{1}{t}\int_0^t \boldsymbol{v}^T(\tau)\boldsymbol{i}(\tau)\,d\tau \quad (25)$$

$$V_p(t) = \sqrt{\frac{1}{T_c}\int_{t-T_c}^{t} \boldsymbol{v}_p^T(\tau)\boldsymbol{v}_p(\tau)\,d\tau} = \sqrt{\frac{1}{t}\int_0^t \boldsymbol{v}_p^T(\tau)\boldsymbol{v}_p(\tau)\,d\tau} \quad (26)$$

$$\boldsymbol{i}_a(t) = \frac{P(t)}{V_p^2(t)}\boldsymbol{v}_p(t) \quad (27)$$

In a power system, the voltage and the current are finite power waveforms, that is

$$P(t) = \lim_{t\to\infty}\frac{1}{t}\int_0^t \boldsymbol{v}^T(\tau)\boldsymbol{i}(\tau)\,d\tau = P \quad (28)$$

$$V_p(t) = \lim_{t\to\infty}\sqrt{\frac{1}{t}\int_0^t \boldsymbol{v}_p^T(\tau)\boldsymbol{v}_p(\tau)\,d\tau} = V_p \quad (29)$$

$$\boldsymbol{i}_a(t) = \frac{P(t)}{V_p^2(t)}\boldsymbol{v}_p(t) \to \frac{P}{V_p^2}\boldsymbol{v}_p(t), \quad \text{as } t\to\infty \quad (30)$$

The generalised non-active power theory is valid for voltage and current of any waveform, and the non-active current can only be completely eliminated when $T_c = t$ and $t \to \infty$ [$\boldsymbol{i}_a(t)$ has the same shape as, and is in phase with, $\boldsymbol{v}_p(t)$ so that the unity power factor is achieved]. However, this is impractical in a power system, and T_c is chosen to have a finite value. T_c is usually chosen as a few multiples of the fundamental period, and this finite T_c will mitigate most of the non-active components in the current.

3.2 Reference voltage, $v_p(t)$

If $P(t)$ and $V_p(t)$ are constant, which can be achieved with $T_c = T/2$ for a periodic system and with $T_c \to \infty$ for a non-periodic system, the active current $\boldsymbol{i}_a(t)$ is in phase with $\boldsymbol{v}_p(t)$, and the waveforms of $\boldsymbol{i}_a(t)$ and $\boldsymbol{v}_p(t)$ have the same shape and they differ only by a scale factor. Theoretically, $\boldsymbol{v}_p(t)$ can be arbitrarily chosen, but in practice, it is chosen based on the voltage $\boldsymbol{v}(t)$, the current $\boldsymbol{i}(t)$ and the desired active current $\boldsymbol{i}_a(t)$. Choices for $\boldsymbol{v}_p(t)$ include the following.

1. $\boldsymbol{v}_p(t) = \boldsymbol{v}(t)$, if $\boldsymbol{v}(t)$ is a pure sinusoid; or the active current $\boldsymbol{i}_a(t)$ is preferred to have the same waveform as $\boldsymbol{v}(t)$.
2. $\boldsymbol{v}_p(t) = \boldsymbol{v}_f(t)$, where $\boldsymbol{v}_f(t)$ is the fundamental positive sequence component of $\boldsymbol{v}(t)$. In power systems, if $\boldsymbol{v}(t)$ is distorted or even unbalanced and a purely sinusoidal $\boldsymbol{i}_a(t)$ is desired, then $\boldsymbol{v}_p(t)$ is chosen as the fundamental positive sequence component of $\boldsymbol{v}(t)$. This ensures that $\boldsymbol{i}_a(t)$ is balanced and does not contain any harmonics.
3. Other references can be chosen to eliminate certain components in current $\boldsymbol{i}(t)$. For example, in a hybrid non-active power compensation system with a STATCOM and a passive *LC* filter, the lower order harmonics in the current are compensated by the STATCOM, and the higher order harmonics will be filtered by the passive filter because of the limit of the switching frequency of the STATCOM. In this case, a reference voltage with the higher order harmonics is chosen such that the resulting non-active current, which will be compensated by the STATCOM, only contains the lower order harmonics.

By choosing different reference voltages, $\boldsymbol{i}_a(t)$ can have various desired waveforms and the unwanted components in $\boldsymbol{i}(t)$ can be eliminated. Furthermore, the elimination of each component is independent of each other. The definitions of the generalised non-active power theory are consistent with the standard definitions for three-phase sinusoidal systems [11].

3.3 General characteristics

In general, the non-active power theory has the following characteristics.

1. $\boldsymbol{i}_a(t)$ and $\boldsymbol{i}_n(t)$ are orthogonal

$$\int_{t-T_c}^{t} \boldsymbol{i}_a^T(\tau)\boldsymbol{i}_n(\tau)\,d\tau = 0 \quad (31)$$

$$I^2(t) = I_a^2(t) + I_n^2(t) \quad (32)$$

The instantaneous active current is in phase with the voltage $\boldsymbol{v}_p(t)$, whereas the instantaneous non-active current is 90° out of phase with $\boldsymbol{v}_p(t)$. The physical meaning of this characteristic is that the active current carries active power and the non-active current carries non-active power.
2. The instantaneous power $p(t)$ can be decomposed into two components, the instantaneous active power $p_a(t)$ and the instantaneous non-active power $p_n(t)$, which satisfy

$$p(t) = p_a(t) + p_n(t) \quad (33)$$

Similar to the instantaneous active current and non-active current, at any moment, the power flowing in the system has two components, that is, the active power component and the non-active power component.
3. If $\boldsymbol{v}_p(t) = \boldsymbol{v}(t)$, the apparent powers $S(t)$, $P_p(t)$ and $Q(t)$ satisfy

$$S^2(t) = P_p^2(t) + Q^2(t) \quad (34)$$

4 Implementation in a STATCOM

In a shunt non-active power compensation system, the compensator is connected to the utility at the point of common coupling and in parallel with the load. It injects a certain amount of current, which is usually the non-active component $\boldsymbol{i}_n(t)$ in the load current $\boldsymbol{i}_l(t)$, into the system so that the utility only needs to provide the active component $\boldsymbol{i}_a(t)$ of the load current. In Fig. 1, $\boldsymbol{i}_l(t)$ is the load current [$\boldsymbol{i}(t)$ in the theoretical development of Sections 2 and 3], $\boldsymbol{i}_s(t)$ the source current and $\boldsymbol{i}_c(t)$ the compensator current. Therefore if the non-active current is completely provided by the compensator, then $\boldsymbol{i}_s(t) = \boldsymbol{i}_a(t)$ and $\boldsymbol{i}_c(t) = \boldsymbol{i}_n(t)$. The theory can be implemented in a three-phase three-wire, three-phase four-wire or single-phase system. The system configuration shown in Fig. 1 is a three-phase three-wire system. The compensator consists of three parts, namely, the non-active current calculator, the controller and the inverter. In the non-active current calculator, the system voltage $\boldsymbol{v}_s(t)$ [denoted as $\boldsymbol{v}(t)$ in Sections 2 and 3] and load current $\boldsymbol{i}_l(t)$ are measured and used to calculate the non-active component of $\boldsymbol{i}_l(t)$, as explained in the previous section. This is the current the shunt compensator needs to inject into the power system ($\boldsymbol{i}_c^*$ in Fig. 1).

In the generalised non-active theory, the compensation system was assumed to be lossless; however, in an actual power system, losses exist in the switches, capacitors and inductors. If the compensator does not have any energy source, the energy stored in the DC link capacitor is drawn to compensate these losses and the DC link voltage v_{dc} drops. Instead, the active power required to meet the losses is drawn from the source by regulating the DC link voltage v_{dc} to the reference V_{dc}^* [15]. Therefore there are two components in the compensator current $\boldsymbol{i}_c$. The first is the non-active component $\boldsymbol{i}_{cn}$ to compensate the non-active component of the load current and the second is the active component $\boldsymbol{i}_{ca}$ to meet the compensator's losses by regulating the DC link voltage v_{dc}, then

$$\boldsymbol{i}_c = \boldsymbol{i}_{cn} + \boldsymbol{i}_{ca} \tag{35}$$

A proportional integral (PI) controller is used to regulate the DC link voltage v_{dc}. The active current required to meet the losses is in phase with $\boldsymbol{v}_s$ (In Fig. 2, $\boldsymbol{i}_{ca}^*$ has a minus sign because its direction is opposite to the convention of $\boldsymbol{i}_c$ in Fig. 1). The amplitude of the active current is controlled by the difference between the reference voltage V_{dc}^* and the actual DC link voltage v_{dc}. $\boldsymbol{i}_{ca}^*$ is calculated by the amplitude modulation of $\boldsymbol{v}_s$ as follows

$$\boldsymbol{i}_{ca}^* = -\boldsymbol{v}_s\left[K_{P1}(V_{dc}^* - v_{dc}) + K_{I1}\int_0^t (V_{dc}^* - v_{dc})\,dt\right] \tag{36}$$

As shown in Fig. 1, the system voltage $\boldsymbol{v}_s$, inverter output voltage $\boldsymbol{v}_c$, compensator current $\boldsymbol{i}_c$ and the coupling inductor

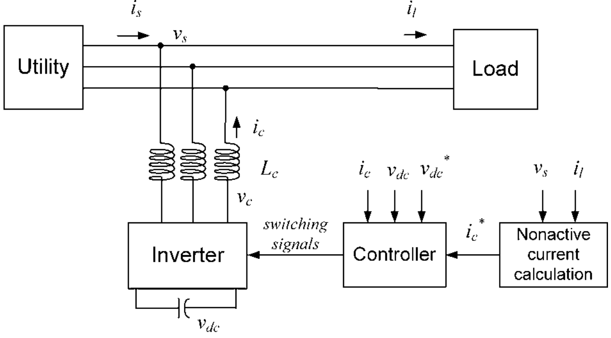

Fig. 1 *System configuration of the STATCOM*

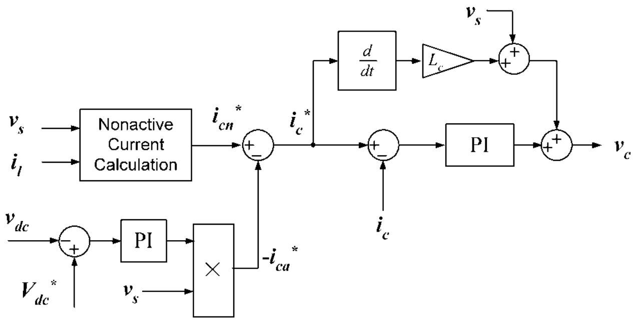

Fig. 2 *Control diagram of the shunt compensation system*

L_c are related as

$$L_c\frac{d\boldsymbol{i}_c}{dt} = \boldsymbol{v}_c - \boldsymbol{v}_s \tag{37}$$

The reference of the compensator non-active current $\boldsymbol{i}_c^*$ and that of the compensator output voltage $\boldsymbol{v}_c^*$ satisfy

$$L_c\frac{d\boldsymbol{i}_c^*}{dt} = \boldsymbol{v}_c^* - \boldsymbol{v}_s \tag{38}$$

where $\boldsymbol{i}_c^*$ is the non-active current calculated on the basis of the generalised non-active power theory presented in Section 2, and $\boldsymbol{v}_c^*$ is the reference voltage of the compensator output voltage

$$\boldsymbol{v}_c^* = L_c\frac{d\boldsymbol{i}_c^*}{dt} + \boldsymbol{v}_s \tag{39}$$

Subtract (37) from (39) to obtain

$$L_c\frac{d(\boldsymbol{i}_c^* - \boldsymbol{i}_c)}{dt} = \boldsymbol{v}_c^* - \boldsymbol{v}_c \tag{40}$$

Using a PI controller and substituting (39) into (40), the output voltage of the compensator $\boldsymbol{v}_c$ is calculated as

$$\boldsymbol{v}_c = \boldsymbol{v}_s + L_c\frac{d\boldsymbol{i}_c^*}{dt} + K_{P2}(\boldsymbol{i}_c^* - \boldsymbol{i}_c) + K_{I2}\int_0^t (\boldsymbol{i}_c^* - \boldsymbol{i}_c)\,dt \tag{41}$$

Fig. 2 is the control diagram of the shunt compensation system. The inner loop is the compensator current control, which controls the output voltage of the compensator according to the required non-active current $\boldsymbol{i}_c^*$. The outer loop controls the active current drawn by the compensator by regulating the DC link voltage v_{dc} and combines the active current $\boldsymbol{i}_{ca}^*$ and the non-active current $\boldsymbol{i}_{cn}^*$.

5 Experimental results

An experimental setup of the STATCOM was built. A dSPACE real-time control platform measured the instantaneous voltage/current and calculated the non-active power/current which was used in the inverter control. A Powerex POW-R-PAK™ configurable IGBT-based three-phase inverter was used in this experiment. Four cases were tested, and the experimental results are presented. A simulation by which both the system voltage and the load current are distorted is also presented at the end of this section. No phase-locked loop is required because the generalised non-active power theory and the control scheme are instantaneous in the *a-b-c* coordinates. The averaging interval in all the experiments and simulations is half of the system period, that is, $T_c = T/2$. The STATCOM is current-limited, so that the current flowing through the STATCOM will not exceed the current rating. The software of the STATCOM will shut down the device once the fault current is detected, and a related contactor will open to protect the STATCOM from being damaged by the fault current.

5.1 Three-phase balanced RL load

A three-phase balanced *RL* load compensation is performed. The three-phase system voltage waveforms $[v_{sa}(t), v_{sb}(t), v_{sc}(t)]^T$ are shown in Fig. 3*a*. The load current waveforms $[i_{la}(t), i_{lb}(t), i_{lc}(t)]^T$ are plotted in Fig. 3*b*, together with the phase *a* system voltage $v_{sa}(t)$, to show the phase angle between the system voltage and the load current. The load current lags the system voltage in an *RL* load. Fig. 3*c* shows the source current $\boldsymbol{i}_s(t)$ after compensation, together with phase *a* voltage. The amplitude of the source current is smaller than the load current, and the source current is in phase with the system voltage. The compensator current $\boldsymbol{i}_c(t)$ is illustrated in Fig. 3*d*. This current, which is about 90° out of phase with the system voltage, contains mostly non-active current.

To further analyse the compensation results, the instantaneous powers [instantaneous power $p(t)$, instantaneous active power $p_a(t)$, and instantaneous non-active power $p_n(t)$], the average powers [average power $P(t)$, average active power $P_a(t)$ and average non-active power $P_n(t)$] and the apparent powers [apparent power $S(t)$, apparent active power $P_p(t)$ and apparent non-active power $Q(t)$] are calculated on the basis of the system voltage and the load current. The average and apparent powers are shown in the second column in Table 1. $P(t) = P_a(t) = P_p(t)$, $P_n(t) = 0$ and $S^2(t) = P_p^2(t) + Q^2(t)$; they are consistent with the discussion in the previous section.

The powers of the experimental results are listed in the fourth column of Table 1.

The instantaneous load power is

$$p_l(t) = \boldsymbol{v}_s^T(t)\boldsymbol{i}_l(t) \tag{42}$$

The instantaneous source power is

$$p_s(t) = \boldsymbol{v}_s^T(t)\boldsymbol{i}_s(t) \tag{43}$$

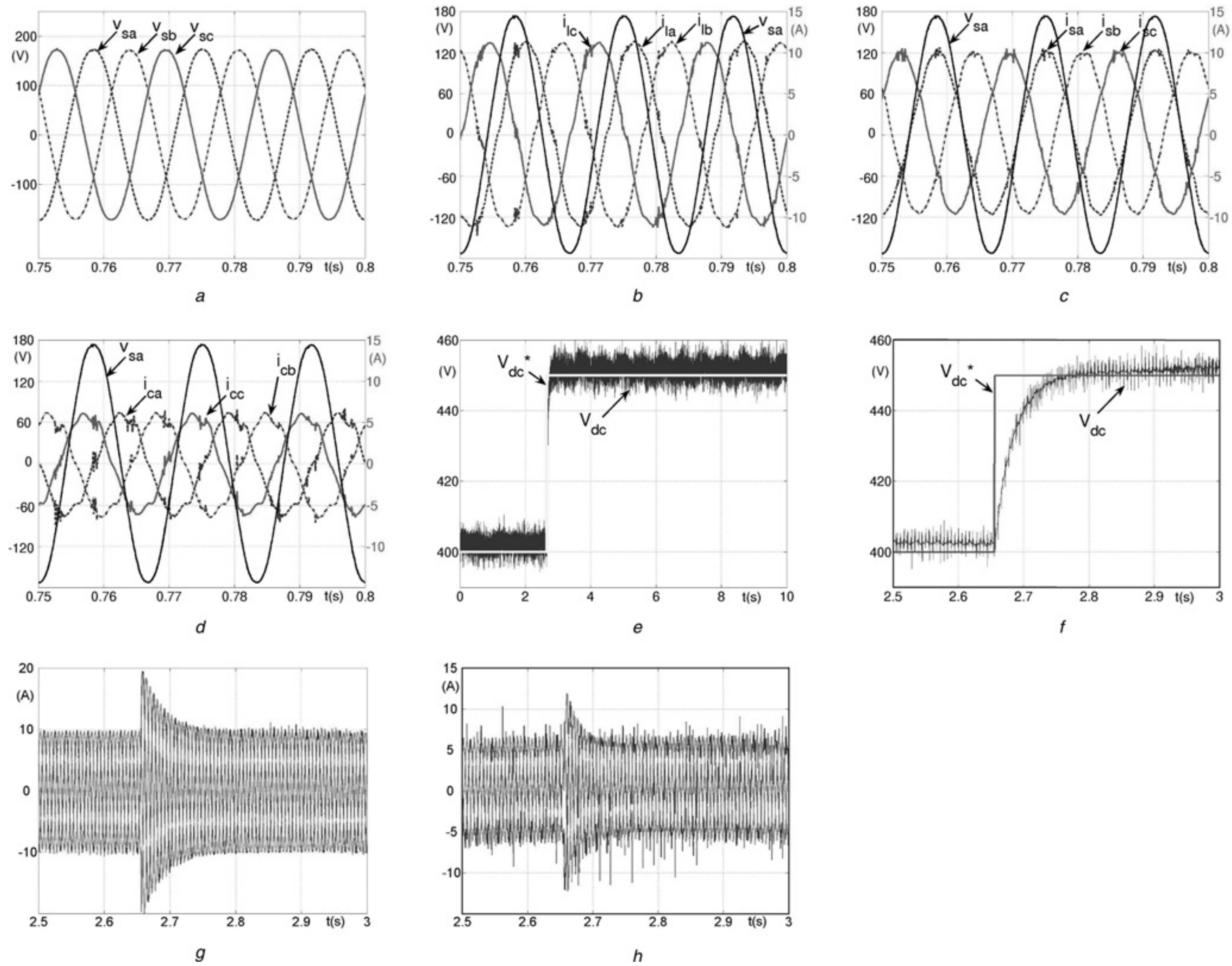

Fig. 3 *Three-phase balanced RL load compensation*

a System voltage $\boldsymbol{v}_s(t)$
b Load current $\boldsymbol{i}_l(t)$
c Source current $\boldsymbol{i}_s(t)$
d Compensator current $\boldsymbol{i}_c(t)$
e DC link voltage change
f DC link voltage change (short period)
g Source current $\boldsymbol{i}_s(t)$
h Compensator current $\boldsymbol{i}_c(t)$

Table 1: Powers of the three-phase balanced *RL* load compensation

Theoretical powers (VA)		Experimental powers (VA)	
average power $P(t)$	2426	average load power $P_l(t)$	2426
average active power $P_a(t)$	2426	average source power $P_s(t)$	2533
average non-active power $P_n(t)$	0	average compensator power $P_c(t)$	−107
apparent power $S(t)$	2933	apparent load power $S_l(t)$	2934
apparent active power $P_p(t)$	2426	apparent source power $S_s(t)$	2537
apparent non-active power $Q(t)$	1649	apparent compensator power $S_c(t)$	1584

The instantaneous compensator power is

$$p_c(t) = \boldsymbol{v}_s^T(t)\boldsymbol{i}_c(t) \tag{44}$$

The average load power is

$$P_l(t) = \frac{1}{T_c}\int_{t-T_c}^{t} \boldsymbol{v}_s^T(\tau)\boldsymbol{i}_l(\tau)\,d\tau \tag{45}$$

The average source power is

$$P_s(t) = \frac{1}{T_c}\int_{t-T_c}^{t} \boldsymbol{v}_s^T(\tau)\boldsymbol{i}_s(\tau)\,d\tau \tag{46}$$

The average compensator power is

$$P_c(t) = \frac{1}{T_c}\int_{t-T_c}^{t} \boldsymbol{v}_s^T(\tau)\boldsymbol{i}_c(\tau)\,d\tau \tag{47}$$

The apparent load power is

$$S_l(t) = V_s(t)I_l(t) \tag{48}$$

The apparent source power is

$$S_s(t) = V_s(t)I_s(t) \tag{49}$$

and the apparent compensator power is

$$S_c(t) = V_s(t)I_c(t) \tag{50}$$

In the experiment, the average source power $P_s(t)$ is slightly higher than the average load power $P_l(t)$, and the average compensator power $P_c(t)$ is negative (see the third and fourth columns of Table 1). This is because the compensator draws a small amount of active current from the utility to compensate the losses in the inverter. Therefore the compensator current contains a small amount of active current, and the average compensator power contains a small amount of active power component. The apparent load power, apparent source power and apparent compensator power do not have the same characteristics as the apparent power, apparent active power and apparent non-active power, as described in (16)-(18). The difference between the second and fourth columns of Table 1 is mainly because of the DC link voltage regulation, which is not considered in the generalised non-active power theory.

Before the compensation of the *RL* load in Fig. 3, the power factor of the load is $P(t)/S(t) = 0.8269$; after compensation, the power factor of the source current is $P_a(t)/P_p(t) = 0.9984$. Therefore the non-active power compensator can improve the load power factor to nearly unity power factor.

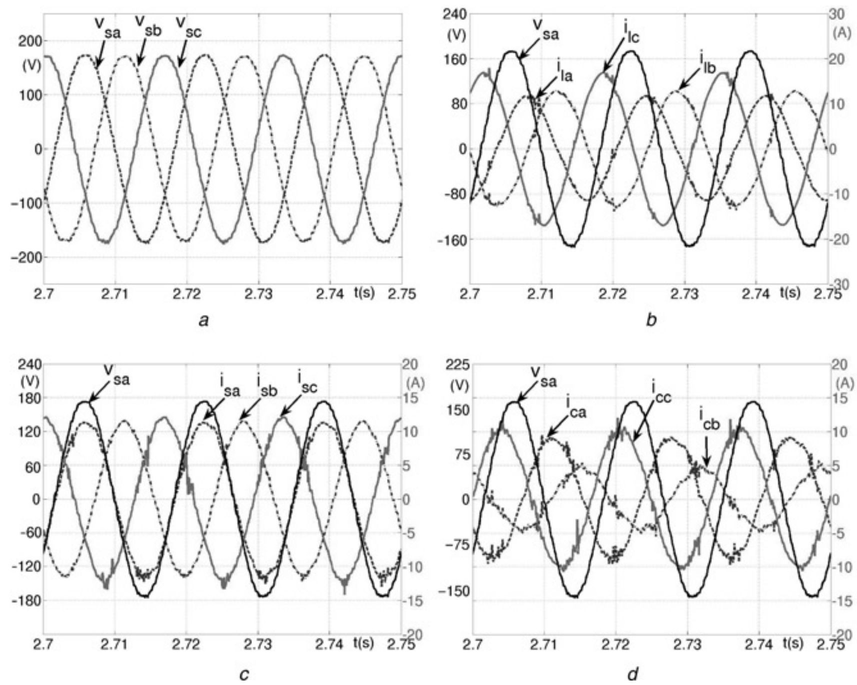

Fig. 4 *Three-phase unbalanced RL load compensation*

a System voltage $\boldsymbol{v}_s(t)$
b Load current $\boldsymbol{i}_l(t)$
c Source current $\boldsymbol{i}_s(t)$
d Compensator current $\boldsymbol{i}_c(t)$

Table 2: rms current values of the unbalanced load compensation

	I_l (A)	I_s (A)
phase *a*	8.06	8.11
phase *b*	9.00	7.95
phase *c*	11.81	8.35
$I_{unbalance}$(%)	38.97	4.92

Figs. 3*e*–*h* show the results of the DC link voltage control [15]. In Fig. 3e, the dashed line waveform is the reference DC voltage, which is 400 V at the beginning, and a step change is made to the reference to 450 V at $t = 2.66$ s. The solid line waveform is the actual DC link voltage measured by the potential transformer (PT). Fig. 3*f* is a zoomed-in view of Fig. 3*e* around the step change. To regulate the DC voltage from 400 to 450 V, the compensator draws a larger amount of active current than usual to charge the capacitor. This is controlled by the DC link voltage PI controller. The dynamic processes of the source current and the compensator current are shown in Figs. 3 *g* and *h*. Because of the constraints of the system, for instance, the current and/or power ratings of the components in the compensator, only a limited current can be drawn from the utility, which causes the actual DC link voltage to reach a new reference value in ≃0.1 s. A higher current rating would allow a faster dynamic response.

5.2 Three-phase unbalanced load

For this test, the inductors of the *RL* load are not equal in each phase; therefore the three-phase load currents are not balanced (Fig. 4*b*). The three-phase system voltages are balanced, as shown in Fig. 4*a*. The rms values of the three-phase load currents are given in the second column of Table 2. Fig. 4*c* shows the source currents after compensation, which are nearly balanced compared with the load current. The compensation current is shown in Fig. 4*d*. The three-phase source current rms values are listed in the third column of Table 2. The unbalance of the three-phase currents is calculated as

$$I_{\text{unbalance}} = \frac{\max\{|I_a - I_b|, |I_b - I_c|, |I_c - I_a|\}}{(I_a + I_b + I_c)/3} \tag{51}$$

The unbalance of the load current is 38.97% and that of the source current is only 4.92% after compensation.

5.3 Diode rectifier load

A three-phase diode rectifier load is a typical nonlinear load in the power system. A resistive load is used on the DC side of the rectifier. The measured load current is shown in Fig. 5*b*, which is highly distorted. Figs. 5*c* and *d* show the source current and compensator current, respectively, when the DC link voltage is 500 V. The source current is still distorted, which is mainly because of the large di/dt in the load current. A large di/dt requires a large DC link voltage for full compensation, that is, a large inverter output voltage, so that the compensator can inject a large amount of current in a very short period of time. However, it is often not practical to have such a high DC link voltage. Increasing the DC link voltage to 600 and 700 V, the resulting source currents are shown in Figs. 5*e* and *f*, respectively. The dashed sinusoidal waveforms in these figures are the phase *a* voltage to show that the currents are in phase with the voltage. By increasing the DC link voltage from 500 to 600 V and then to 700 V, the

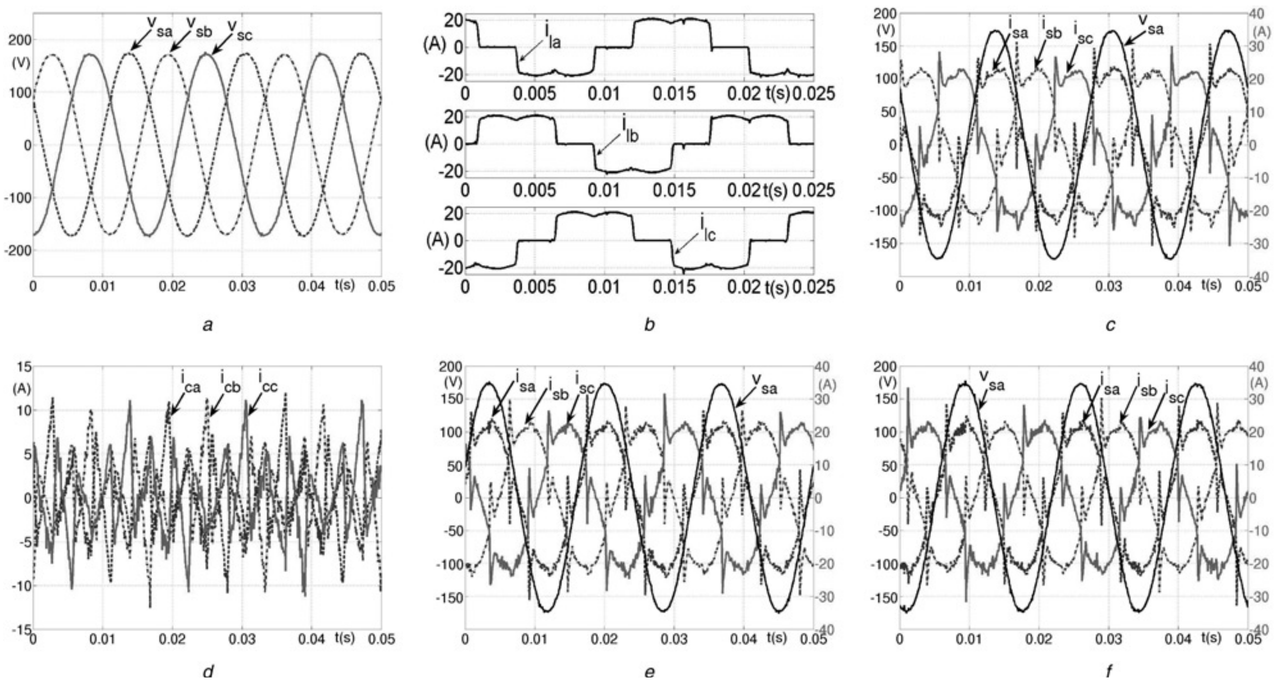

Fig. 5 *Three-phase rectifier load compensation*

a System voltage $\boldsymbol{v}_s(t)$
b Load current $\boldsymbol{i}_l(t)$
c Source current $\boldsymbol{i}_s(t)$, $V_{dc} = 500$ V
d Compensator current $\boldsymbol{i}_c(t)$, $V_{dc} = 500$ V
e Source current $\boldsymbol{i}_s(t)$, $V_{dc} = 600$ V
f Source current $\boldsymbol{i}_s(t)$, $V_{dc} = 700$ V

Table 3: THD of the currents in the rectifier load compensation

	$V_{dc} = 500$ V		$V_{dc} = 600$ V		$V_{dc} = 700$ V	
	THD of i_l(%)	THD of i_s(%)	THD of i_l(%)	THD of i_s(%)	THD of i_l(%)	THD of i_s(%)
phase *a*	29.90	25.40	30.19	21.55	30.07	18.51
phase *b*	29.96	25.40	30.09	21.28	30.76	19.29
phase *c*	29.96	25.17	30.38	20.90	30.02	18.73
average	29.94	25.32	30.22	21.24	30.28	18.84

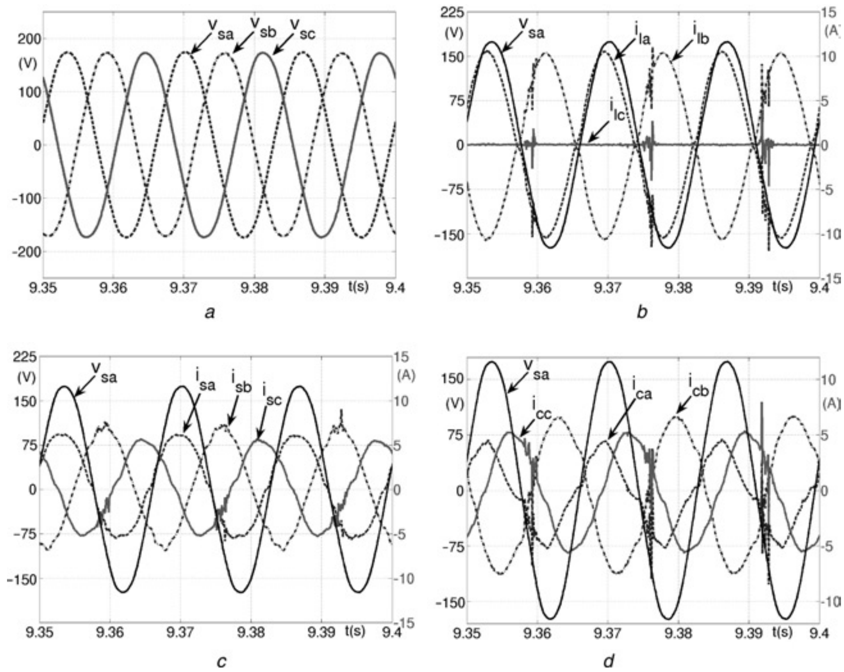

Fig. 6 *Single-phase RL load compensation*

a System voltage $\boldsymbol{v}_s(t)$
b Load current $\boldsymbol{i}_l(t)$
c Source current $\boldsymbol{i}_s(t)$
d Compensator current $\boldsymbol{i}_c(t)$

waveforms of the source current are improving, that is, closer to a sinusoid.

Table 3 lists the THD of the load current and source current in the three cases. The THD of the load current is $\simeq$30% and that of the source current is 25.32% (average value of the three phases) when the DC link voltage is 500 V, and it decreases to 21.24% and 18.84% as the DC link voltage is increased to 600 and 700 V, respectively.

5.4 Single-phase load

An *RL* load is connected between phase *a* and phase *b* in the three-phase system while phase *c* is left open (unloaded). The voltage and the load current are shown in Figs. 6*a* and *b*, respectively. The load currents in phase *a* and phase *b* are equal in magnitude and opposite in phase, and the current in phase *c* is zero. The rms values of the three-phase load currents are listed in the second column of Table 4. The rms value of the phase *c* load current is slightly greater than zero because of the measurement error as a result of IGBT switching. The source current and the compensator current are shown in Figs. 6*c* and *d*, respectively. The magnitudes of phase *a* and phase *b* source currents are reduced and there is a current in phase *c*. The rms values of the three-phase source currents are shown in the third column of Table 4. The values of phase *a* and phase *b* are reduced and the three phases are more balanced after compensation. The unbalance of the load current is 137.17%, and after compensation, the unbalance of the source current is reduced to 22.42%.

5.5 Distorted voltage and current

If the system voltage is distorted, choosing a different reference voltage will result in a different source current. Usually, sinusoidal system current is desired even with distorted voltage; therefore the reference is chosen as the fundamental component of the system voltage. This case is simulated in MatLab, and the system source voltage $\boldsymbol{v}_s(t)$ and the load current $\boldsymbol{i}_l(t)$ are shown in Figs. 7*a* and *b*, respectively. The reference voltage $\boldsymbol{v}_p(t)$ is shown in

Table 4: Rms current values of the single-phase load compensation

	I_l (A)	I_s (A)
phase *a*	7.20	4.44
phase *b*	7.22	4.97
phase *c*	0.43	3.97
$I_{unbalance}$(%)	137.17	22.42

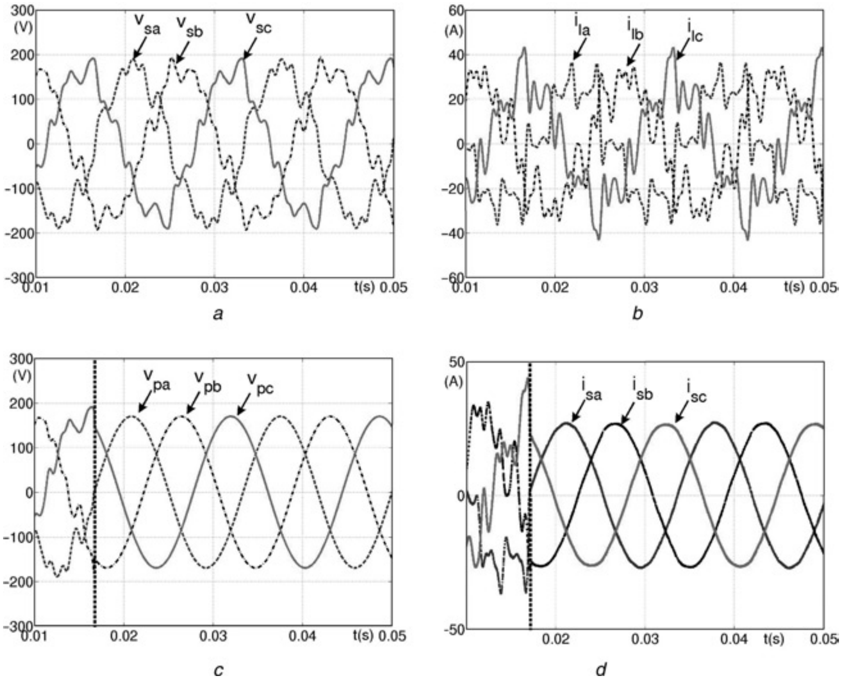

Fig. 7 Three-phase harmonic load compensation with sinusoidal $\boldsymbol{v}_p$

a System voltage $\boldsymbol{v}_s(V)$
b Load current $\boldsymbol{i}_l(A)$
c Reference voltage $\boldsymbol{v}_p(V)$
d System current $\boldsymbol{i}_s(A)$

Fig. 7*c*. From $t = 0$ to $t = 0.017$ s (the left region of the dashed line), the reference voltage is still the system voltage, whereas from $t = 0.017$ to $t = 0.05$ s (the right region of the dashed line), the reference voltage is the fundamental component of the system voltage, which is sinusoidal. The source current $\boldsymbol{i}_s(t)$ is shown in Fig. 7*d*, which is sinusoidal and in phase with the reference voltage after $t = 0.017$ s.

6 Conclusion

A generalised non-active power theory was presented in this article for non-active power compensation. The instantaneous active current, the instantaneous non-active current, the instantaneous active power and the instantaneous non-active power were defined for a power system and did not have any limitations such as the number of the phases. The voltage and the current were sinusoidal or non-sinusoidal, periodic or non-periodic. By changing the reference voltage and the averaging interval, this theory had the flexibility to define non-active current and non-active power in different cases.

A STATCOM demonstrated the versatility of this theory. A control scheme was developed to regulate the DC link voltage of the STATCOM and to generate the switching signals for the inverter based on the required non-active current. The experimental results showed that the theory proposed in this article was applicable to non-active power compensation in a three-phase balanced or unbalanced *RL* load, a diode rectifier load and a single-phase load.

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