Analysis of Var Benefits with Application to Var Planning

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***Abstract*– This paper will first discuss the quantitative economic benefits from reactive power (Var) compensation. The benefits can be categorized as reduced losses (B_1), exchanging reactive power flow to real power flow (B_2), and increased transfer capability (B_3). Then, the benefits in the three categories are applied to Var planning considering different locations and amounts of Var compensation. The study on Var planning presented in this paper is a straightforward, two-step approach: 1) it utilizes an Optimal Power Flow (OPF) model to update the Total Transfer Capability (TTC) in order to give a more accurate evaluation of Var benefits; 2) it then performs three OPFs for each Var location and amount combination to evaluate the Var benefits in three categories. Although this approach may be time-consuming, it does give a full spectrum and insightful information about the benefits under different categories if a Var compensator is installed at a specific location in various amounts. Hence, the sensitivity of economic benefits under different categories will be easily obtained. This approach may be used for future researches for benchmarking. It is also interesting to observe that the test results from a seven-bus system in this paper shows that it is not always economically efficient if Var compensation increases continuously.**

***Index Terms*–Optimal power flow (OPF), reactive power planning, total transfer capability (TTC), (RPP), voltage stability margin, Var economic benefits.**

I. INTRODUCTION

THE US power industry has been under great pressure to serve load economically since deregulation was initiated over a decade ago. Reactive power is critical to support voltage and regulate power factor in electric power systems. However, the reactive power in US power systems was not very well planned and managed, as evidenced by the Great 2003 Blackout that occurred in northeastern US and Canada in August 2003. The official final report of the Blackout indicated that "deficiencies in corporate policies, lack of adherence to industry policies, and inadequate management of reactive power and voltage caused the blackout [1]." Reactive power including its planning process has received tremendous interest and re-examination after the Blackout involving power utilities, independent system operators (ISOs), researchers, and the government. For example, various reactive power planning groups have been formed in ISOs.

Most Var planning objectives were to provide the least cost of new reactive power supplies with feasible voltage magnitudes as constraints. Variants of these objectives have been also presented by many researchers. Nevertheless, the Optimal Power Flow (OPF) model fits the need of Var planning naturally [2]. References [3]-[5] treat minimizing Var source cost as objective. In [7]-[12], minimizing weighted sum of Var cost and real power losses is the objective. And in [13]-[15], rather than the real power losses, minimizing weighted sum of Var cost and production cost is adopted as the objective. This is because minimization of the total fuel cost already includes the cost reduction due to the minimization of real power loss, which is consistent with the economic benefits analysis in this paper.

The main function of shunt reactive power compensation is for voltage support to avoid voltage collapse. Voltage itself is a poor indicator of proximity to system collapse conditions. Thus, the incorporation of voltage stability margin has become essential when the location and size of new Var sources need to be determined during Var planning. References [16] and [17] have incorporated the static voltage stability margin in the Var planning, which provided more realistic solutions for the Var planning problem. But two sets of constraints, one for normal operating state, and the other for critical state corresponding to the voltage collapse point in the P-V curve are required. In addition, the objective is to minimize the total Var installation quantity; no derived system benefits are taken into consideration in the two papers.

This paper will first discuss the quantitative economic benefits under three categories. The three benefits are referred to as B_1, B_2 and B_3. Here B_1 is the benefit from system loss, B_2 is the benefit from exchanging reactive power flow to real power flow, and B_3 is the benefit corresponds to the increased power transfer limit due to increased voltage stability. The discussion shows that three OPF runs for a given location and amount of Var compensation can decide the three benefits.

This paper then applies the benefits under three categories to Var planning with the consideration of the updated tie-line transfer capability due to Var compensation. The study on Var planning presented in this paper is a straightforward, two-step approach: 1) it utilizes an Optimal Power Flow (OPF) model to update the Total Transfer Capability (TTC) in order to give a more accurate evaluation of Var benefits; 2) it then performs three OPF runs for each Var location and amount combination to evaluate the Var benefits in three categories. Although this approach may be time-consuming, it does give a full spectrum and insightful information about the benefits under different categories if a Var compensator is installed at a specific location in various amounts. Hence, the sensitivity of economic benefits under different categories will be easily obtained. This approach may be used for future researches for benchmarking. It is also interesting to observe that the test results from a seven-bus system in this paper shows that it is

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not always economically efficient if Var compensation increases continuously

This paper is organized as follows. Section II illustrates the possible benefits under three categories from local Var compensators using a simple two-bus system. Section III presents a more rigorous approach using economic dispatch in OPF to identify the benefits in three categories. Section IV presents TTC model to calculate a key parameter, *tie line MVA transfer capability limits*, in the evaluation of reactive power benefits. Section V presents Var economic benefits-based OPF trial candidates approach for reactive power planning. Section VI presents the test results for a seven-bus system with Var compensation, and Section VII presents the conclusion.

II. QUANTITATIVE EVALUATION OF REACTIVE POWER BENEFIT IN A TWO-BUS SYSTEM

Before reactive power planning is studied, it is necessary to clearly illustrate the benefits obtained from reactive power compensation and their quantitative evaluation. The optimal location chosen is based on maximizing the quantitative benefits evaluation. This section demonstrates a possible quantitative approach to assess the "hidden" benefits from Var sources at the demand side [18]. These benefits are illustrated with a simple two-bus model in this section and then presented with a more complicated model using Optimal Power Flow in section III.

A two-bus system shown in Fig. 1 is used to illustrate a systematic methodology for capturing the hidden benefits. In Fig. 1, there are a generation center with a low-cost generation unit of \$20/MWh cost, a load center with a large amount of load, and a high-cost generation unit of \$25/MWh cost, and a tie line with maximum transfer capability of 100 MVA at the receiving end connecting the two areas. The net load of the load center is 100 MVA with 0.9 lagging power factor, which implies 90 MW and 43.59 MVar (P_2 and Q_2, respectively).

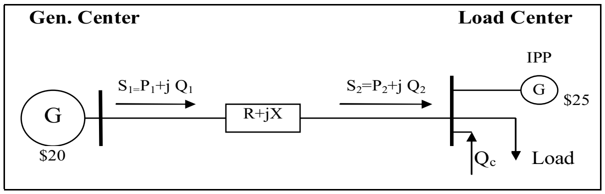

Fig. 1. A two-bus system.

The other parameters are as follows: the power base is 100 MVA; the voltage at the generation center bus is fixed at $1.0\angle 0^o$ per unit; and the line impedance is 0.02 + j0.2 per unit. The local compensation device will constantly inject Q_c = 14.01 MVar to lift the load power factor from 0.9 to 0.95, i.e., $P_2' = P_2$ = 90 MW and $Q_2' = Q_2 - Q_c$ = 43.59-14.01 = 29.58 MVar. The three economic benefits are discussed below.

A. Benefit from Reduced Losses (B$_1$)

Injection of reactive power at the receiving end reduces the reactive power through the tie line and therefore reduces the line current. Since the real power loss is I^2R, the loss will be reduced if the current is reduced. With the consideration of the load-side voltage magnitude remains unchanged and very close to 1.0, the original line loss and the power at the delivery end before the Q_c compensation are given as follows.

$$P_{loss} = I^2 R = \frac{P_2^2 + Q_2^2}{V^2} R = \frac{0.9^2 + 0.4359^2}{1.0^2} \cdot 0.02 = 0.02\, p.u. = 2\, MW$$

$$P_1 = P_2 + P_{loss} = 90 + 2 = 92\, MW$$

After Q_c is connected, the power losses and delivery end power are as follows.

$$P'_{loss} = I^2 R = \frac{(P'_2)^2 + (Q'_2)^2}{V^2} R = \frac{0.9^2 + 0.2958^2}{1.0^2} \cdot 0.02 = 0.018\, p.u. = 1.80\, MW$$

$$P'_1 = P'_2 + P'_{loss} = 90 + 1.80 = 91.80\, MW$$

Therefore, the total loss savings at the delivery end is 0.2 MW (92-91.8). This loss reduction represents reduced total generation. Therefore, the savings in dollars per MVar-year is \$2,501/MVar-year [(\$20/MWh x 0.2 MW x 8760hr)/14.01 MVar].

B. Benefit from Exchanging Reactive Power Flow to Real Power Flow (B$_2$)

As previously assumed, the tie line is congested due to the maximum transfer capability of 100 MVA at the receiving end. If this is the case, it is still assumed that the limit of S_2 remains at 100 MVA. Since the reactive power flow, Q_2, has been reduced due to local compensation, this makes it possible to have more real power delivered from the lower-cost generator while the 100 MVA limit is still respected because of $P_2 = \sqrt{S_2^2 - Q_2^2}$. This benefit of transferring more cheap real power while keeping the same transfer capability is classified as the benefit of exchanging reactive power flow to real power flow, as in the title of this subsection.

The new real power transferred over the tie-line is given as

$$P_2 = \sqrt{100^2 - (Q_2 - Q_C)^2} = \sqrt{100^2 - 29.58^2} = 95.52\, MW$$

Hence, the additional deliverable real power is 5.52 MW. Ignoring the additional loss due to the 5.52 MW, this is the amount of additional lower-cost real power from the generation center to the load center. The economic benefit to the load-serving utility will be the 5.52 MW times the price difference between the two generators. Assuming the tie line is congested during 2 peak months, the savings per MVar-year due to B_2 for the load center is \$2,837/MVar-year [(\$25/MWh -\$20/MWh) x 5.52MW x 60day x 24hr / 14.01MVar].

C. Benefit from Increased Maximum Transfer Capability (B$_3$)

In the previous analysis, the maximum transfer capability is assumed to be unchanged. However, it is very possible that the local Var compensation in the stressed area may increase the maximum transfer capability constrained by voltage stability. This is shown in Fig. 2.

There are various ways to calculate the change of transfer capability with respect to a change of system conditions [19]-[20] including local Var injection. Here, the equation of the maximum real power transfer in a two-bus model [21] is employed as follows:

$$P_{\max} = \frac{E^2(-k + \sqrt{1 + k^2})}{2X}, \quad where\, k = \frac{Q}{P}$$

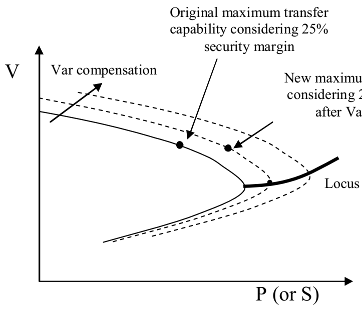

Fig. 2. The original and new transfer capability considering security margin.

It can be easily verified that the maximum transfer capacity has been improved by 15.5%. Therefore, the load center may receive 103.95 MW (90 x 1.155), which means it may receive another 8.43 MW (103.95-95.52) of lower-cost power from the generation center due to the increase of the transfer capability. Ignoring the line loss caused by this transfer capability increase, with the previously assumed 2 months of peak load, the benefit B_3 in \$/MVar-year is \$4,384 /MVar-year [(\$25/MWh -\$20/MWh) x 8.43MW x 60day x 24hr / 14.01MVar].

III. BENEFITS FROM VAR SOURCE IN A MULTI-BUS SYSTEM

The previous section illustrates the three benefits with a simple two-bus model; this section presents a generic formulation to assess the economic benefits of Var compensation via comparisons of three different cases of optimal generation dispatch. The dispatch is performed for the three cases using Optimal Power Flow (OPF) with respect to transmission limits and inter-tie transfer capability limits [22]. The three cases are as follows:

Base Case: Base system without Var compensation ($Q_c = 0$);
Case 1: Compensation is available at a given bus in a given amount and the original interface transfer limit is maintained;
Case 2: Compensation is available as in Case 1 and a new interface transfer limit is applied.

The objective of the OPF for the above three cases is to minimize the production cost. The constraints include the limits of the transmission networks. The dispatch formulation in the OPF model can be written as follows:

Min: $\sum f(P_{Gi})$ (1)

Subject to:

$P_{Gi} - P_{Li} - P(V,\theta) = 0$ *(Real power balance)*

$Q_{Gi} + Q_{Ci} - Q_{Li} - Q(V,\theta) = 0$ *(Reactive power balance)*

$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max}$ *(Generation real power limits)*

$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max}$ *(Generation reactive power limits)*

$V_i^{\min} \le V_i \le V_i^{\max}$ *(Voltage limits)*

$Q_{ci}^{\min} \le Q_{ci} \le Q_{ci}^{\max}$ *(Compensation limits)*

$|LF_l| \le LF_l^{\max}$ *(Line flow thermal limits)*

$\sum_{l\in Lt} S_l \le \sum_{l\in Lt} S_l^{\max}$ *(Tie line MVA transfer capability limits)*

where

i — the set of buse;
l — the set of lines;
Lt— the set of tie lines;
P_{Gi} — generator active power output;
P_{Li} — load active power;
Q_{Gi} — generator reactive power output;
Q_{Li} — load reactive power;
Q_{ci} — Var source installed at bus i ;
V_i — bus voltage;
LF_l — transmission line flow;
S_l — line MVA flow.

After the optimal dispatches are performed for the three cases, the benefits B_1, B_2 and B_3 may be identified using the following approach, assuming z_0, z_1, z_2 are the fuel cost for the Base Case, Case 1, and Case 2 respectively.

$$B_2 = \sum_{\text{congested hours}} (C_L - C_G)\cdot \Delta P_{exch} \quad (2)$$

$$B_1 = z_0 - z_1 - B_2 \quad (3)$$

$$B_3 = z_1 - z_2 \quad (4)$$

$$B_t = B_1 + B_2 + B_3 = z_0 - z_2 \quad (5)$$

where

B_t= the total benefit from local Var compensation;
B_1 = the benefit from reduced loss;
B_2 = the benefit from exchanging reactive power flow to real power flow without considering change of transfer capability;
B_3 = the benefit from the increased transfer capability;
C_G = the average cost of the generators at the generation center;
C_L = the average cost of the generators at the load center;
ΔP_{exch}= the exchange of reactive power flow to real power flow;
ΔP_{loss} = the reduced loss.

1. Perform OPF for Base Case and Case 1.
2. Calculate the total reduced MW generation from Base Case to Case 1. This MW amount is $\Delta P_{loss} + \Delta P_{exch}$.
3. Find the reduced system losses, ΔP_{loss}. Then, ΔP_{exch} can be easily obtained.
4. Perform OPF for Case 2.
5. Apply (2) – (5) to calculate the three economic benefits, B_1, B_2, and B_3.

IV. OPF MODEL FOR TIE-LINE TRANSFER CAPABILITY

In the previous section, there is a necessary parameter *tie line transfer limits* in the OPF model. In the two-bus system, the P_{max} equation is used to obtain the change of tie line transfer capability with respect to the local Var injection. However, the P_{max} equation is not suitable for the multi-bus system. Then the calculation of the total transfer capability (TTC) becomes a key point in the evaluation of reactive power benefits, which is also a discussion topic in the literature. Various ways to calculate Point of Collapse (PoC) of a P-V curve are introduced in [23] such as continuation power flow (CPF) and OPF. In this section, OPF is applied to obtain the TTC because it is easy to incorporate various limits into the OPF model such as generator real power and reactive power limits, and bus voltage limits, which is difficult to realize for CPF. The TTC formulation in the OPF model, though similar to some previous works, is presented here for integrity and illustration. It can be written as follows:

Max:

$$f(P_{Gi}(i \in Source), P_{Li}(i \in Sink), Q_{Li}(i \in Sink)) = \sum_{i \in Sink} P_{Li} - \sum_{i \in Sink} P_{Li}^0$$

Subject to:

$$P_{Gi} - P_{Li} - P(V,\theta) = 0 \qquad \textit{(Real power balance)}$$

$$Q_{Gi} + Q_{Ci} - Q_{Li} - Q(V,\theta) = 0 \qquad \textit{(Reactive power balance)}$$

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \qquad \textit{(Generation real power limits)}$$

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \qquad \textit{(Generation reactive power limits)}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \qquad \textit{(Voltage limits)}$$

$$Q_{ci}^{\min} \le Q_{ci} \le Q_{ci}^{\max} \qquad \textit{(Compensation limits)}$$

$$\left|LF_l\right| \le LF_l^{\max} \qquad \textit{(Line flow thermal limits)}$$

$$P_{Li} \ge P_{Li}^0 \qquad (i \in Sink)$$

$$Q_{Li} \ge Q_{Li}^0 \qquad (i \in Sink)$$

$$P_{Gi} \ge P_{Gi}^0 \qquad (i \in Source)$$

$$P_{Gi} = P_{Gi}^0 + \frac{\left(\sum_{i \in Source} P_{Gi} - \sum_{i \in Source} P_{Gi}^0\right) \times (P_{Gi}^{\max} - P_{Gi}^0)}{\sum_{i \in Source} (P_{Gi}^{\max} - P_{Gi}^0)} \qquad (i \in Source)$$

$$P_{Li} = P_{Li}^0 + \frac{\left(\sum_{i \in Sink} P_{Li} - \sum_{i \in Sink} P_{Li}^0\right) \times P_{Li}^0}{\sum_{i \in Sink} P_{Li}^0}$$

$(i \in Sink)$

$$P_{Li} / P_{Li}^0 = Q_{Li} / Q_{Li}^0 \qquad (i \in Sink)$$

where

$P_{Li}{}^0, Q_{Li}{}^0, P_{Gi}{}^0$ *= initial operation point;*

Typically, a generation center is named a *Source* area, and a load center is named as a *Sink* area. The objective is to maximize the load demand increase from the initial operation point in a *Sink* area. The real power outputs of generators in the *Source* area increase following a specified pattern, which is the ratio of reserve real power of generator *i* to the total reserve real power of the generators in the *Source* area. The reserve real power is the power available for use to balance the load demand increase, which can be expressed as $P_{Gi}^{\max} - P_{Gi}^0$.

The real power loads in the *Sink* area increase by the ratio $P_{Li}^0 / \sum_{i \in Sink} P_{Li}^0$. And the complex load is adjusted with constant power factor. The real power outputs of generators in the *Source* area and the real/ reactive load in the *Sink* area can be adjusted in order to obtain the maximum transfer capability.

Assuming the Q_{ci} is zero, the tie line transfer capability limit for Base Case can be achieved by running the TTC OPF model. Then assigning the Q_{ci} a specified value, an increased tie line transfer capability limit may be obtained for Case 2. Both limits can be put into the three OPF models in section II.

V. OPTIMAL VAR LOCATION AND SIZE SELECTION BASED ON ECONOMIC BENEFITS SENSITIVITY ANALYSIS

The previous sections have discussed how to evaluate three Var economic benefits and how to obtain a necessary parameter *tie line transfer limits* in the evaluation of reactive power benefits by a TTC model. This section will focus on optimal location and size decision strategy based on maximum Var economic benefits considering technical requirement such as tie line transfer capability limits.

A. Trial candidates method

In order to decide the optimal Var quantity, sensitivity analysis of Var economic benefits with respect to the size of the Var compensator is necessary, which has fundamental economic interpretation of the $ benefits associated with marginally increasing the Var quantity.

The sensitivity to the Var quantity is a good indicator not only for size selection, also for location decision. In the economic benefits analysis, the location is fixed. However, if some other buses as potential candidates for Var compensator installation are tried for the benefits sensitivity analysis, a series of benefits versus Q_c curves at different buses may be obtained.

The optimal location can be achieved by a comparison of the total benefit and the optimal Var size as shown in Fig. 3 at all candidate buses, which is called *trial candidates* method. The bus with the highest total benefit *and* the smallest optimal Var size is definitely the optimal location because of its highly efficient compensation. However, the highest total benefit does not necessarily correspond to the smallest optimal Var size, usually a larger optimal Var size matches the highest total benefit. In this case, yearly based cost-benefit analysis or other approach can be employed to identify a tradeoff between costs and benefits. The actual tradeoff may depend on the investor's need and is beyond the scope of this paper. Nevertheless, with a set of B_t versus Q_c curve at different buses, it is not difficult to perform a cost-benefit analysis to select the most appropriate compensation location and amount.

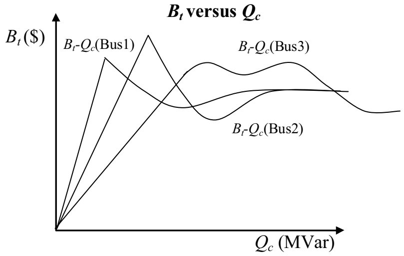

Fig. 3. Identify the optimal Var size from B_t versus Q_c curve.

The economic benefits with respect to the Var quantity may be obtained by increasing the variable of Var size step by step and solving the associated OPF nonlinear problems such as Base Case, Case 1, and Case 2. Please note that an additional OPF needs to be run to obtain the new TTC with Q_c compensation such that the new TTC may be used in Case 2 run. The whole process needs to be repeated at different locations to eventually obtain B_t-Q_c curve at different buses.

This approach has three features as follows compared with the previous work in the economic benefit analysis for Var compensation. The first feature lies in the consideration of economic impact due to the *change* of the tie line transfer limit corresponding to different quantity of Var compensation, which is not clearly shown in the literatures. Secondly, this approach maximizes the sum of three benefits compared with maximizing only $B_1 + B_2$ in the literature.

B. Trial candidates method GAMS (General Algebraic Modeling System) implementation procedure

The general scheme implemented with GAMS is summarized in this subsection. The basic procedure as applied to sensitivity analysis is shown in Fig. 4. The process starts by repeating to solve the TTC model with different Q_c quantity from Q_{c1}-Q_{cn}, the output is the tie line transfer limit L_1-L_n corresponding to Q_{c1}-Q_{cn}. The benefits calculation includes Base Case, Case 1, and Case 2 introduced in section III.

The total fuel cost output is z_0 in Base Case, in which there is no Var compensation. Thus, z_0 will not change in the whole process. If Case 1 is repeated at different compensation levels from Q_{c1} to Q_{cn}, different total fuel costs, $z_{1,1}$-$z_{1,n}$, may be obtained. Then Case 2 may be repeated with (Q_{c1}, L_1) to (Q_{cn}, L_n) as input, the corresponding output of total fuel costs in Case 2 may be written as $z_{2,1}$-$z_{2,n}$. Finally, $B_{i,1}$-$B_{i,n}$ (i = 1,2,3) may be obtained following the procedure introduced in section III. As stated earlier, the important feature in this process lies in the change of tie line transfer limit corresponding to different quantity of Var compensation.

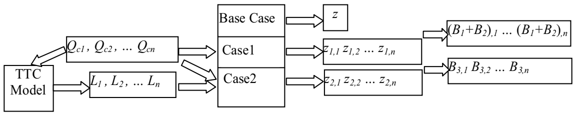

Fig. 4. General scheme for sensitivity analysis of economic benefits.

VI. CASE STUDY WITH RESULTS

A. Test System

In this section the seven-bus test system from PowerWorld [24] is used to demonstrate the optimal location and size selection for Var compensation. The diagram of the test system is shown in Fig. 5. The data for the loads, generation, transmission thermal limits and voltage limits are shown in Table 1. In order to study the increased maximum transfer capability for the tie lines, the test system is divided into two areas, the Top Area and the Bottom Area, as shown in Fig. 5 and Table 2. The Top Area is a load center, and the Bottom Area is a generation center. The generators in the load center are owned by IPPs and are more expensive than those in the generation center.

The interface tie lines between the two areas are line 6-2 and line 7-5. The voltage stability limit (the nose point of the P-V curve) is 464 MVA, which is lower than the sum of their thermal limits (500 MVA). If the voltage stability margin is assumed to be 25%, then the interface transfer limit is 464 *75%=348 MVA for Base Case. The OPF models for the three cases are solved by the Nonlinear Programming (NLP) solver MINOS.

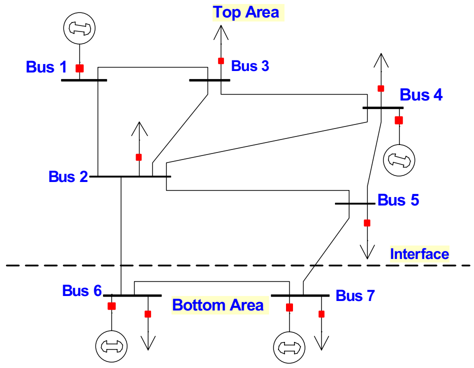

Fig. 5. Diagram of a seven-bus test system.

Table I. Parameters of the test system

<table>
<tr><th colspan="11">Power base: 100MVA</th></tr>
<tr><th colspan="11">Voltage base: 138kV</th></tr>
<tr><th colspan="11">Load</th></tr>
<tr><td>Bus</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>P_L (MW)</td><td>0</td><td>100</td><td>190</td><td>150</td><td>200</td><td>50</td><td>80</td></tr>
<tr><td>Q_L (MVar)</td><td>0</td><td>40</td><td>75</td><td>50</td><td>60</td><td>20</td><td>40</td></tr>
<tr><th colspan="11">Generator fuel consumption coefficient</th></tr>
<tr><td>Bus</td><td>1</td><td>4</td><td>6</td><td>7</td></tr>
<tr><td>a ($/hr)</td><td>798.92</td><td>814.03</td><td>515.34</td><td>400.41</td></tr>
<tr><td>b ($/MW*hr)</td><td>20</td><td>19</td><td>14</td><td>15</td></tr>
<tr><td>Fuel Cost ($/MW*hr)</td><td>20</td><td>19</td><td>14</td><td>15</td></tr>
<tr><th colspan="11">Active power generation limits (MW)</th></tr>
<tr><td>Bus</td><td>1</td><td>4</td><td>6</td><td>7</td></tr>
<tr><td>P_G^{max}</td><td>150</td><td>200</td><td>300</td><td>300</td></tr>
<tr><td>P_G^{min}</td><td>70</td><td>50</td><td>60</td><td>0</td></tr>
<tr><th colspan="11">Reactive power generation limits (MW)</th></tr>
<tr><td>Bus</td><td>1</td><td>4</td><td>6</td><td>7</td></tr>
<tr><td>Q_G^{max}</td><td>100</td><td>100</td><td>100</td><td>100</td></tr>
<tr><td>Q_G^{min}</td><td>-100</td><td>-100</td><td>-100</td><td>-100</td></tr>
<tr><th colspan="11">Transmission line thermal limits (MVA)</th></tr>
<tr><td>Line</td><td>1-2</td><td>1-3</td><td>2-3</td><td>2-4</td><td>2-5</td><td>4-3</td><td>5-4</td><td>6-2</td><td>6-7</td><td>7-5</td></tr>
<tr><td>Limit</td><td>120</td><td>100</td><td>100</td><td>100</td><td>100</td><td>120</td><td>80</td><td>250</td><td>100</td><td>250</td></tr>
<tr><th colspan="11">Voltage limits (p.u.)</th></tr>
<tr><td colspan="11">V_{max} =1.05 and V_{min}=0.95 for every bus.</td></tr>
</table>

Table II. Load and Generations in Two Areas

Area	Bus	Gen. Cap. (MW)	Load (MW)	Margin (MW)
Load Center	1, 2, 3, 4, 5	350	640	-290
Gen. Center	6, 7	600	130	470

B. Results

Assuming Bus 2, Bus 3, and Bus 5 are three Var compensation location candidates. By applying the scheme in section V, Fig. 6, Fig. 7 and Fig. 8 provide the whole picture for the economic benefits changing tendency with Var compensation at Bus 2, Bus 3, and Bus 5, respectively. The TTC OPF model becomes infeasible if the Q_c is greater than 200MVar, so the upper limit of Q_c size in this case is set to 200MVar. If 1 MVar step change is chosen, 200 points are calculated to draw the following graphs in this case.

There are some commonalities found in Fig. 6, Fig. 7, and Fig. 8. The shape of Benefit 1, or B_1, from reduced losses vs. Var compensation is similar to a hill. The uphill part is due to the reduction of current flow from the injection of reactive power at the load center, therefore the lower losses raise B_1. The reason for the downhill part is the over injected reactive power that can not be consumed by the loads has to be balanced by generators. Therefore, the overprovided reactive power has to be transferred through the lines; as a result, the line current increases, then the I^2R losses increases.

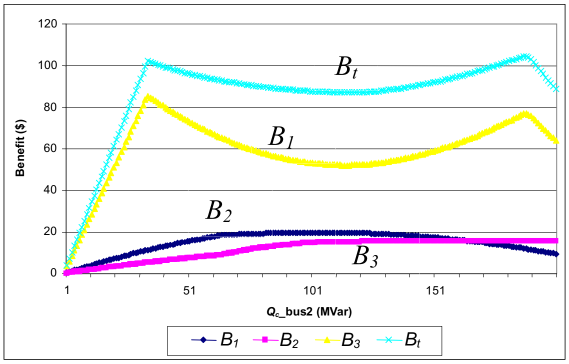

Fig. 6. B_1, B_2, B_3, and B_t versus Var compensation at Bus 2.

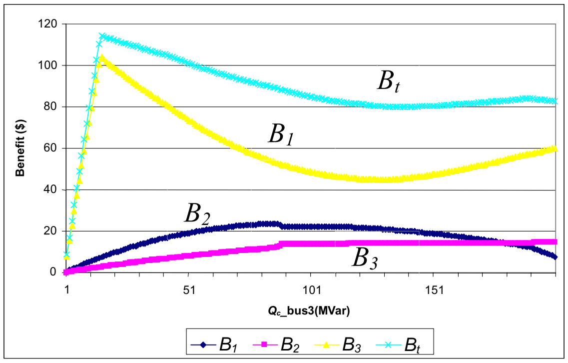

Fig. 7. B_1, B_2, B_3, and B_t versus Var compensation at Bus 3.

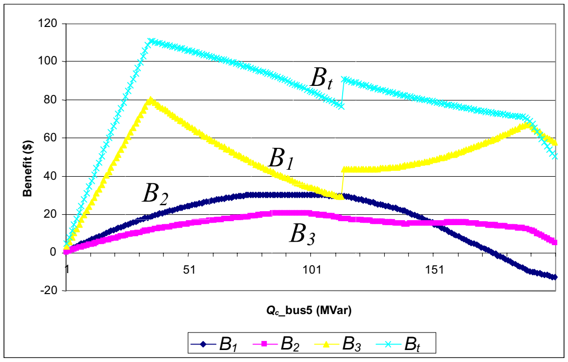

Fig. 8. B_1, B_2, B_3, and B_t versus Var compensation at Bus 5.

Benefit 2 from exchanging reactive power flow to real power flow vs. Var compensation increases from zero until saturation at some Q_c quantity in Fig. 6 and Fig. 7.

The shape of B_3 from increased tie line transfer limit vs. Var compensation is positively related to TTC vs. Var compensation graph, because the greater tie line transfer capability the more cheap power can be delivered from generation center to load center, therefore, the more benefit 3. After B_3 reaches its peak value, it slowly drops to the minimum value, and then slowly increases again. Usually, B_3 and TTC are considered to increase with more Var compensation, but it is only true in some specific Q_c range. On the contrary, it becomes smaller with more Var compensation in some other Q_c range. This trend affects the total benefit B_t greatly.

In reactive power planning, the total benefit B_t is generally the most important; the criteria of location and size selection is to get maximum total benefit. B_t_bus2, B_t_bus3 and B_t_bus5 are shown in one picture for convenient comparison in Fig. 9.

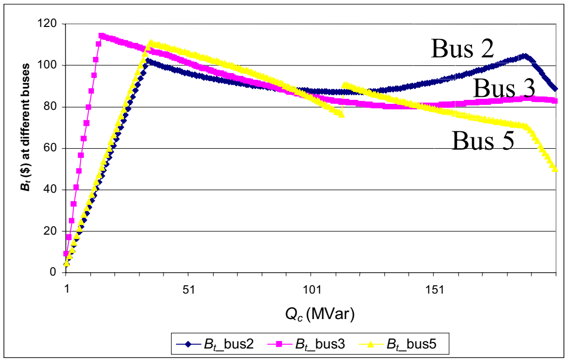

Fig. 9. Total benefit B_t at candidate buses versus Var compensation.

B_t_bus2, B_t_bus3 and B_t_bus5 rapidly increase from 0 and reach their maximum value $102.04/hr, $114.15/hr and $110.99/hr, when Q_c_bus2 = 34 MVar, Q_c_bus3 = 15 Mvar, and Q_c_bus5 = 35 MVar, immediately followed by a slow decline. After that, B_t_bus2 gradually increases to another peak value $104.53/hr at Q_c_bus2 = 187 MVar. The two peak values of B_t_bus2 are very close, but the corresponding Var size of the second peak is much greater than that of the first peak, thus the second peak is not an economic choice. It is obvious that installation of 15 Mvar Q_c at Bus 3 is the best choice, since the smallest Var size corresponds the highest total benefit B_t_bus3 = $114.15/hr. Certainly, there may be more complicated scenarios in which the maximum benefit occurs at a large compensation amount, a cost-benefit analysis or other approaches based on decision makers' needs may be employed, as stated in Section V.

The above analysis shows if the Var is over-compensated more than 15 MVar, the potential economic benefits will decrease. In one word, continuously increasing Var is not necessarily better; the benefits may decrease at some point with the Var compensation increasing.

The above simulation is a test for one hour. If multiple hours need to be simulated, we may simply employ a chronological simulation or some estimation from several typical hours as discussed in Section II.

VII. CONCLUSIONS

In this paper, the economic benefits in three categories from Var sources are discussed through a quantitative approach. The benefits are then applied to Var planning to show the different Var benefits under three categories with respect to various locations and amounts of Var compensation. The conclusions and need for future work based on this research are summarized as follows:

- The major and quantifiable economic benefit may be classified into three categories: reduced losses, exchanging reactive power flow to real power flow, and increased transfer capability.
- Based on the sensitivity of Var benefits, the economic efficiency may not grow as the Var compensation amount grows. The benefits may decrease at some point as the Var compensation increases.
- An important feature in this approach lies in the impact to economic benefit from the new tie line transfer limits corresponding to different amount of Var compensation, which is not clearly shown in the literature.
- B_t versus Q_c curves at different candidates may be adopted to decide the optimal location and Var quantity, which is "trial candidates" approach. Although this straightforward approach may be time-consuming, the results from this approach may be used for benchmarking purposes for future researches.
- Four OPF models involved in this approach, they are Base Case, Case 1, Case 2, and TTC models.
- In the future, a combined model from Base Case, Case 1, Case 2, and TTC models is needed to achieve an efficient model.
- Maximizing three benefits and minimizing Var installation cost can be easily implemented by including Var cost in the objective function of OPF runs for Base Case, Case 1 and Case 2.

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