Identification of the Rotor Time Constant in Induction Machines without Speed Sensor

M. Li*, J.N. Chiasson*, M. Bodson**, L.M. Tolbert*
*The University of Tennessee, ECE Department, Knoxville, USA
**The University of Utah, ECE Department, Salt Lake City, USA

Abstract—A differential-algebraic method is used to estimate the rotor time constant $T_R$ of an induction motor without measurements of the rotor speed/position. The method consists of solving for the roots of a polynomial equation in $T_R$ whose coefficients depend only on the stator currents, stator voltages, and their derivatives. Experimental results are presented.

Index Terms—Rotor Time Constant, Sensorless Speed Observer, Induction Motor.

I. INTRODUCTION

Induction motors are very attractive in many applications owing to their simple structure, low cost, and robust construction. Field-oriented control is now used to obtain high performance drive of the induction motor because it gives control characteristics similar to separately excited DC motors. Implementation of a (rotor-flux) field-oriented controller requires knowledge of the rotor speed and the rotor time constant $T_R$ to estimate the rotor flux linkages. There has been considerable work done in the last several years to implement a field-oriented controller without the use of a speed sensor [1][2][3][4][5][6]. However, many of these methods still require the value of $T_R$, which can change with time due to ohmic heating; that is, to be able to update the value of $T_R$ to the controller as it changes is valuable. The work presented here uses an algebraic approach to identify the rotor time constant $T_R$ without the motor speed information. It is most closely related to the ideas described in [7][8][9][10][11]. Specifically, it is shown that $T_R$ satisfies a polynomial equation whose coefficients are functions of the stator currents, the stator voltages, and their derivatives. A zero of this polynomial is the value of $T_R$. It is further shown $T_R$ is not identifiable by this technique under steady-state conditions. It is also true (and shown here) that a standard least-squares approach cannot identify $T_R$ under steady-state conditions. In [4], the speed $\omega$ and $T_R$ are identified assuming constant speed but not (sinusoidal) steady state. In [12], the speed is assumed constant, but the flux magnitude is perturbed by a small amplitude sinusoidal signal to identify $T_R$.

The paper is organized as follows. Section II introduces a space vector model of the induction motor. Section III uses this model to develop a differential-algebraic equation that $T_R$ must satisfy. Section IV shows that in steady state, $T_R$ is not identifiable by either the differential-algebraic method or a standard linear least-squares method. Section V presents the experimental results, while Section VI gives the conclusions and future work.

II. MATHEMATICAL MODEL OF INDUCTION MOTOR

The starting point of the analysis is a space vector model of the induction motor given by (see e.g., pp. 568 of [13])

\begin{align*}
\frac{d}{dt} i_S &= \frac{\beta}{T_R} (1 - j n_p \omega T_R) \psi_r - \gamma i_S + \frac{1}{\sigma L_S} u_S \\
\frac{d}{dt} \psi_r &= -\frac{1}{T_R} (1 - j n_p \omega T_R) \psi_r + \frac{M}{T_R L_S} \\
\frac{d \omega}{dt} &= \frac{n_p M}{J L_R} \text{Im} \left\{ i_S \psi_r \right\} - \frac{\tau_L}{J},
\end{align*}

where $i_S \triangleq i_{Sa} + j i_{Sb}$, $\psi_r \triangleq \psi_{Ra} + j \psi_{Rb}$, and $u_S \triangleq u_{Sa} + j u_{Sb}$. Here, $\theta$ is the position of the rotor, $\omega = d\theta/dt$ is the rotor speed, $n_p$ is the number of pole pairs, $i_{Sa}$, $i_{Sb}$ are the (two-phase equivalent) stator currents, $\psi_{Ra}$, $\psi_{Rb}$ are the (two-phase equivalent) rotor flux linkages, $R_S$, $R_R$ are the stator and rotor resistances, respectively, $M$ is the mutual inductance, $L_S$ and $L_R$ are the stator and rotor inductances, respectively, $J$ is the moment of inertia of the rotor, and $\tau_L$ is the load torque.

The symbols $T_R = \frac{L_R}{R_R}$, $\sigma = 1 - \frac{M^2}{L_S L_R}$, $\beta = \frac{M}{\sigma L_S L_R}$, $\gamma = \frac{R_S}{T_R}$ have been used to simplify the expressions. $T_R$ is referred to as the rotor time constant, while $\sigma$ is called the total leakage factor.

III. DIFFERENTIAL-ALGEBRAIC APPROACH TO $T_R$ ESTIMATION

The idea of the differential-algebraic approach is to solve (1) and (2) for $T_R$ [14][15]. However, equations (1) and (2) are only four equations while there are six unknowns, namely $\psi_{Ra}$, $\psi_{Rb}$, $d\psi_{Ra}/dt$, $d\psi_{Rb}/dt$, $\omega$, and $T_R$. Equation (3) is not used because it introduces the additional unknown $\tau_L$. To find two more independent equations, equation (1) is differentiated to obtain

\begin{align*}
\frac{d^2}{dt^2} i_S &= \beta (1 - j n_p \omega T_R) \frac{d}{dt} \psi_r - j n_p \beta \psi_r \frac{d \omega}{dt} \\
&\quad - \gamma \frac{d}{dt} i_S + \frac{1}{\sigma L_S} \frac{d}{dt} u_S.
\end{align*}

Using the (complex-valued) equations (1) and (2), one can solve for $\psi_r$ and $\frac{d}{dt} \psi_r$ in terms of $\omega$, $i_S$ and $u_S$ and substitute
the resulting expressions into (4) to obtain
\[
\frac{d^2}{dt^2} L_S = -\frac{1}{T_R} \left(1 - jn_πωT_R\right) \left(\frac{d}{dt} L_S + \frac{\beta M}{\sigma L_S} \sqrt{\frac{2}{T_R}}\frac{d\bar{u}_S}{dt}\right) \\
+ \frac{\beta M}{\sigma L_S} \left(1 - jn_πωT_R\right) \bar{u}_S - \frac{1}{\sigma L_S} \frac{d}{dt} \bar{u}_S \\
- \frac{1}{\sigma L_S} \frac{d}{dt} \bar{u}_S = 0.
\]
(5)

Solving (5) for \(d\omega/dt\) gives
\[
\frac{d\omega}{dt} = \frac{(1 - jn_πωT_R)^2}{jn_πT_R^2} + \frac{1}{jn_πT_R} \times \\
\frac{\beta M}{\sigma L_S} \left(1 - jn_πωT_R\right) \bar{u}_S - \frac{1}{\sigma L_S} \frac{d}{dt} \bar{u}_S - \frac{d^2}{dt^2} \bar{u}_S.
\]
(6)

The left-hand side of (6) is real, so the right-hand side must also be real. Note by (1) that \(d\bar{u}_S/dt + \gamma \bar{u}_S - \sigma \bar{u}_S/\sigma L_S = \frac{\beta M}{\sigma L_S} \left(1 - jn_πωT_R\right) \bar{u}_S\) so that the right-hand side of (6) is singular if and only if \(\frac{\beta M}{\sigma L_S} \left(1 - jn_πωT_R\right) \bar{u}_S \neq 0\). Other than at startup, \(\frac{\beta M}{\sigma L_S} \left(1 - jn_πωT_R\right) \bar{u}_S \neq 0\) in normal operation of the motor. Separating the right-hand side of (6) into its real and imaginary parts, the real part has the form
\[
\frac{d\omega}{dt} = a_2 (u_{sa}, u_{sb}, i_{sa}, i_{sb}) ω^2 + a_1 (u_{sa}, u_{sb}, i_{sa}, i_{sb}) ω \\
+ a_0 (u_{sa}, u_{sb}, i_{sa}, i_{sb}).
\]
(7)
The expressions for \(a_2 (u_{sa}, u_{sb}, i_{sa}, i_{sb}), a_1 (u_{sa}, u_{sb}, i_{sa}, i_{sb})\), and \(a_0 (u_{sa}, u_{sb}, i_{sa}, i_{sb})\) are lengthy in terms of \(u_{sa}, u_{sb}, i_{sa}, i_{sb}\), and their derivatives as well as of the machine parameters including \(T_R\). As a consequence, they are not explicitly presented here. Their steady-state expressions are given in [6].

On the other hand, the imaginary part of the right-hand side of (6) must be zero. In fact, the imaginary part of (6) is a second degree polynomial equation in \(ω\) of the form
\[
q(ω) = q_2 (u_{sa}, u_{sb}, i_{sa}, i_{sb}) ω^2 + q_1 (u_{sa}, u_{sb}, i_{sa}, i_{sb}) ω \\
+ q_0 (u_{sa}, u_{sb}, i_{sa}, i_{sb}).
\]
(8)
and, if \(ω\) is the speed of the motor, then \(q(ω) = 0\).

The \(q_i\) are functions of \(u_{sa}, u_{sb}, i_{sa}, i_{sb}\), and their derivatives as well as of the machine parameters including \(T_R\). The expressions for \(q_2 (u_{sa}, u_{sb}, i_{sa}, i_{sb}), q_1 (u_{sa}, u_{sb}, i_{sa}, i_{sb})\), and \(q_0 (u_{sa}, u_{sb}, i_{sa}, i_{sb})\) are also lengthy and not explicitly presented here. (Their steady-state expressions are given in [6].) If the speed was measured, then (8) would be equal to zero and could then be solved for \(T_R\). However, in the problem being considered, \(ω\) is not known. To eliminate \(ω\), \(q(ω)\) in (8) is differentiated to obtain
\[
\frac{d}{dt} q(ω) = (2q_2ω + q_1) \frac{d\omega}{dt} + q_2ω^2 + q_1ω + q_0
\]
(9)
where \(d\omega/dt\) in (9) is replaced by the right-hand side of (7) so that \(q(ω)\) in (9) may be written as
\[
\frac{dq(ω)}{dt} = g(ω) ≜ 2q_2ω^3 + (2q_2a_1 + q_1a_2 + q_1)ω^2 \\
+ (2q_2a_0 + q_1a_1 + q_1)ω + q_0.
\]
(10)
\(g(ω)\) is a third-order polynomial in \(ω\) for which the speed of the motor is one of its zeros. Dividing \(g(ω)\) in (10) by \(q(ω)^2\) in (8), \(g(ω)\) may be rewritten as
\[
g(ω) = \frac{1}{q_2} \left( \left(2q_2ω^2 + 2q_2a_1 - q_1a_2 + q_1\right)ω \right) \\
+ r_1 (u_{sa}, u_{sb}, i_{sa}, i_{sb}) ω + r_0 (u_{sa}, u_{sb}, i_{sa}, i_{sb}).
\]
(11)

If \(ω\) is equal to the speed of the motor, then both \(g(ω) = 0\) and \(q(ω) = 0\), and one obtains
\[
r(ω) = r_1 (u_{sa}, u_{sb}, i_{sa}, i_{sb}) ω + r_0 (u_{sa}, u_{sb}, i_{sa}, i_{sb}).
\]
(12)

This is now a first-order polynomial in \(ω\) which uniquely determines the motor speed \(ω\) as long as \(r_1\) (the coefficient of \(ω\)) is nonzero. (It is shown in Appendix VII-A that \(r_1 ≠ 0\) in steady state.) Solving for the motor speed \(ω\) using (11), one obtains
\[
ω = -r_0/r_1.
\]
(13)

Next, replace \(ω\) in (8) by the expression in (15) to obtain
\[
q_2r_1^2 - q_1r_0r_1 + q_0r_1^2 ≠ 0.
\]
(16)
The expressions for \(q_i, r_i\) are in terms of motor parameters (including \(T_R\) as well as the stator currents, voltages, and their derivatives. Expanding the expressions for \(q_0, q_1, q_2, r_0, r_1\), one obtains a twelfth-order polynomial equation in \(T_R\), which can be written as
\[
\sum_{i=0}^{12} C_i (u_{sa}, u_{sb}, i_{sa}, i_{sb}) T_R^i = 0.
\]
(17)

Solving equation (17) gives \(T_R\). The coefficients \(C_i (u_{sa}, u_{sb}, i_{sa}, i_{sb})\) of (17) contain third-order derivatives of the stator currents and second-order derivatives of the stator voltages making noise a concern. For short time intervals in which \(T_R\) does not vary, (17) must hold identically with \(T_R\) constant. In order to average out the effect of noise on the \(C_i\), (17) is integrated over a time interval \([t_1, t_2]\) to obtain
\[
\sum_{i=0}^{12} \left( \int_{t_1}^{t_2} C_i (u_{sa}, u_{sb}, i_{sa}, i_{sb}) dt \right) T_R^i = 0.
\]
(18)
There are 12 solutions satisfying (18). However, simulation results have always given 10 conjugate solutions. The remaining two solutions include the correct value of $T_R$ while the other one was either negative or close to zero. The method is to compute the coefficients $\frac{1}{T_R} \int_0^t f_i(t) C_i \, dt$ and then compute the roots of (18). Among the positive real roots is the correct value of $T_R$. Experimental results using this method are presented in Section IV.

IV. IDENTIFIABILITY OF $T_R$ IN STANDE STATE

A. Differential-algebraic approach

The polynomial (18) is now considered with the machine in steady state so that, in particular, the speed is constant. That is, $u_{sa} + ju_{sb} = U_s e^{j\omega s t}$ and $i_{sa} + ji_{sb} = I_s e^{j\omega s t}$ are substituted into (8) and (14). In steady state, the motor speed in (15) becomes (see Appendix VII-A and [16])

$$\omega = -\frac{r_0}{T_1} + \omega_S (1 - \frac{1}{n_p})$$

(19)

where $S \triangleq (\omega_S - n_\text{p} \omega)/\omega_S$ is the normalized slip and $\omega_S$ is the electrical frequency. Substituting the steady-state expressions for $q_2$, $q_1$, and $q_0$ as well as the expression (19) for $\omega$ into (8), one obtains $q_2 \omega^2 + q_1 \omega + q_0 = 0$.

That is, in steady state (8) and (14) hold independent of the value of $T_R$ and thus so does (17) making $T_R$ unidentifiable in steady state by this method.

B. Linear least-squares approach

Véllez-Reyes et al [3][4] have used least-squares methods for simultaneous parameter and speed identification in induction machines. In the approach used herein, $\omega$ is taken to be zero so that a linear (in the parameters) regressor model can be obtained. Specifically, consider the mathematical model of the induction motor in (5). Assuming constant speed, $\frac{d\omega}{dt} = 0$ so that this equation reduces to

$$\frac{d^2 i_{sb}}{dt^2} = -\frac{1}{T_R} (1 - j n_\text{p} \omega T_R) \left( \frac{di_{sb}}{dt} + \gamma i_{sb} - \frac{1}{\sigma L_S} u_{sb} \right)$$

$$+ \frac{\beta M}{T_R} (1 - j n_\text{p} \omega T_R) \frac{di_{sb}}{dt} - \gamma \frac{di_{sb}}{dt} + \frac{1}{\sigma L_S} \frac{du_{sb}}{dt}$$

(20)

where $i_S = i_{sa} + ji_{sb}$ and $u_S = u_{sa} + ju_{sb}$. Decomposing equation (20) into its real and imaginary parts gives

$$\frac{d^2 i_{sa}}{dt^2} = -\frac{1}{T_R} \left( \frac{di_{sa}}{dt} + \frac{R_S}{\sigma L_S} i_{sa} + \frac{1}{\sigma L_S} u_{sa} \right)$$

$$+ n_\text{p} \omega \left( -\frac{di_{sb}}{dt} + \frac{R_S}{\sigma L_S} i_{sb} + \frac{1}{\sigma L_S} u_{sa} \right)$$

$$- \gamma \frac{di_{sb}}{dt} + \frac{1}{\sigma L_S} \frac{du_{sb}}{dt}$$

(21)

and

$$\frac{d^2 i_{sb}}{dt^2} = -\frac{1}{T_R} \left( \frac{di_{sb}}{dt} + \frac{R_S}{\sigma L_S} i_{sb} + \frac{1}{\sigma L_S} u_{sa} \right)$$

$$- n_\text{p} \omega \left( -\frac{di_{sa}}{dt} + \frac{R_S}{\sigma L_S} i_{sa} + \frac{1}{\sigma L_S} u_{sa} \right)$$

$$- \gamma \frac{di_{sa}}{dt} + \frac{1}{\sigma L_S} \frac{du_{sa}}{dt}$$

The goal here is to estimate $T_R$ without knowledge of $\omega$. So, it is now assumed the motor parameters are all known except for $T_R$. The set of equations (21) and (22) may then be rewritten in regressor form as

$$y(t) = W(t) K$$

(23)

where $K \in \mathbb{R}^2$, $y \in \mathbb{R}^2$, and $W \in \mathbb{R}^{2 \times 2}$ are given by

$$y(t) = \begin{bmatrix} L_S \frac{du_{sa}}{dt} - u_{sa} + R_S i_{sa} & \sigma L_S \frac{du_{sb}}{dt} - u_{sb} + R_S i_{sb} \\ L_S \frac{du_{sb}}{dt} - u_{sb} + R_S i_{sb} & -\sigma L_S \frac{du_{sa}}{dt} - u_{sa} + R_S i_{sa} \end{bmatrix} \cdot W(t) = \begin{bmatrix} L_S \frac{du_{sa}}{dt} - u_{sa} + R_S i_{sa} & \sigma L_S \frac{du_{sb}}{dt} - u_{sb} + R_S i_{sb} \\ L_S \frac{du_{sb}}{dt} - u_{sb} + R_S i_{sb} & -\sigma L_S \frac{du_{sa}}{dt} - u_{sa} + R_S i_{sa} \end{bmatrix}$$

The regressor system (23) is linear in the parameters. The standard linear least-squares approach is to let (i.e., collect data at $t = 0, T, 2T, \ldots, NT$, multiply (23) on the left by $W^T(nT)$, sum $W^T(nT)y(nT) = W^T(nT)W(nT)K$ from $t = 0$ to $t = NT$, and finally compute the solution to

$$R_W K = R_{Y_W}$$

(24)

where

$$R_W = \sum_{n=0}^{N} W^T(nT)W(nT), \quad R_{Y_W} = \sum_{n=0}^{N} W^T(nT)y(nT).$$

A unique solution to (24) exists if and only if $R_W$ is invertible. However, $R_W$ is never invertible in steady state as is now shown. To proceed, define

$$D(t) = \begin{bmatrix} i_{sb}(t) & -i_{sa}(t) \\ i_{sa}(t) & i_{sb}(t) \end{bmatrix}.$$

In steady state where $u_{sa} + ju_{sb} = U_s e^{j\omega s t}$ and $i_{sa} + ji_{sb} = I_s e^{j\omega s t}$, det$(D(t)) = i_{sa}(t)^2 + i_{sb}(t)^2 = |I_s|^2$, $D(t)^T D(t) = |I_s|^2 I_{2 \times 2}$. Multiply both sides of (23) on the left by $D(t)$ to obtain

$$D(t) y(t) = D(t) W(t) K$$

or

$$\begin{bmatrix} R_S \omega_S |I_s|^2 - \omega_S P & \sigma L_S |I_s|^2 - \omega_S Q \\ -\omega_S L_S |I_s|^2 + Q & R_S |I_s|^2 - P \end{bmatrix} K$$

(25)
where $P \triangleq u_{sa}^t S_a + u_{sb}^t S_b$ and $Q \triangleq u_{sb}^t S_a - u_{sa}^t S_b$ are the real and reactive powers, respectively, whose steady-state expressions are given by (30) and (31) in the Appendix. Using (30) and (31) to replace $P$ and $Q$ in (25), one obtains

$$
\dot{D} \triangleq D(t) W(t) = \frac{|L_s|^2 (1 - \sigma) \omega_S L_S}{1 + S^2 \omega_S^2 T_R^2} \begin{bmatrix}
S^2 \omega_S^2 T_R^2 & S \omega_S T_R \\
S \omega_S T_R & 1
\end{bmatrix},
$$

$$
\dot{Y} \triangleq D(t) y(t) = \omega_S |L_s|^2 \left(1 - \sigma\right) \omega_S L_S \begin{bmatrix}
S \omega_S T_R \\
1
\end{bmatrix}.
$$

That is, in steady state, $\dot{D} \triangleq D(t) W(t) \in \mathbb{R}^{2 \times 2}$ and $\dot{Y} \triangleq D(t) y(t) \in \mathbb{R}^2$ are constant matrices. Further, it is easily seen that the determinant of $\dot{D} \triangleq D(t) W(t)$ is zero. Also,

$$
R_{DW} \triangleq \sum_{n=1}^{N} (D(nT) W(nT))^T (D(nT) W(nT)) = |L_s|^2 W_R.
$$

$R_{DW}$ is singular because $D(t) W(t)$ is constant and singular. It then follows that $R_W$ is also singular using steady-state data. Further,

$$
R_{DY} \triangleq \sum_{n=1}^{N} (D(nT) W(nT))^T (D(nT) y(nT)) = |L_s|^2 W_{R_Y}.
$$

Thus $R_W$ and $R_{DY}$ are given by

$$
R_W = R_{DW} / |L_s|^2 = ND^T \dot{D} / |L_s|^2
$$

$$
= N |L_s|^2 \left(1 - \sigma\right) \omega_S L_S^2 \begin{bmatrix}
S^2 \omega_S^2 T_R^2 & S \omega_S T_R \\
S \omega_S T_R & 1
\end{bmatrix},
$$

$$
R_{DY} = R_{DW} / |L_s|^2 = ND^T \dot{Y} / |L_s|^2
$$

$$
= \omega_S N |L_s|^2 \left(1 - \sigma\right) \omega_S L_S^2 \begin{bmatrix}
S \omega_S T_R \\
1
\end{bmatrix},
$$

where again $\dot{D}$ and $\dot{Y}$ are from (26) and (27), respectively.

By inspection of (28) and (29), $K = [0 \omega_S]^T$ is one solution to (24). The null space of $R_W$ is generated by $[-1/T_R \omega_S S \omega_S]^T$ so that all possible solutions are given by $[0 \omega_S]^T + \alpha [-1/T_R \omega_S S \omega_S]^T$ for some $\alpha \in \mathbb{R}$. In summary, solving (24) using steady-state data leads to an infinite set of solutions so that $T_R$ is not identifiable using the linear regressor (23) with steady-state data.

V. EXPERIMENTAL RESULTS

To demonstrate the viability of the speed sensorless estimator (18) for $T_R$, experiments were performed. A three-phase, 0.5 hp, 1735 rpm ($\nu_p = 2$ pole-pair) induction motor was driven by an ALLEN-BRADLEY PWM inverter to obtain the data. Given a speed command to the inverter, it produces PWM voltages to drive the induction motor to the commanded speed. Here a step speed command was chosen to bring the motor from standstill up to the rated speed of 188 rad/s. The stator currents and voltages were sampled at 10 kHz. The real-time computing system RTLAB from OPAL-RT with a fully integrated hardware and software system was used to collect data [17]. Filtered differentiation (using digital filters) was used for the derivatives of the voltages and currents. Specifically, the signals were filtered with a third-order Butterworth filter whose cutoff frequency was 100 Hz. The voltages and currents were put through a $3 - 2$ transformation to obtain their two-phase equivalent values.

Using the data $\{u_{sa}, u_{sb}, i_{sa}, i_{sb}\}$ collected between 0.84 sec to 0.91 sec, which includes the time the motor accelerates, the quantities $du_{sa}/dt$, $du_{sa}/dt$, $di_{sa}/dt$, $di_{sb}/dt$, $d^2 i_{sa}/dt^2$, $d^2 i_{sb}/dt^2$, $d^3 i_{sa}/dt^3$, $d^3 i_{sb}/dt^3$ are calculated and used to evaluate the coefficients $C_i, i = 1, 2, \ldots, 12$ in equation (18). Solving (18), one obtains the 12 solutions

$$
T_{R1} = +0.1064
$$

$$
T_{R2} = -0.0186
$$

$$
T_{R3} = -0.0576 + j0.0593
$$

$$
T_{R4} = -0.0037 - j0.0166
$$

$$
T_{R5} = -0.0072 + j0.0103
$$

$$
T_{R6} = -0.0072 - j0.0103
$$

$$
T_{R7} = +0.0125 + j0.0077
$$

$$
T_{R8} = +0.0125 - j0.0077
$$

$$
T_{R9} = +0.0065 + j0.0018
$$

$$
T_{R10} = +0.0065 - j0.0018
$$

$T_R$ must be a real positive number, so $T_R = 0.1064$ is the only possible choice. This value compares favorably with the value of $T_R = 0.11$ obtained using the method of Wang et al. [18], which requires a speed sensor.

To illustrate the identified $T_R$, a simulation of the induction motor model was carried out using the measured voltages as input. Then the simulation’s output [stator currents computed according to (1) and (2)] are used to compare with the measured (stator currents) outputs. Figure 1 shows the sampled two-phase equivalent current $i_{sb}$ and its simulated response $i_{sb-sim}$. The phase $a$ current $i_{sa}$ is similar, but shifted by $\pi/(2\nu_p)$. The resulting phase $b$ current $i_{sb-sim}$ from the simulation corresponds well with the actual measured current $i_{sb}$. Note that in equation (1) $\gamma = \frac{R_s}{\alpha L_s} + \frac{3M}{T_R}$ also depends on $T_R$.

VI. CONCLUSIONS AND FUTURE WORK

This paper presented a differential-algebraic approach to the estimation of the rotor time constant of an induction motor without using a speed sensor. The experimental results demonstrated the practical viability of this method. Though the method is not applicable in steady state, neither is a standard linear least-squares approach. Future work includes studying an on-line implementation of the estimation algorithm and using such an online estimate in a speed sensorless field-oriented controller.

VII. APPENDIX: STEADY-STATE EXPRESSIONS

In the following, $\omega_S$ denotes the stator frequency and $S$ denotes the normalized slip defined by $S \triangleq (\omega_S - \nu_p \omega) / \omega_S$. With $u_{sa} + j u_{sb} = U_S e^{j\omega_S t}$ and $i_{sa} + j i_{sb} = I_S e^{j\omega_S t}$, it is
shown in [19] that under steady-state conditions, the complex phasors $\bar{U}_S$ and $\bar{L}_S$ are related by ($S_p \triangleq \frac{R_p}{\sigma \omega_S L_R} = \frac{1}{\sigma \omega_S T_R}$)

$$L_S = \frac{U_S}{\bar{R}_S + j \omega_S L_S \left(\left(1 + \frac{S_p}{S_R}\right) / \left(1 + \frac{S_p}{S_S}\right)\right)} = \frac{R_S + (1 - \sigma) S \omega_S^2 L_T R + j \omega_S L_S (1 + \sigma) S \omega_S^2 T_R^2}{1 + S \omega_S^2 T_R^2},$$

and straightforward calculations (see [6]) give

$$P \triangleq u_{S\delta}i_{S\delta} + u_{S\beta}i_{S\beta} = R_e \left(\frac{U_S}{L_S}\right) = \frac{|L_S|^2}{U_S} \left(\frac{R_S + (1 - \sigma) S \omega_S^2 L_T R}{1 + S \omega_S^2 T_R^2}\right),$$

$$Q \triangleq u_{S\beta}i_{S\delta} - u_{S\delta}i_{S\beta} = I_m \left(\frac{U_S}{L_S}\right) = \frac{|L_S|^2}{U_S} \left(\frac{\omega_S L_S (1 + \sigma) S \omega_S^2 T_R^2}{1 + S \omega_S^2 T_R^2}\right).$$

A. Steady-State Expression for $r_1$ and $r_0$

It is now shown that the steady-state value of $r_1$ in (12) is nonzero. Substituting the steady-state values of $q_2$, $q_1$, $q_0$, $a_2$, $a_1$, and $a_0$ shown in [6] (noting that $q_1 = 0$ and $q_2 = 0$ in steady state) into (12) gives

$$r_1 = -\frac{|L_S|^6}{\left(1 + S^2 \omega_S^2 T_R^2\right)} n_p^3 \left(\frac{1}{1 - \sigma}\right)^6 L_S^3 \times \omega_S^3 \left(1 + T_R^2 \omega_S^2 (1 - S)^2\right)^2 \frac{1}{\text{den}} \times \frac{1}{\sigma^4},$$

$$r_0 = \frac{|L_S|^6}{\left(1 + S^2 \omega_S^2 T_R^2\right)} n_p^3 \left(\frac{1}{1 - \sigma}\right)^6 L_S^3 \times \omega_S^3 \left(1 - S\right) \left(1 + \omega_S^2 T_R^2 \times (1 - S)^2\right)^2 \frac{1}{\text{den}} \times \frac{1}{\sigma^4},$$

where

$\text{den} \triangleq n_p T_R |L_S|^4 \left(\frac{(1 - \sigma) S^2 \omega_S^2 T_R^2 - S \omega_S^2 L_T R}{\sigma T_R} \right)^2 + \left(\frac{1 - \sigma}{\sigma} \right) \left(\frac{\omega_S}{1 + S^2 \omega_S^2 T_R^2}\right)^2.$

Recall from Section III [following (6)] that $\text{den} = 0$ if and only if $\left|\omega_S\right| = 0$. It is then seen that $r_1 \neq 0$ in steady state.

REFERENCES


