Compensation-Based Non-Active Power Definition

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***Abstract*—This paper presents a new definition of nonactive current from which the definitions of instantaneous active and nonactive power are also derived. The definitions are consistent with the traditional power definitions and valid for single-phase and polyphase systems, as well as periodic and nonperiodic waveforms. The definitions are applied to a shunt compensation system. The paper elaborates on the compensation of three different cases of nonperiodic current: single-phase disturbance, three-phase subharmonics, and three-phase stochastic current. Simulation results give credibility to the applicability of the definition for a diversity of load currents. According to different compensation cases and the goals to be achieved, different averaging time intervals for the compensator are chosen which will determine the compensator's energy storage requirement and the extent of residual distortion in the source current.**

***Index Terms*—Active filter, nonactive power, reactive power, shunt compensator.**

I. Introduction

THE WIDESPREAD use of nonlinear loads and power electronics converters, such as cycloconverters and line-commutated three-phase thyristor-based rectifiers, has increased the generation of nonsinusoidal and nonperiodic currents and voltages in power systems [1], [2]. An arc furnace is an example of a nonlinear load that may draw rapidly changing nonsinusoidal currents. Problems such as voltage flicker and harmonic penetration associated with arc furnace loads have been reported in several papers [3]–[5]. A transient disturbance may also be considered as one kind of nonperiodic current, e.g., a sudden large load current such as starting a motor or a fault.

Several papers have dealt with the definition, characterization, and compensation of nonactive current/power [6]–[11]. However, most of the previous efforts have focused on *periodic*, nonsinusoidal currents instead of *nonperiodic* currents. The diversity of the features of nonperiodic currents makes their compensation quite difficult, and theoretically, their compensation is very different from that of periodic distorted currents. However, in practice, these two cases may be quite similar to each other [11]. Generally speaking, after compensation, a sinusoidal source current with a constant rms magnitude is preferred for both cases.

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If a shunt active filter is used as the compensator, then it must inject the current components that are the difference between the desired source current and the demanded load current. The work here defines active and nonactive currents from a compensation point of view and, in particular, examines how to apply this definition when nonperiodic load currents are present in the power system [12], [13].

II. Definition of Non-Active Current

Instantaneous power is defined as the time rate of energy generation, transfer, or utilization. It is a physical quantity and satisfies the principle of conservation of energy. For a single-phase circuit, it is defined as the instantaneous product of voltage and current

$$p(t) = v(t)i(t). \tag{1}$$

For a polyphase circuit with M phases, each phase's instantaneous power is still expressed as (1), and instantaneous total power is the sum of the active powers of each the individual phases

$$p(t) = \sum_{i=1}^{M} p_i(t) = \sum_{i=1}^{M} v_i(t)i_i(t). \tag{2}$$

Non-active power can be thought of as the useless power that causes increased line current and losses as well as greater generation requirements for utilities. For a single-phase circuit, nonactive power is the power that circulates back and forth between the source and loads and yields zero average active power over one period (or a certain interval) of the wave $p(t)$; therefore, for the single-phase case the nonactive power is based on average or rms values. For a polyphase circuit, nonactive power is the power that circulates among phases as well as the power that circulates between source and load. The power that circulates among phases can be formulated on an instantaneous basis, whereas the power that circulates between source and load can only be formulated on average or rms values.

Some nonactive power theories are based on average values and restricted to the frequency domain, while others are formulated in the time domain on an instantaneous base. No matter what mathematical means are used, the goal of these theories is to improve the power factor and to minimize power losses and disturbances by identifying, measuring, and eliminating the useless (nonactive) power.

For a single or polyphase power system, a shunt compensator to minimize the nonactive power/current required of the source can be configured as shown in Fig. 1. The shunt compensator is assumed to consist only of passive components (inductor/capacitor) and/or switching devices and no external power source.

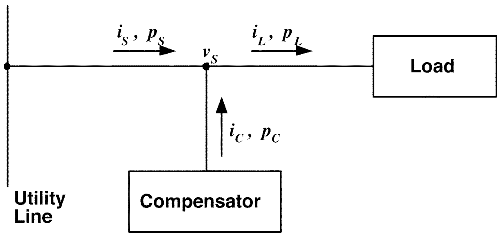


Fig. 1. Shunt compensator configuration.

As a consequence, (neglecting the compensator's power loss), conservation of energy requires the active power of the compensator to average zero. In more detail, let $p_S(t)$, $p_L(t)$, and $p_C(t)$ denote the instantaneous power of the source, load and compensator, respectively, and their average values over a time interval T_C be given by

$$P_X(t) = \frac{1}{T_C} \int_{t-T_C}^{t} p_x(\tau) d\tau \quad \text{where } X = S, L, \text{ or } C. \tag{3}$$

Then

$$P_S(t) + P_C(t) = P_L(t) \tag{4}$$

and

$$P_S(t) = P_L(t), \quad P_C(t) = 0 \text{ as } t \rightarrow \infty. \tag{5}$$

In (3), T_C is the averaging interval which may be zero, one-half of the voltage's fundamental cycle, one full fundamental cycle, or multiple cycles, depending on the compensation objectives and the passive components' energy storage capacity. T_C is the period chosen to average the active power of the load and does not cause any time delay on the system response. Equations (3), (4) and (5) must hold true regardless of whether the system is single-phase or polyphase and regardless of whether the compensation is by passive or active means. Based on these physical and practical limitations, nonactive power/current can be defined and formulated.

The definition given in this paper is an extension of Fryze's idea of nonactive current/power [9]. The instantaneous active current $i_p(t)$ and nonactive current $i_q(t)$ are

$$i_p(t) \equiv \frac{P_L(t)}{V_P^2(t)} v_P(t), \quad i_q(t) = i_L(t) - i_p(t) \tag{6}$$

where $P_L(t)$ is the average load active power over the interval $[t - T_C, t]$, and $V_P(t)$ is the rms value of the reference voltage $v_p(t)$ over the interval $[t - T_C, t]$

$$\begin{aligned} P_L(t) &= \frac{1}{T_C} \int_{t-T_C}^{t} p_L(\tau) d\tau \\ &= \frac{1}{T_C} \int_{t-T_C}^{t} v(\tau) i(\tau) d\tau \\ &= \frac{1}{T_C} \int_{t-T_C}^{t} v(\tau) i_p(\tau) d\tau \end{aligned} \tag{7}$$

TABLE I
PARAMETERS FOR DIFFERENT COMPENSATION OBJECTIVES

Compensation Objective	v_P	T_C	Resulting Source Current
Single-phase or polyphase reactive current	v	$T/2$ or T	Unity pf and sinusoidal for sinusoidal v_s
Single-phase or polyphase reactive current and harmonic current	v_f	$T/2$ or T	Sinusoidal regardless of v_s distortion
Instantaneous reactive power for polyphase system	v	$T_C \rightarrow 0$	Instantaneously unity pf for polyphase system
Non-periodic disturbance current	v_f	$n(T/2)$	Reduced amplitude and near sine wave with unity pf
Subharmonic current	v_f	nT	Pure sine wave or smoothed sine wave with unity pf
Stochastic non-periodic current	v_f	nT	Smoothed sine wave with near-unity pf

$$V_P(t) = \sqrt{\frac{1}{T_C} \int_{t-T_C}^{t} v_P^2(\tau) d\tau}. \tag{8}$$

The technique is to simply have the compensator provide $i_q(t)$ so the source need only provide $i_p(t)$. The average nonactive power is defined as

$$Q(t) = \frac{1}{T_C} \int_{t-T_C}^{t} v(\tau) i_q(\tau) d\tau \quad \text{where } Q(t) = P_C(t) = 0, \text{ as } t \rightarrow \infty. \tag{9}$$

$v_P(t)$ is the reference voltage whose specification depends on the compensation objectives. For example, this specification can be the terminal voltage $v_s(t)$ itself or it may be the fundamental component of $v_s(t)$ (i.e., $v_P(t) = v_f(t)$ where $v_s(t) = v_f(t) + v_h(t)$, $v_f(t)$ is the fundamental, and $v_h(t)$ is the harmonic component. The definitions (7) and (8) are valid for single- and polyphase circuits. However, in the case of polyphase circuits, the voltages and currents are expressed in vector form, which for a three-phase system is

$$\begin{aligned} v &= [v_a,\ v_b,\ v_c]^T, \\ i &= [i_a,\ i_b,\ i_c]^T, \quad \text{and} \\ v^2 &= [v_a,\ v_b,\ v_c] \cdot [v_a,\ v_b,\ v_c]^T \\ &= v_a^2 + v_b^2 + v_c^2. \end{aligned}$$

It was shown in [12] that this new definition has the following features:

1) flexibility to meet many different compensation objectives;
2) valid for nonsinusoidal and nonperiodic systems;
3) valid for single-phase and polyphase systems.

Table I illustrates that by choosing a different voltage reference and time averaging intervals, different source currents will result. Because of its flexibility in regards to compensation objectives, this definition is quite suitable for the mitigation of nonperiodic currents. This will be shown in the next section.

III. COMPENSATION OF PERIODIC CURRENTS

For compensation of periodic currents with fundamental period T, using a compensation period T_C that is a multiple of $T/2$ is enough for complete compensation of the nonactive current such that the source current has a unity power factor. This is seen by examining (8) and noting that the average rms value of a periodic quantity does *not* depend on the time averaging interval T_C if it is an integer multiple of $T/2$. Even if the system consists of harmonics and/or unbalanced components in voltages and/or

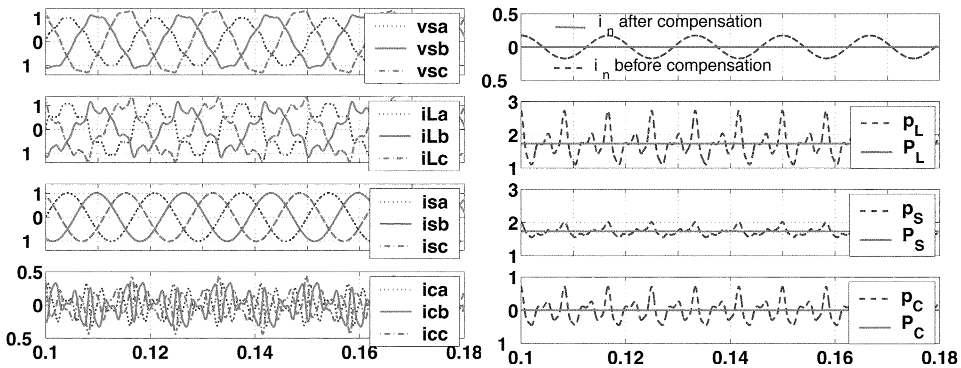
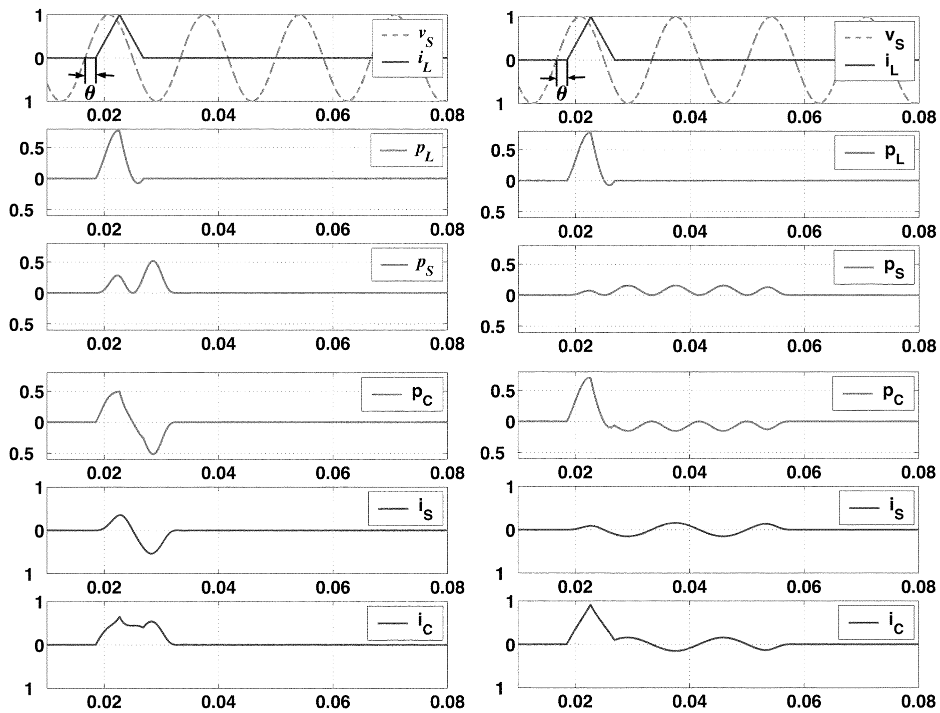


Fig. 2. Three-phase periodic current compensation ($T_C = T/2$).



(a) (b)

Fig. 3. Simulation results for disturbance type nonperiodic current compensation (a) $T_C = T/2$, (b) $T_C = 2T$.

currents, the system can be completely compensated. Using the fundamental positive-sequence components of the phase voltages as reference voltages, the source currents will follow the reference voltages by choosing $T_C = nT/2$, where n is an integer.

For the example shown in Fig. 2, both the voltage v_S and load current i_L are unbalanced and have harmonics. After compensation, however, the source current i_S is balanced, sinusoidal, and in phase with the reference voltage. It also shows that the average powers $P_S(t) = P_L(t)$ and $P_C(t) = 0$. With the compensation, the amplitude and variation of the instantaneous source power p_S is much smaller than that of the load power p_L.

IV. Compensation of Non-Periodic Currents

For the purposes of discussion, in this paper nonperiodic currents are those currents without a period or those currents whose fundamental period differs from that of the voltage waveform. These currents include load subharmonic current (frequency is lower than the voltage fundamental frequency f_f), current whose fundamental frequency is not a multiple of f_f, or current that contains some nonperiodic components. Unlike in periodic compensation where T_C does not have much influence on the compensation as long as it is a multiple of 1/2 period of the voltage fundamental frequency period, here the choice of T_C is a critical factor as will be shown in the following sections.

Theoretically, T_C can be chosen arbitrarily in the case of nonperiodic currents. However, it is desirable for the interval to be an integer multiple of the line frequency period because of the desire that the source current be sinusoidal and have the same frequency as the source voltage. In general, the period of the line voltage is not the same as the period of a quasiperiodic load current, or there is no period in the case of nonperiodic load current. Thus, choosing different T_C will result in quite different source currents and compensator currents. Simulation results of

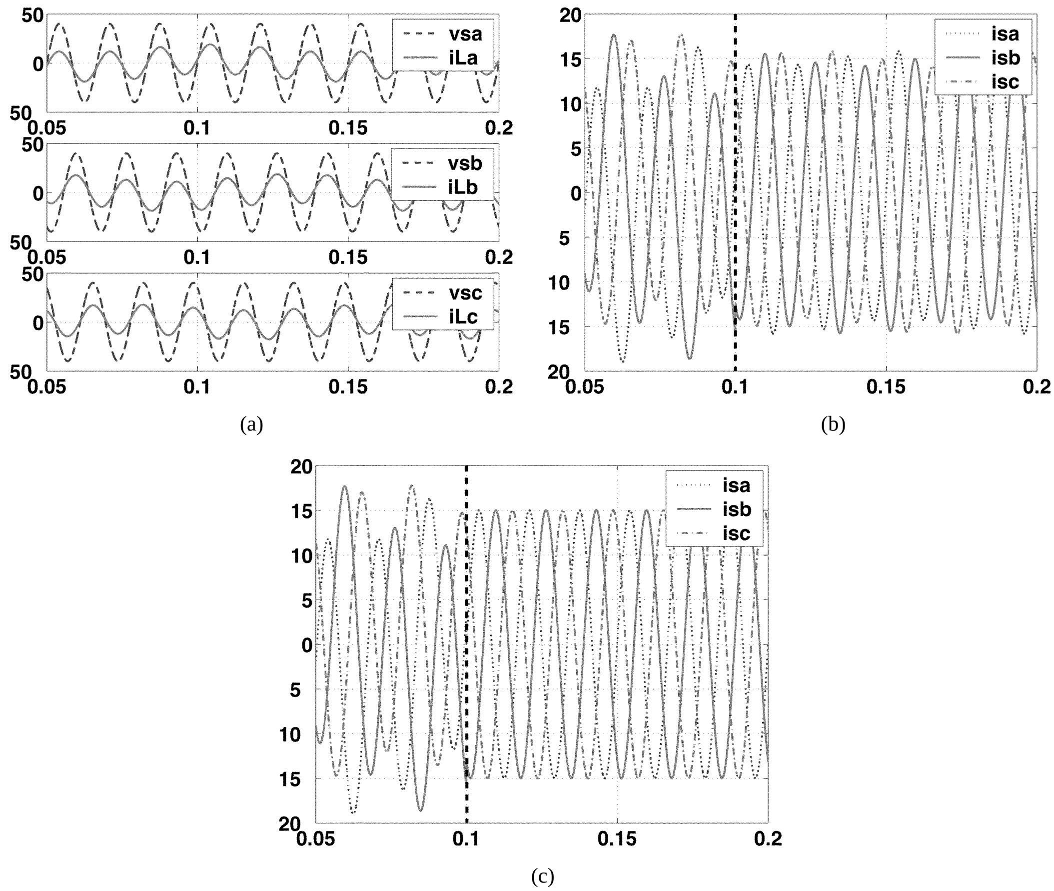


Fig. 4. Subharmonic current compensation. (a) 3-phase load current and voltage waveforms; (b) source currents, compensation at $\mathrm{t} = 100\ \mathrm{ms}\ (T_C = T)$; (c) source currents, compensation at $\mathrm{t} = 100\ \mathrm{ms}\ (T_C = 2.5\ T)$.

the compensation of three different kinds of nonperiodic currents are given in the following subsections.

A. Non-Periodic Disturbance Currents

The duration of a nonperiodic current may be a fraction of the line frequency cycle, or it may be several cycles. Outside of this time period, the current may be zero or a pure sine wave. Fig. 3 shows the simulation result of a single-phase disturbance current (pulse of duration that is 1/2 of the voltage period) for two different compensation cases: $T_C = T/2$ [Fig. 3(a)) and $T_C = 2T$ (Fig. 3(b)] where $v_P = v_f$ for both cases. θ is the phase angle between the source voltage and initiation of the load current pulse, and in this case, $\theta = 30°$ as shown in Fig. 3.

Because the disturbance energy is a fixed value, choosing a different time averaging interval T_C results in different average active power P_L; thus, a different magnitude of source current and compensator current will result. Larger values of T_C result in smaller peak values of $|i_s|$, i.e., a smaller disturbance seen from the source side. However, the compensator current rating will increase accordingly. Compensator design engineers must weigh the tradeoff between minimizing the source current against the cost of additional energy storage devices (capacitance) to accomplish this.

The simulations show that by increasing T_C from $T/2$ to $2T$, the source current decreases significantly (from 60–20% of the magnitude of the load current) without a significant increase in the compensator current (from 60–90% of the magnitude of the load current). Thus, one can substantially decrease the source current with what may be a cost-effective increase in the compensator energy storage requirements. While these simulations are done for a single-phase case, the same result would be expected in three-phase cases.

B. Subharmonic or Quasi-Periodic Currents

The main feature of this group of nonperiodic currents is that the currents may have a repetitive period. The currents generated by power electronics converters may fall into this group. In the simulation shown in Fig. 4, the quasiperiodic current is composed by adding one subharmonic component (12 Hz) to the fundamental current (60 Hz), and the total harmonic distortion (THD) of the load current is 26.7%. When the fundamental frequency f_f of the source voltage is an odd multiple of the subharmonic frequency f_{sub}, the minimum T_C for complete compensation is 1/2 of the common period of both f_f and f_{sub}, for example, if $f_f = 50$ Hz, and $f_{sub} = 20$ Hz, then the common period is 0.1 s (10 Hz). When f_f is an even multiple of f_{sub}, the minimum T_C for complete compensation is the common period of both f_f and f_{sub}.

Fig. 4 shows the source current with compensation initiating at $t = 100$ ms. The subharmonic frequency is 12 Hz and the fundamental is 60 Hz, which is 5 times that of the subharmonic frequency. Thus, the subharmonic component can be completely compensated by choosing $T_C = 2.5T$, and the source current is a pure sine wave. Choosing T_C smaller than 1/2 of the common

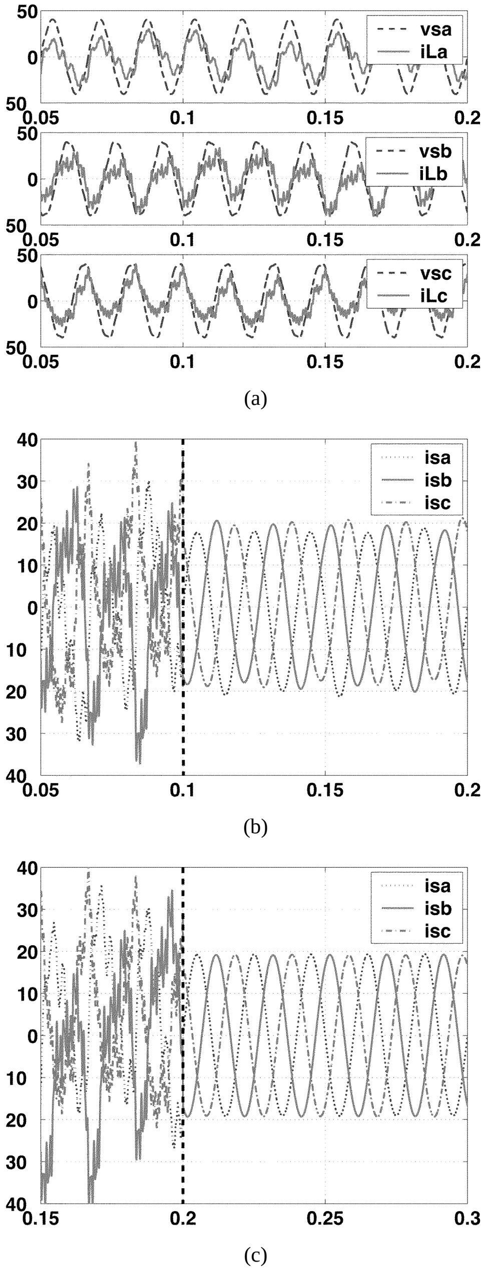


Fig. 5. Stochastic current compensation: (a) 3-phase load current and voltage waveforms; (b) source currents, compensation at $\mathrm{t} = 100 \text{ ms}$ ($T_C = T$); (c) source currents, compensation at $\mathrm{t} = 200 \text{ ms}$ ($T_C = 10T$).

period will result in a source current that still contains some subharmonic components. When $T_C = T$, the THD of the source current is 2.8% [Fig. 4(b)] because not all of the subharmonic component is eliminated; however, the source current THD has been reduced by 89.5% compared to the THD of the load current.

C. *Stochastic Non-Periodic Currents*

The load currents of arc furnaces are typically quite irregular as seen by the simulated arc furnace current shown in Fig. 5(a). It is impossible to choose a specific T_C to make the source current be a sine wave. In the case of nonperiodic waveforms, one can mathematically consider the period to be infinite. As can be seen from (3) and (4), as T_C goes to infinity, both P_L and V_p become constant and i_p tracks the voltage reference v_p. If v_p is chosen as the fundamental component of the source voltage v_f, then the source current will be sinusoidal. Of course, choosing the time interval to be infinite is not feasible in a practical application. However, it may still be possible to find some repetitive period in the current waveform that has most of the rms content of the waveform. Choosing that period as T_C may result in an acceptable source current which is quite close to a sine wave.

In Fig. 5(a), the THD of i_{La}, i_{Lb}, and i_{Lc} are 75.7%, 47.5%, and 52.7%, respectively. When $T_C = T$ (Fig. 5(b)), the THD of the source current is 6.9%. All of the harmonics with frequencies that are an integral multiple of the fundamental frequency are completely eliminated, and harmonics with frequencies that are not an integral multiple of the fundamental frequency are also mitigated. When T_C increases to $10T$, the source current is nearly a sine wave, with a THD of only 0.6%. In this way, the voltage flicker and harmonic penetration problem [4], [5] associated with this nonperiodic current waveform can be mitigated.

V. Conclusions

This paper presents a new definition of nonactive power/current, which is consistent with the conventional definition of power and applicable to single phase or polyphase, periodic or nonperiodic, and balanced or unbalanced electrical systems. Furthermore, by combining the definition and a conventional shunt active power filter, the application presented in this paper accomplishes the compensation of a variety of nonperiodic currents in power systems. Simulation results give credibility to the applicability of the definition for a diversity of load currents. According to different compensation cases and the goals to be achieved, different averaging time intervals for the compensator are chosen, which determines the compensator's ratings and the extent of residual distortion in the source current.

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