AC vs. DC Distribution: Maximum Transfer Capability

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Abstract—Many studies comparing AC and DC systems have focused on efficiency, stability, and controllability, but have not compared the maximum transfer capability. In this paper, the maximum transfer capability of an AC system and two DC systems, one with two lines and another with three, is determined through the continuation power flow method and compared. The results reveal that significant gains can be achieved by moving to a DC system with three lines.

***Index Terms*-DC power systems, power system modeling, power distribution, maximum power transfer, continuation power flow method**

I. Introduction

The increase in demand for DC (direct current) applications such as distribution generation is the driving force for the research on alternative distribution systems other than the AC (alternating current) systems. Also, the increasing demand of electrical energy by consumers has pushed engineers to find solutions for transferring exceedingly larger amounts of power from generation to load with high efficiencies.

Furthermore, the power industry is also under pressure to reduce the impact on the environment by adopting alternative energies. These alternative energies are prevailingly DC. However, the power infrastructure is presently based on an AC system. To convert the DC to AC for distribution, inverters are implemented resulting in losses in the power system. If the system were completely based on DC, these losses could possibly be reduced resulting in a more efficient power system.

Two studies have investigated the merit of DC distribution in terms of efficiency. In [1], the author compares several AC and DC systems for data centers. Based on the results, the author concludes that a high voltage (HV) DC distribution system is more efficient that AC, but no components need this high voltage. The author constructed another DC system, coined a hybrid DC system, which contained multiple DC voltages. However, the author observed that the converter losses reduced the efficiency to that below what is found in an equivalent AC system. In [2], a small-scale DC powered residential systems was also investigated. The authors note that although conduction losses in a DC system appear to be lower, the efficiency of power converters during partial loading is a concern and can ultimately lead to higher losses in DC.

Cost reduction, weight, and electrical isolation were the key elements for choosing DC in [3]. A DC zonal distribution system for a Navy ship is investigated where each zone of the distribution system has a separate distribution system providing protection to the overall ship power systems when an attack has occurred. However, due to the many converters possible stability issues were introduced. Nevertheless, the authors in [4] describes a method to eliminate this problem. Here the authors discuss buffering and a control algorithm for the converters as ways to elevate the negative incremental impedance associated with converter instability.

Control of the bus voltage is discussed in [5]. The authors choose to investigate a droop control algorithm over master slave since no communication is necessary between converters for droop control. The authors implement the existing converters in the DC system to perform load sharing with the droop control algorithm.

Although much insight has been achieved in stability, controllability, and efficiency of DC distribution systems, little has been conducted in terms of maximum power transfer capability. The only studies that have focused on maximum transfer capability with any reference to DC are those referring to HVDC [6].

The maximum power transfer capability of a bus is generally provided by the P-V (power versus voltage) curve of the bus. The nose of the curve (otherwise known as the point of collapse) provides an indication of the maximum load that can be placed on that bus before the system collapses.

In an AC transmission and sub-transmission system the line reactance plays a large role in the amount of power that can be transmitted. In fact, the resistance tends to be ignored due to the size of the reactance in comparison. Although line resistance in distribution is more considerable than in higher voltage networks, it is still less significant than reactance in general. However, in a DC distribution system the reactive power requirements and line reactance are non-existent. This insinuates that a DC system could have a much larger power transfer capability compared to that of AC.

In this paper an examination of an AC system maximum power transfer on a bus is compared to that of two equivalent DC systems. One DC system is composed of two lines, one positive and one negative, while a second DC system contains a positive, negative, and neutral. This second system is ideal in replacing an AC system since AC systems also have three lines.

II. MODELS

The model under examination consists of a four bus system with elements taken from [8] and is shown in Fig 1. Although many distribution systems are radial, the test system here is chosen as networked system because 1) more urban distribution systems are moving toward networked or weakly meshed; 2) the network system is more generic and may represent a combination of distribution system and sub-transmission, which may be converted to DC system eventually. So, the networked four-bus system is a generic model that can be used as a guideline for future study. It should be noted that the research work assumes that the line limits are ignored, because the purpose of this paper is to study the impact to maximum transfer capability constrained by voltage stability.

The test system has a generator connected to bus 1 and loads connected to all the buses. The system parameters are provided in Tables I and II.

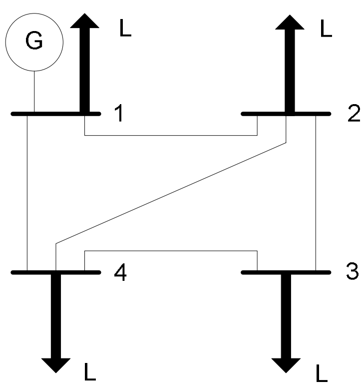

Fig. 1. Power system under study.

TABLE I. POWER SYSTEM GENERATION AND LOAD DATA

Bus No.	|V| (kV)	Pgen (kW)	Qgen (kVAR)	Pload (kW)	Qload (kVAR)
1	14.5	--	--	25	10
2	13.8	0	0	15	5
3	13.8	0	0	27.5	11
4	13.8	0	0	25	15

TABLE II. POWER SYSTEM IMPEDANCE DATA

Line	To Bus	From Bus	R (Ω)	X (Ω)	B (Ω)
1	1	2	38.09	352.31	17.14
2	1	4	59.04	493.23	19.04
3	2	3	11.43	47.61	0
4	2	4	135.21	609.4	28.57
5	3	4	142.08	127.6	0

To determine the maximum power transfer capability of each system type, P-V curves for each bus, excluding the bus with the generator, are to be constructed. The following subsections A and B review the standard methodology for power flow and continuation power flow to construct P-V curves for AC systems; then, the subsections C to E present the corresponding models to construct P-V curves for DC systems.

A. AC Load Flow

To create the P-V curve, a steady-state load flow analysis must be conducted for each point on the curve. However, the load flow system equations are non-linear, therefore demanding an iterative approach.

The basic method employs Kirchoff's laws that require that the sum of the power leaving or entering a bus must be zero. The equations for the net power leaving a bus are given as

$$\Delta P_i = P_{Gi} - P_{Li} - V_i \sum_{j=1}^{Nbus} V_j Y_{ij} \cos(\delta_i - \delta_j - \phi_{ij}) = 0 \tag{1}$$

$$\Delta Q_i = Q_{Gi} - Q_{Li} - V_i \sum_{j=1}^{Nbus} V_j Y_{ij} \sin(\delta_i - \delta_j - \phi_{ij}) = 0 \tag{2}$$

where P represents the real power, Q the reactive power, V the voltage, $Y\angle\phi$ the admittance matrix values, δ the voltage angles [7,8].

These equations are non-linear and can be linearized in terms of $\Delta\delta$ and ΔV by taking the derivative with respect to each variable

$$\frac{\Delta P}{\Delta\delta}\Delta\delta + \frac{\Delta P}{\Delta V}\Delta V = \Delta P \tag{3}$$

$$\frac{\Delta Q}{\Delta\delta}\Delta\delta + \frac{\Delta Q}{\Delta V}\Delta V = \Delta Q \tag{4}$$

Placing these equation into matrix form, the following set of linear equations can be obtained

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \delta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \delta} & \frac{\partial \Delta Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = J\,x \tag{5}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = b \tag{6}$$

$$Jx = b \tag{7}$$

where J is known as the Jacobian matrix and x is the solution [7,8]. Essentially, an initial guess for the unknown variables is made, generally 1 p.u. for the voltages and 0 for the phase angle. Using the initial guess, the values in the Jacobian and b matrices are determined and the solution to $Jx = b$ is found. The solution x is added to the previous state (guess) and the process is repeated with each new solution substituted for the original guess until the solution converges [7, 8].

$$\begin{bmatrix} \delta^{k+1} \\ V^{k+1} \end{bmatrix} = \begin{bmatrix} \Delta\delta^{k} \\ \Delta V^{k} \end{bmatrix} + \begin{bmatrix} \delta^{k} \\ V^{k} \end{bmatrix} \quad (8)$$

B. AC Continuation Power Flow

In the development of the PV curve, the continuation power flow method is employed to gain computational efficiency. Instead of incrementing the load power and performing load flow until convergence, this method predicts a solution and then corrects the solution as shown in Fig 2.

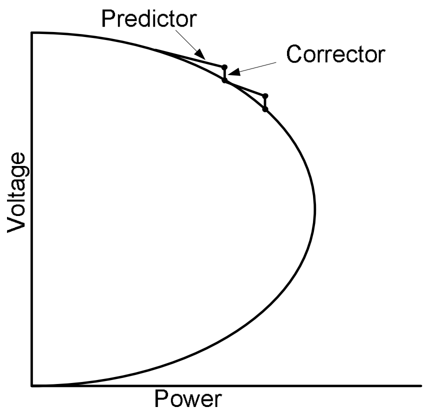

Fig. 2. Predictor and corrector on PV curve.

Through this method, an initial guess (predictor) is made so that the load flow (corrector) does not have to start with values that are far from the solution, thereby allowing an approximate solution with only one iteration of load flow [8, 9].

This approach employs an additional unknown variable λ, which represent the loading parameter.

$$F(\delta, V, \lambda) = 0 \quad (9)$$

Taking the derivative of F to linearize the system of equations results in

$$\frac{\partial F}{\partial \delta} d\delta + \frac{\partial F}{\partial V} dV + \frac{\partial F}{\partial \lambda} d\lambda = 0 \quad (10)$$

Or in matrix form

$$\begin{bmatrix} & & \vdots & \\ & J_{LF} & \vdots & K \\ \cdots & \cdots & \cdots & \cdots \\ & & e & \end{bmatrix} \begin{bmatrix} \partial\delta \\ \partial V \\ \partial\lambda \end{bmatrix} = Jx = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (11)$$

where J_{LF} represents the Jacobian that is developed in section A, K represents a vector of zeros except for the position of the unknown bus power (λ), and e is a vector of all zeros except for the position location of the unknown variable λ [8, 9].

Thus, the predicted value for the unknown values is computed through the following equation

$$\begin{bmatrix} \delta \\ V \\ \lambda \end{bmatrix}^{predictor} = \begin{bmatrix} \delta_k \\ V_k \\ \lambda_k \end{bmatrix} + \sigma \begin{bmatrix} \partial\delta \\ \partial V \\ \partial\lambda \end{bmatrix} \quad (12)$$

where σ represents the step size or incremental size of the loading parameter [8, 9]. Once the predictor values have been uncovered, these values are implemented in the basic load flow algorithm, discussed in section A, for one iteration. This provides the corrected values. This process is repeated until the nose of the curve is reached.

C. DC Load Flow

For DC systems, the basic load flow equations are similar to those of the AC load flow except that there is no voltage angles (δ), no reactive power (Q), and the lines are composed of purely real elements. Hence, only one equation exists for the net power leaving a bus and is given by

$$\Delta P_i = P_{Gi} - P_{Li} - V_i \sum_{j=1}^{Nbus} \frac{V_j}{R_{ij}} = 0 \quad (13)$$

As with the AC model equations, the DC equation is linearized in terms of ΔV by taking the derivative. In matrix form this is

$$\left[\frac{\partial \Delta P}{\partial V}\right] \Delta V = Jx \quad (14)$$

$$\Delta P = b \quad (15)$$

$$Jx = b \quad (16)$$

The updated solution for each iteration is then

$$V^{k+1} = \Delta V^{k} + V^{k} \quad (17)$$

D. DC Continuation Power Flow

As with the DC system load flow, the continuation power flow method in DC is analogous to that of the AC system except there is no reactive power and phase angle. Hence, the equations are only based on the voltage and the loading parameter

$$F(V, \lambda) = 0 \quad (18)$$

Linearizing the system of equations through performing the partial derivative of each variable results in

$$\frac{\partial F}{\partial V} dV + \frac{\partial F}{\partial \lambda} d\lambda = 0 \quad (19)$$

Composed in matrix form, this appears as

$$\begin{bmatrix} & \vdots & & \\ J_{LF} & \vdots & K & \\ \cdots & \cdots & \cdots & \cdots \\ & e & & \end{bmatrix} \begin{bmatrix}\partial V \\ \partial \lambda\end{bmatrix} = Jx = \begin{bmatrix}0 \\ \vdots \\ 0 \\ 1\end{bmatrix} \tag{20}$$

where J_{LF} represents the Jacobian that is developed in section *C*, *K* represents a vector of zeros except for the position of the unknown bus power, and *e* is a vector of all zeros except for the position location of the unknown variable λ.

Thus the predictor is given as

$$\begin{bmatrix}V \\ \lambda\end{bmatrix}^{predictor} = \begin{bmatrix}V_k \\ \lambda_k\end{bmatrix} + \sigma \begin{bmatrix}\partial V \\ \partial \lambda\end{bmatrix} \tag{21}$$

The corrector is determined through one iteration of the basic load flow described in section C. This process is repeated until the nose of the curve is reached.

In summary, Table III and Table IV show the equations necessary for load flow and the continuation power flow method for both AC and DC.

TABLE III. POWER FLOW EQUATIONS

No.	AC	DC
Load Flow	$J = \begin{bmatrix}\frac{\partial \Delta P}{\partial \delta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \delta} & \frac{\partial \Delta Q}{\partial V}\end{bmatrix}$ $x = \begin{bmatrix}\Delta\delta \\ \Delta V\end{bmatrix}$ $b = \begin{bmatrix}\Delta P \\ \Delta Q\end{bmatrix}$ $Jx = b$	$J = \begin{bmatrix}\frac{\partial \Delta P}{\partial V}\end{bmatrix}$ $x = \Delta V$ $b = \Delta P$ $Jx = b$
Correction	$\begin{bmatrix}\delta^{k+1} \\ V^{k+1}\end{bmatrix} = \begin{bmatrix}\Delta\delta^k \\ \Delta V^k\end{bmatrix} + \begin{bmatrix}\delta^k \\ V^k\end{bmatrix}$	$V^{k+1} = \Delta V^k + V^k$

It is very important to note that the Jacobian matrix in the DC system model is not a sub-matrix of and significantly different from AC Jacobian matrix, although they appears to be similar by definition (partial derivative of ΔP with respect to *V*). The reason is that the flows in DC systems are determined by line resistances only, while the flows in AC systems are mainly determined by line reactances. This will eventually lead to the significant difference in P-V curves (or the maximum transfer capability) of the two systems.

TABLE IV. CONTINUATION POWER FLOW EQUATIONS

No.	AC
Continuation Power Flow Method (AC)	$\begin{bmatrix} & \vdots & & \\ J_{LF} & \vdots & K & \\ \cdots & \cdots & \cdots & \cdots \\ & e & & \end{bmatrix} \begin{bmatrix}\partial \delta \\ \partial V \\ \partial \lambda\end{bmatrix} = \begin{bmatrix}0 \\ \vdots \\ 0 \\ 1\end{bmatrix}$
Correction (AC)	$\begin{bmatrix}\delta \\ V \\ \lambda\end{bmatrix}^{predictor} = \begin{bmatrix}\delta_k \\ V_k \\ \lambda_k\end{bmatrix} + \sigma \begin{bmatrix}\partial \delta \\ \partial V \\ \partial \lambda\end{bmatrix}$
Continuation Power Flow Method (DC)	$\begin{bmatrix} & \vdots & & \\ J_{LF} & \vdots & K & \\ \cdots & \cdots & \cdots & \cdots \\ & e & & \end{bmatrix} \begin{bmatrix}\partial V \\ \partial \lambda\end{bmatrix} = \begin{bmatrix}0 \\ \vdots \\ 0 \\ 1\end{bmatrix}$
Correction (DC)	$\begin{bmatrix}V \\ \lambda\end{bmatrix}^{predictor} = \begin{bmatrix}V_k \\ \lambda_k\end{bmatrix} + \sigma \begin{bmatrix}\partial V \\ \partial \lambda\end{bmatrix}$

E. DC Circuit Models

When performing analysis on a three phase AC system, the typical approach is to convert to a single line diagram in which one impedance per phase is examined. However, since DC does not have phases, this cannot be done. Instead two resistances are needed in the model for a two-line DC model as shown in Fig. 3.

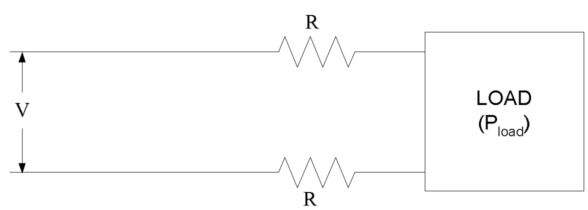

Fig. 3. DC two-line model.

For the three line DC model however, the line resistance can be reduced by half since the center line is a considered as a neutral and the current passing through this conductor should be zero as shown in Fig. 4.

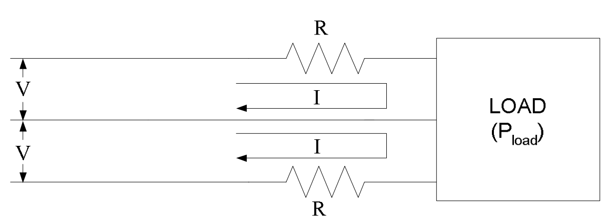

Fig. 4. DC three-line model.

Hence, the actual model appears more like what is shown in Fig. 5. Only one resistance is considered and the power consumed by each portion is reduced by half. This represents ½ of the circuit, and to obtain the full power transferred, the result must be doubled.

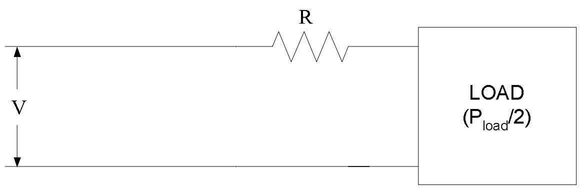

Fig. 5. DC three-line equivalent model.

Another perception of this model is that the voltage of the system depicted in Fig. 3 has been doubled as shown in Fig. 6 and that the resistance is still 2R. Either of these methodologies results in the same computational solution.

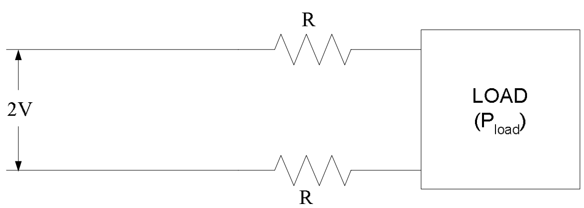

Fig. 6. DC three-line equivalent model.

III. Modeling Results

For the power system modeled, the P-V curves of bus 2, bus 3, and bus 4 for the AC system, DC system with two lines, and DC system with three lines were mapped by the methods described. The results are shown in Figs. 7-9.

The different buses have different maximum power limits which are based solely on the amount of impedance in the lines between the generation and loads. From observation, the line reactance is much larger than the resistance, thereby reducing the maximum power transfer capability of AC compared to DC.

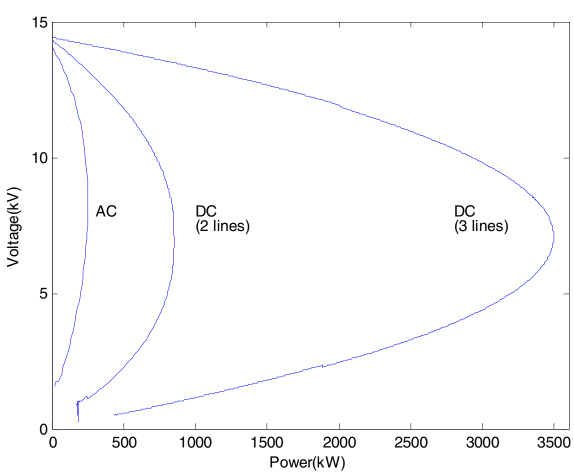

Fig. 7. Bus 2 PV curves

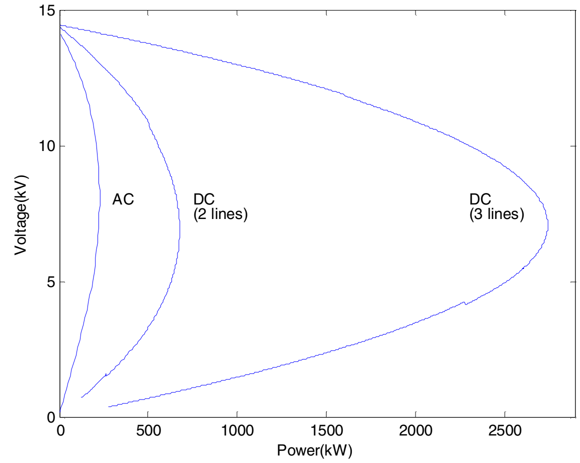

Fig. 8. Bus 3 PV curves

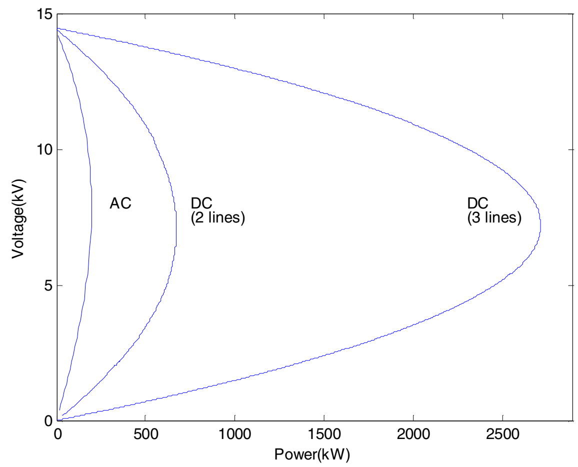

Fig. 9. Bus 4 PV curves.

As shown from the PV curves, a two-line DC system has a significant gain over an AC system. Nevertheless, the addition of another line essentially doubles the voltage of the DC system increasing the transmitting power by four times. This should be expected considering that

$$P_{2LINE} = \frac{V^2}{R} \longrightarrow P_{3LINE} = \frac{(2V)^2}{R} = \frac{4(V)^2}{R}$$

The maximum power that can be transferred in this example for each bus and each type of power system is provided in Table V. This represents the value of the nose of the curve.

TABLE V. Maximum Power Transfer for Different Power Systems

	Maximum Power Transfer		
Power System Type	AC	Two-Line DC	Three-Line DC
Bus No.	(kW)	(kW)	(kW)
Bus 2	252	857	3530
Bus 3	227	678	2750
Bus 4	196	674	2720

As clearly evident, a significant increase in power transfer is achievable in moving from AC to DC. To best represent the gains, Table VI shows the percent difference between the AC and DC systems.

TABLE VI. PERCENT POWER TRANSFER CAPABILITY INCREASE FOR DC SYSTEMS COMPARED TO AC SYSTEMS

	Increase of Power Transfer Capability	
Bus No.	Two-Line DC	Three- Line DC
Bus 2	240%	1301%
Bus 3	199%	1111%
Bus 4	244%	1288%

IV. CONCLUSION

Since alternative energies tend to be DC sources, investigations into the merit of DC must be performed to compare the advantages and disadvantages of DC to the conventional AC power system. Studies in the past have focused on efficiency, stability, and controllability. Nevertheless, analysis on the maximum power transfer capability of DC in comparison to AC has not been examined.

This study has shown that through the use of DC, the transmittable power can be increased by as much as 10 times. This increase could help alleviate many of the congestion problems currently faced in power systems along with allow for power flow optimization that are more economical since constraints on the amount of power flow may no longer be reached.

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BIOGRAPHIES

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He presently is a Ph.D. student in electrical engineering at The University of Tennessee. His research interests include alternative energies, power systems, and power electronics interface with utilities.

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Since 1991, he worked on several electrical distribution projects at the three U.S. Department of Energy plants in Oak Ridge, Tennessee. In 1999, he joined the Department of Electrical and Computer Engineering at the University of Tennessee, Knoxville, where he is presently an associate professor. He is an adjunct participant at the Oak Ridge National Laboratory. He does research in the areas of electric power conversion for distributed energy sources, reactive power compensation, multilevel converters, hybrid electric vehicles, and application of SiC power electronics.

Dr. Tolbert is a registered Professional Engineer in the state of Tennessee. He is the recipient of a National Science Foundation CAREER Award and the 2001 IEEE Industry Applications Society Outstanding Young Member Award.

Burak Ozpineci (S 1992 – M 2002 – SM 2005) received the B.S. degree in electrical engineering from the Middle East Technical University, Ankara, Turkey, in 1994, and the M.S. and Ph.D. degrees in electrical engineering from the University of Tennessee, Knoxville, in 1998 and 2002, respectively.

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Dr. Ozpineci was the Chair of the IEEE PELS Rectifiers and Inverters Technical Committee and Transactions Review Chairman of the IEEE Industry Applications Society Industrial Power Converter Committee. He was the recipient of the 2006 IEEE Industry Applications Society Outstanding Young Member Award, 2001 IEEE International Conference on Systems, Man, and Cybernetics Best Student Paper Award, and 2005 UT-Battelle (ORNL) Early Career Award for Engineering Accomplishment.