

# Definitions and Compensation of Non-Active Current in Power Systems

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**Abstract** -Many definitions have been formulated to characterize, detect, and measure active and non-active current and power for non-sinusoidal and non-periodic waveforms in electric systems. This paper presents definitions and compensation of non-active current from the compensation standpoint and provides guidance on how to determine compensation objectives and select detection parameters. The proposed definition is valid to both single and multi-phase power systems. Clear and easy guidance to determine objectives and design parameters of compensation systems is provided.

## I. INTRODUCTION

Because of the widespread use of nonlinear loads and electronic power converters, non-sinusoidal and non-periodic loads and voltage distortion are becoming more common in today's electrical systems. Many papers have dealt with the definition, identification, characterization, detection, measurement, and compensation of such non-sinusoidal and non-periodic current and power [1-11]. Tolbert and Habetler have compiled a comprehensive technical survey of the published literature on the topic [12].

Instantaneous active power is defined as the time rate of energy generation, transfer, or utilization. It is a physical quantity and satisfies the principle of conservation of energy. For a single-phase circuit, it is defined as the instantaneous product of voltage and current:

$$p(t) = v(t)i(t). \quad (1)$$

Active power  $P$  is the time average of the instantaneous power over one period of the wave  $p(t)$ . For a polyphase circuit with  $M$  phases, each phase's instantaneous active power is still expressed as (1) and instantaneous total active power is the sum of the active powers of the individual phases:

$$p(t) = \sum_{i=1}^M p_i(t) = \sum_{i=1}^M v_i(t)i_i(t). \quad (2)$$

Non-active power can be thought of as the useless power that causes increased line current and losses, greater generation requirements for utilities, and other effects/burdens to power systems and connected/related equipment.

For a single phase circuit with inductors, capacitors, and/or nonlinear elements, non-active power is the power that circulates back and forth between the source and loads

and yields zero average active power over one period of the wave  $p(t)$ . Therefore, the non-active power for single phase circuits is based on average or rms values. For a polyphase circuit, non-active power is the power that circulates back and forth between the sources and loads and the power that circulates among the phases. The source-load circulating power yields zero average over one period of the wave  $p(t)$  because of unbalanced storage elements in the circuit. The phase circulating power contributes no total instantaneous active power to the circuits because of balanced storage elements. Therefore, the non-active power for polyphase circuits includes two components: one is based on average or rms values and the other is based on instantaneous value. Some theories are based on average values and restricted to frequency domain, while some others are formulated on time domain and instantaneous base. No matter what mathematical means are used, the goal of these theories is to improve the power factor and to minimize power losses and disturbances by identifying, measuring, and eliminating the useless power. This paper presents definitions of non-active power or non-active current from the compensation standpoint. Almost all existing non-active power theories and definitions can be extended and deduced from the definitions presented.

## II. COMPENSATION SYSTEM AND DEFINITION OF NON-ACTIVE POWER

### A. Compensation Systems

For a single or polyphase power system, a shunt compensator to minimize the useless power/current can be configured as in Fig. 1. Assuming that the shunt compensator only consists of passive (inductor and/or capacitor) and/or switching devices without any external power source and neglecting the compensator's power loss, then the active power of the compensator should average zero according to the principle of conservation of energy. That is,

$$P_s(t) = P_L(t), P_C(t) = 0, \text{ for } t \rightarrow \infty. \quad (3)$$

$$P_X(t) = \frac{1}{T_C} \int_{-T_C}^T p_X(\tau) d\tau \text{ and } X = S, L, \text{ or } C. \quad (4)$$

In (4),  $T_C$  is the averaging interval which can be zero, one fundamental cycle, one-half cycle, or multiple cycles, depending on compensation objectives and the passive components' energy storage capacity. Suffixes "S", "L", and "C" represent the source, load, and compensator quantities

as shown in Fig. 1, respectively. Equations (3) and (4) must hold true regardless of single-phase or polyphase, passive compensation or active compensation. Based on these physical and practical limitations, non-active power and non-active current can be defined and formulated.

### B. Definitions of Non-Active Power and Current

We extend Fryze's idea of non-active current/power as follows:

$$i_p(t) = \frac{P(t)}{V_p^2(t)} v_p(t), \quad i_q(t) = i(t) - i_p(t), \quad (5)$$

where

$$V_p(t) = \sqrt{\frac{1}{T_C} \int_{-T_C}^t v_p^2(\tau) d\tau}. \quad (6)$$

$i_p(t)$  is the active current and  $i_q(t)$  is the non-active current.  $P(t)$  is the average active power over the interval  $[t-T_C, t]$ , which can be calculated from (4).  $V_p(t)$  is the rms value of the voltage,  $v_p(t)$  over the interval  $[t-T_C, t]$ , which is expressed by (6).  $v_p(t)$  is the reference voltage depending on compensation objectives, which can be

- 1) the terminal voltage itself, i.e.,  $v_p(t) = v(t)$ ; or
- 2) the fundamental component of  $v(t)$ , i.e.,  $v_p(t) = v_f(t)$  where  $v(t) = v_f(t) + v_h(t)$  and  $v_f(t)$  is the fundamental and  $v_h(t)$  is the harmonic component, respectively; or
- 3) something else.

These definitions, (5) and (6) are valid for single- and poly-phase circuits. However, for poly-phase circuits, voltages and currents are expressed as a vector, e.g., for a three-phase system,  $v = [v_a, v_b, v_c]^T$ ,  $i = [i_a, i_b, i_c]^T$ , and  $v^2 = [v_a, v_b, v_c] \cdot [v_a, v_b, v_c]^T = (v_a^2 + v_b^2 + v_c^2)$ .

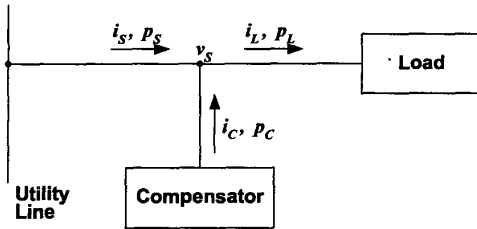


Fig. 1. A shunt compensator configuration.

### III. DISCUSSION, DEDUCTION, AND COMPENSATION

Equation (5) provides the basic definitions of active and non-active current, from which most of the existing non-active power theories and definitions based on time-domain can be extended and deduced. The following discusses some deductions and compensation examples.

#### A. Sinusoidal Single-Phase Circuits

For a single phase circuit with sinusoidal waveforms, e.g.,  $v_s = V_s \sin(\omega t)$  and  $i_L = I_L \sin(\omega t - \alpha)$ , the active and non-active currents are consistent with the traditional active and reactive currents and can be derived from (4), (5) and (6) by the following steps:

- (i) choose  $T_C$  to be one or half fundamental cycle, i.e.,  $T_C = 2\pi/\omega$  or  $T_C = \pi/\omega$ ;
- (ii) calculate the average load active power,  $P_L$ , according to (4);
- (iii) calculate the rms value of the voltage,  $v_s$ , according to (6); and
- (iv) calculate the load active current and non-active current,  $i_{Lp}$  and  $i_{Lq}$  according to (5).

The result is

$$i_{Lp}(t) = I_L \cos \alpha \sin(\omega t), \text{ and } i_{Lq}(t) = -I_L \sin \alpha \cos(\omega t). \quad (7)$$

Equation (7) clearly shows the consistence with the traditional reactive power theory. An algorithm for active compensators can be easily implemented because the definitions as formulated in (4), (5) and (6) are all real-time based. In addition, it is easy to show that the compensator needs zero average power when it injects the non-active current because  $P_C = 0$  when  $i_C = i_{Lq}$ . After compensation, the source current,  $i_s$ , will only contain the load active current,  $i_{Lp}$ .

#### B. Non-Sinusoidal Single-Phase Circuits

For a single-phase circuit with non-sinusoidal waveforms, the active and non-active currents, which can be derived from (4), (5) and (6) using the steps described in section III.A, are consistent with the traditional Fryze's active and non-reactive currents when choosing  $v_p = v_s$  in (5) and (6). For example, if  $v_s = v_f(t) + v_h(t) = V_{sf} \sin(\omega t) + V_{sh} \sin(\omega_h t + \beta_h)$  and  $i_L = I_{Lf} \sin(\omega t - \alpha) + I_{Lh} \sin(\omega_h t + \beta_h - \alpha_h)$  then

$$i_{Lp}(t) = \frac{V_{sf} I_{Lf} \cos \alpha + V_{sh} I_{Lh} \cos \alpha_h}{V_{sf}^2 + V_{sh}^2} \cdot [V_{sf} \sin(\omega t) + V_{sh} \sin(\omega_h t + \beta_h)], \quad (8)$$

$$i_{Lq}(t) = -I_{Lf} \sin \alpha \cos(\omega t) - I_{Lh} \sin \alpha_h \cos(\omega_h t + \beta_h) + \frac{V_{sh}^2 I_{Lf} \cos \alpha - V_{sf} V_{sh} I_{Lh} \cos \alpha_h}{V_{sf}^2 + V_{sh}^2} \cdot \sin(\omega t) + \frac{V_{sf}^2 I_{Lh} \cos \alpha_h - V_{sf} V_{sh} I_{Lf} \cos \alpha}{V_{sf}^2 + V_{sh}^2} \cdot \sin(\omega_h t + \beta_h). \quad (9)$$

Again, the average power of  $i_{Lq}$  is zero, which satisfies the requirements in (3) for a compensator. However, it is observed from (8) that the active current contains harmonics because of the voltage distortion, which means that the source current will not become sinusoidal after the compensator injects (compensates) the non-active current expressed in (9). In most cases, it is desirable that compensation of non-active current results in a pure sine wave source current. In order to achieve that, one should choose  $v_p = v_{sf}$  in (5) and (6). By doing so, one has

$$i_{Lp}(t) = \left( I_{Lf} \cos \alpha + \frac{V_{sh} I_{Lh} \cos \alpha_h}{V_{sf}} \right) \cdot \sin(\omega t), \quad (10)$$

$$i_{Lq}(t) = -I_{Lf} \sin \alpha \cos(\omega t) - \frac{V_{sh} I_{Lh}}{V_{sf}} \cos \alpha_h \cdot \sin(\omega t) + I_{Lh} \sin(\omega_h t + \beta_h - \alpha_h). \quad (11)$$

Equation (10) shows that the active current is a sine wave, and (11) shows that the non-active current contains all harmonic current and fundamental reactive current. After compensation, the source current will become sinusoidal and active. In addition, it is noticeable that the active and non-active currents expressed in (10) and (11) still meet the compensation energy conservation requirements expressed in (3). This is a very important property of the definitions given in (4), (5), and (6), which is also necessary in order to implement compensation in Fig. 1 because the average active power from the compensator has to be zero.

### C. Single-Phase Circuits with Non-Sinusoidal and Non-Periodic Current

It is more convenient using simulations to study a load current with a non-sinusoidal and non-periodic waveform. Fig. 2 shows a case study, where the load generates a non-periodic pulse current. The calculation is based on (4), (5), and (6) with  $T_C = T/2$  (one-half fundamental cycle). The simulation results clearly demonstrate the following points: (1) the active load current,  $I_{Lp}$ , is sinusoidal and in phase with the voltage although the load current,  $I_L$ , is highly distorted and non-periodic; (2) the calculated load non-active current,  $I_{Lq}$ , is highly distorted and out of phase with the source voltage; (3) the average load power,  $PL_q$ , generated from the load non-active current approaches zero as times goes. Therefore, when a compensator is used to compensate for the load non-active current it will consume average zero power and maintain the requirement in (3). In addition, the source current will become sinusoidal and in phase with the voltage. In Fig. 2, the simulation results showed fastest response speed with  $T_C = T/2$ , and as a result, the power drawn from the source,  $p_s$ , is a pulse that is not friendly to the power system while the required energy storage and rating of the compensator is kept minimal (The required energy storage is determined by the average

active power,  $PL_q$  or  $PC$ , that results from the compensation current or the load non-active current and rating is determined by the current,  $I_{Lq}$  or  $IC$  in the figure). However, one can choose a longer averaging interval, e.g.,  $T_C = 2T$  as shown in Fig. 3, which resulted in a much slower response. As a result, the power drawn from the source,  $p_s$ , is a smoothed constant that is much more friendly to the power system (a good or model citizen of the grid) while the required energy storage ( $PL_q$  or  $PC$ ) and rating ( $I_{Lq}$  or  $IC$ ) of the compensator is much higher.

### D. Three-Phase Circuits

The definitions described in (5) and (6) are valid for a three-phase system as well regardless of whether the voltage and current waveforms are sinusoidal or non-sinusoidal, periodic or non-periodic, and balanced or unbalanced. Results are similar to those in the previous subsections. For polyphase ( $M$ -phase) systems, there is one interesting concept, instantaneous reactive (or instantaneous non-active) power and current, which do not exist in single phase situations. The instantaneous reactive power is the power that circulates among the phases. This instantaneous non-active power theory [2, 3, 10] can be deduced from (5) and (6) as well. In (4) and (6), choosing  $T_C \rightarrow 0$  yields the instantaneous non-active current (power) theory. That is,

$$i_p(t) = \frac{P(t)}{V_p^2(t)} v_p(t), \quad i_q(t) = i(t) - i_p(t), \quad (12)$$

where

$$v_p(t) = [v_{1p}, v_{2p}, \dots, v_{Mp}]^T, \quad i(t) = [i_1, i_2, \dots, i_M]^T, \quad (13)$$

and

$$V_p^2(t) = v_p^2(t) = v_{1p}^2(t) + v_{2p}^2(t) + \dots + v_{Mp}^2(t). \quad (14)$$

The above definitions are identical to Willems' formulation [3] when choosing the reference voltage to be the terminal voltage of the system, i.e.,  $v_p(t) = v(t)$ . For a three-phase system,  $v(t) = [v_a, v_b, v_c]^T$ ,  $i(t) = [i_a, i_b, i_c]^T$ , and  $v(t) \cdot v(t) = v^2 = [v_a, v_b, v_c] \cdot [v_a, v_b, v_c]^T = (v_a^2 + v_b^2 + v_c^2)$ . When choosing  $v_p(t) = v(t)$ , the definitions in (12) become

$$\begin{cases} i_p(t) = \frac{P(t)}{v(t) \cdot v(t)} v(t), \quad \text{and} \\ i_q(t) = i(t) - i_p(t) = \frac{(v(t) \times i(t)) \times v(t)}{v(t) \cdot v(t)} \end{cases} \quad (15)$$

which are identical to Peng's formulation [10]. Further assuming that there are no zero-sequence components in both voltage and current, the definitions in (15) reduce to Akagi's formulation [2] by transforming the three-phase ( $a, b, c$ ) quantities to two phase ( $\alpha, \beta$ ) ones.

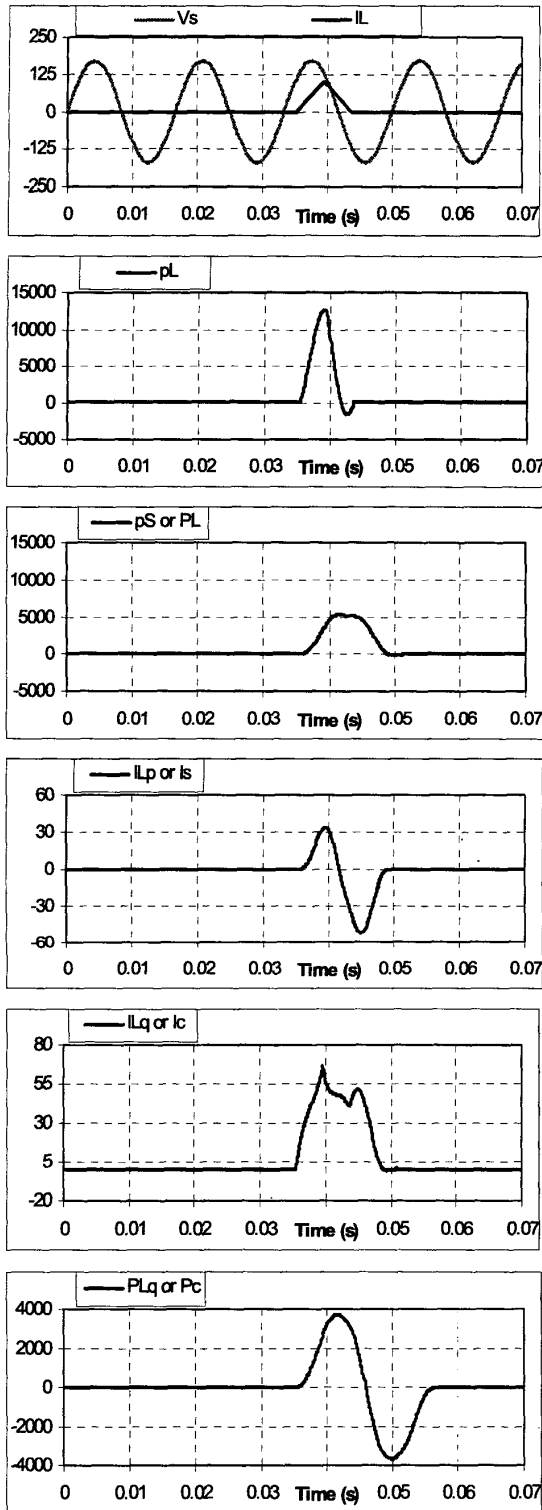


Fig. 2. Simulation results for non-periodic current compensation with  $T_C=172$ .

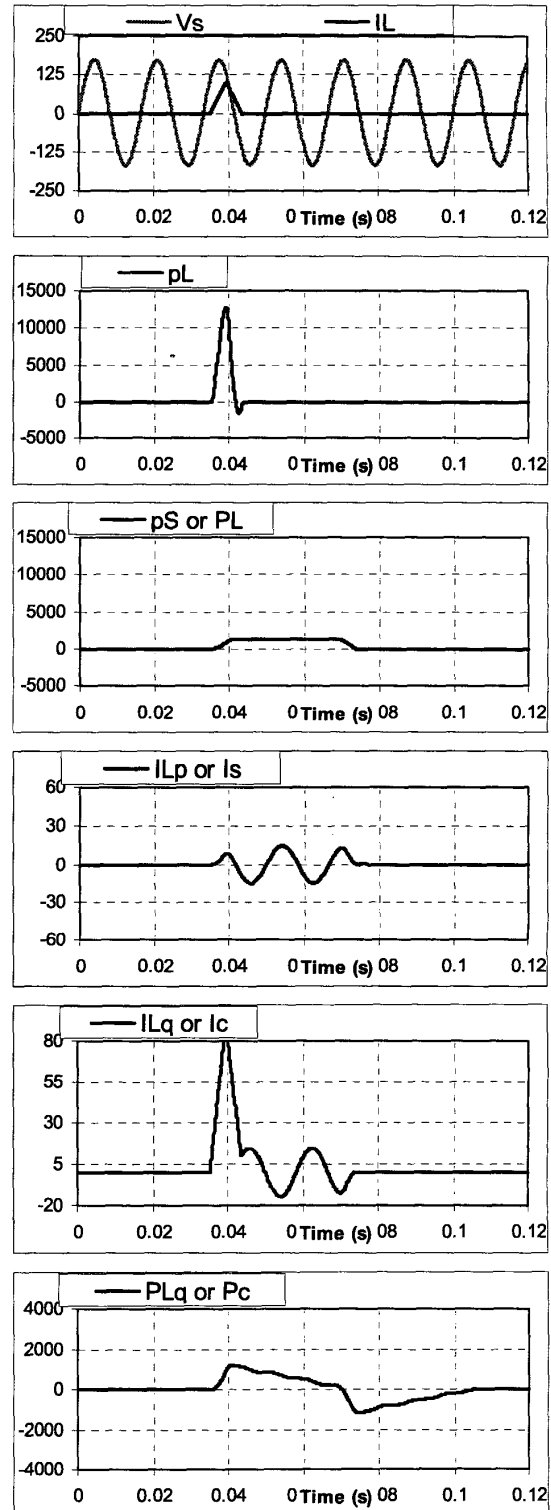


Fig. 3. Simulation results for non-periodic current compensation with  $T_C=27$ .

Again, the definitions given in (4), (5) and (6) have great flexibility to meet all compensation objectives. Regardless of whether the voltage and current are sinusoidal or non-sinusoidal, balanced or unbalanced, and periodic or non-periodic, the definitions give a straightforward method to calculate any non-active current component that requires compensation. Consider a three phase four wire system (Fig. 4):  $v_S = [v_{Sa}, v_{Sb}, v_{Sc}]^T$ ,  $i_L = [i_{La}, i_{Lb}, i_{Lc}]^T$ , and  $i_C = [i_{Ca}, i_{Cb}, i_{Cc}]^T$ . If the compensation objective is to make the source current sinusoidal and balanced, one can calculate compensation current as follows:

- Separate the voltage into four components: fundamental positive-sequence,  $v_{Sf+}$ ; fundamental negative-sequence,  $v_{Sf-}$ ; zero sequence,  $v_{S0}$ ; and harmonic component,  $v_{Sh}$ , i.e.,

$$v_S = v_{Sf+} + v_{Sf-} + v_{S0} + v_{Sh}, \text{ and } v_{S0} = \frac{1}{3} (v_{Sa} + v_{Sb} + v_{Sc}) \cdot [1, 1, 1]^T. \quad (6)$$

- Choose  $T_C$ =one-half or one fundamental cycle and  $v_P = v_{Sf+}$  in (4), (5) and (6), i.e.,

$$T_C = T/2 \text{ or } T_C = T, \text{ and } v_P = v_{Sf+}. \quad (17)$$

- Calculate the load non-active current,  $i_{Lq}$ , as (5) and let  $i_C = i_{Lq}$ .

From (4), (5), (6), (16), and (17), it is easy to show that the compensator consumes zero average power,  $P_C=0$ , and satisfies the requirements in (3). Fig. 5 shows simulation results of a case in which both the source voltage and load current are distorted and unbalanced and contain zero-sequence. The results clearly show that: (1) the source current,  $i_{Sa}$ ,  $i_{Sb}$ , and  $i_{Sc}$ , becomes sinusoidal; (2) the source current becomes balanced instantaneously as indicated by zero source neutral current,  $i_{Sn}$ ; and (3) the average power of the compensator,  $P_C$ , equals zero. If the system's terminal phase voltages are balanced and almost sinusoidal, it would be sufficient just by choosing the phase voltages (or through a moderate filtering) as the reference voltage, i.e.,  $v_P = v_S$  in (4), (5), and (6).

#### IV. CONCLUSION AND DISCUSSION

In this paper, definitions of active and non-active power and current have been given from the compensation standpoint. Their definitions are consistent with the traditional reactive and non-active concept for single phase circuits. In addition, the instantaneous reactive power theories can be deduced from the proposed definitions for polyphase systems. The definitions also have the flexibility that any compensation objective can be achieved by choosing an appropriate averaging interval ( $T_C$ ) and reference voltage. Table 1 gives some examples of how to choose them for different compensation objectives. For example, one can choose different averaging interval ( $T_C = nT$ , where  $n$  can be  $\frac{1}{2}$ , 1, 2, ...and so on) for non-periodic or disturbance currents as indicated in Table 1. To make loads cause less

disturbance and be more friendly to the power system, one can choose longer averaging intervals, which, however, requires larger compensator capacity and/or bigger energy storage components when the load is unbalanced for poly-phase systems. On the other hand, shorter averaging intervals give faster response and minimize the required VA and energy storage ratings of compensators, which, however, results in less friendly and more disturbance to the power system by the load. Therefore, this cost-versus-performance issue is a trade off that one has to make when designing a compensator.

Table 1. Examples Showing Different Compensation Objectives and Their  $v_P$  and  $T_C$  Selections

Compensation Objectives	$v_P$	$T_C$	Resulted Source Current
Single-phase reactive current	$v_S$	$T/2$ or $T$	Unity PF and sinusoidal under sine $v_S$
Single-phase reactive & harmonic current	$v_{Sf}$	$T/2$ or $T$	Unity PF and sinusoidal regardless distortion of $v_S$
Non-periodic & disturbance current	$v_{Sf}$	$NT$	Smoothed sine-wave with unity PF
Instantaneous reactive power in polyphase system	$v_S$	$T_C \rightarrow 0$	Instantaneously unity PF for polyphase
Etc.			

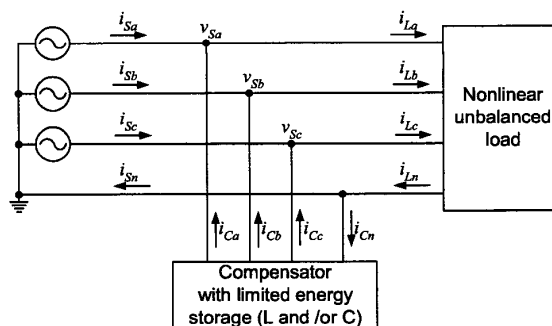


Fig. 4. A compensation system for three-phase four-wire system.

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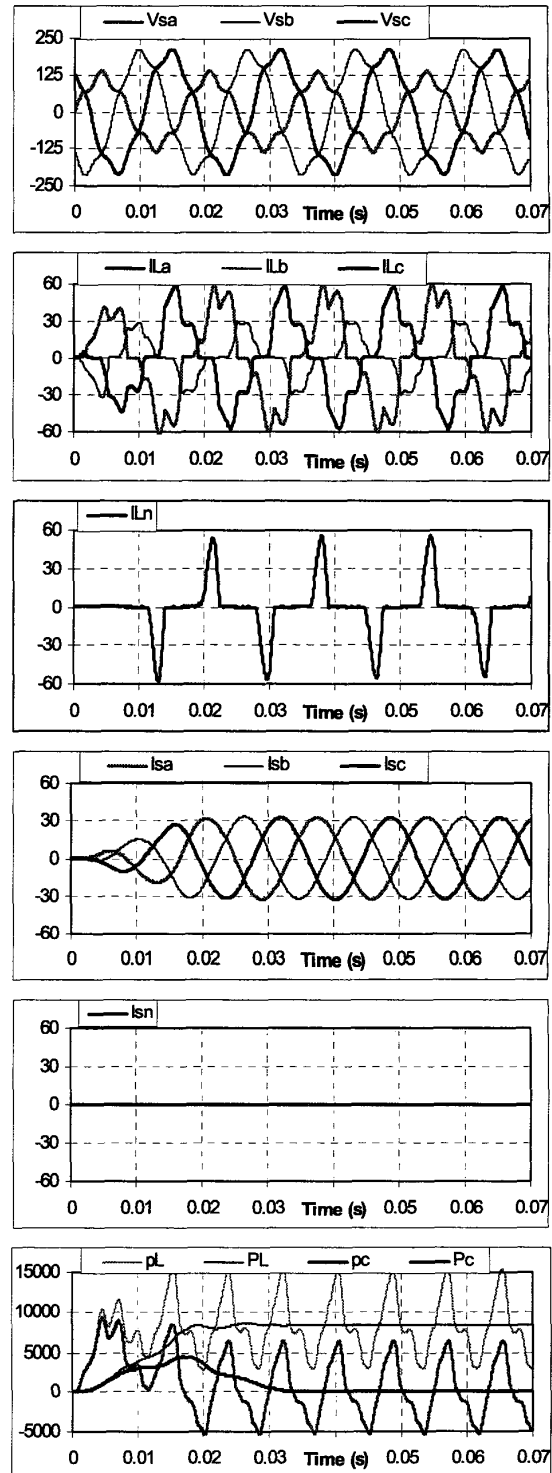


Fig. 5. Simulation results for a distorted and unbalanced 3-phase 4-wire system.