Reduced Switching Frequency Computed PWM Method for Multilevel Converter Control

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***Abstract*— This paper presents two computed PWM methods for 11-level multilevel converters to eliminate the specified harmonics in the output voltage to decrease total harmonic distortion (THD). The first method uses the fundamental switching scheme to eliminate low order harmonics, and uses the active harmonic elimination method to eliminate higher order harmonics. The second method uses these schemes in the reverse order, the fundamental switching scheme to eliminate higher order harmonics, and the active harmonic elimination method to eliminate low order harmonics. The computational results show that the difference between the THD of the two methods is small, but the second method has lower switching frequency. An experimental 11-level H-bridge multilevel converter was used to implement the algorithm and to validate the two computed PWM methods. The experimental results show that the two methods can effectively eliminate the specific harmonics as expected, while the second method results in a significantly lower switching frequency.**

***Keywords* — Multilevel converter, harmonic elimination, computed PWM control**.

I. Introduction

The multilevel converter is a promising technology for utility applications of power electronics because of its low electromagnetic interference (EMI) and high efficiency with low frequency control method [1][2]. There are four kinds of control methods for multilevel converters. They are the fundamental frequency switching method, space vector control method, traditional PWM control method, and space vector PWM method [3]. The benefit of the fundamental frequency switching method and space vector control method are their low switching frequency compared to the other two control methods. However, compared to the traditional PWM method and space vector PWM method, the fundamental frequency switching method and space vector control method have high low order harmonics with low modulation index.

Generally, traditional PWM methods are widely used. But they do not completely eliminate any number of high order harmonics of the output voltage [4]-[14]. To address the problem of having high order harmonics at low modulation indices, the active harmonic elimination method has been proposed [15]. The active harmonic elimination method uses a fundamental frequency switching scheme in which the switch angles are determined using elimination theory [16]-[18] to eliminate low order harmonics. Then the specifically chosen higher harmonics (e.g., the odd non triplen harmonics) are eliminated by using an additional switching angle (one for each higher harmonic) to generate the negative of the harmonic to cancel it.

The active harmonic elimination in [15] has a disadvantage in that it uses a high switching frequency to eliminate higher order harmonics. The active harmonic elimination method is not restricted by the number of unknowns in the equations of the harmonic content, and there are two steps to eliminate the harmonics. It is possible to use the fundamental frequency method to eliminate high order harmonics, and use the active harmonic method to eliminate low order harmonics to decrease the required switching frequency. This paper compares the total harmonic distortion (THD) and the switching frequency for two possible cases for an 11-level multilevel converter control.

The computational results show that different control methods have different effects on the THD and switching frequencies.

An experimental 11-level H-bridge multilevel converter with a first-on first-off switching strategy (used to balance loads between several dc sources) is employed to validate the methods. The experimental results show that the method can effectively eliminate the specific harmonics, and the output voltage waveforms have low THD as expected in theory.

II. Fundamental Frequency Switching Method for an 11-Level Multilevel Converter

A typical 11-level multilevel converter output with fundamental frequency switching scheme is shown in Fig. 1. The Fourier series expansion of the output voltage waveform as shown in Fig. 1 is

$$V(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4V_{dc}}{n\pi}(\cos(n\theta_1) + \cos(n\theta_2) + \ldots + \cos(n\theta_5))\sin(n\omega t) \quad (1)$$

where n is the harmonic number of the output voltage of the multilevel converter. Ideally, given a desired fundamental voltage V_1, one wants to determine the switching angles θ_1, ···, θ_5 so that $V(\omega t) = V_1 sin(\omega t)$, and specific higher harmonics of $V(n\omega t)$ are equal to zero. For a three-phase application, the triplen harmonics in each phase need not be canceled as they automatically cancel in the line-to-line voltages. In this paper, to eliminate the 5th, 7th, 11th, 13th, 17th, 19th, 23rd, 25th, 29th and 31st harmonics, two control methods are proposed here to reduce THD and maintain low switching frequency.

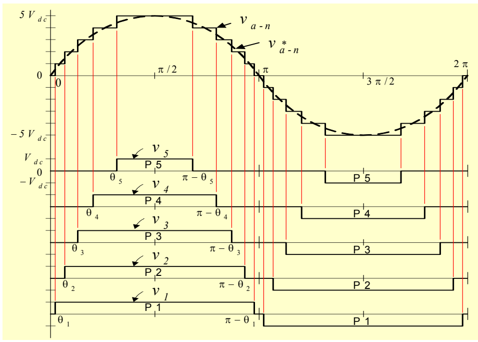

Fig. 1. Output waveform of multilevel converters by fundamental frequency switching scheme.

A. Method I:

The 5th, 7th, 11th, and 13th order harmonics are cancelled by the fundamental frequency switching method [19], and the 17th, 19th, 23rd, 25th, 29th and 31st harmonics are eliminated by the active harmonic elimination method [15]. To find the switching angles for the fundamental frequency switching method to eliminate the 5th, 7th, 11th, and 13th order harmonics, the following equations must be solved.

$$\begin{aligned}
&\cos(\theta_1)+\cos(\theta_2)+\cos(\theta_3)+\cos(\theta_4)+\cos(\theta_5)=m\\
&\cos(5\theta_1)+\cos(5\theta_2)+\cos(5\theta_3)+\cos(5\theta_4)+\cos(5\theta_5)=0\\
&\cos(7\theta_1)+\cos(7\theta_2)+\cos(7\theta_3)+\cos(7\theta_4)+\cos(7\theta_5)=0\\
&\cos(11\theta_1)+\cos(11\theta_2)+\cos(11\theta_3)+\cos(11\theta_4)+\cos(11\theta_5)=0\\
&\cos(13\theta_1)+\cos(13\theta_2)+\cos(13\theta_3)+\cos(13\theta_4)+\cos(13\theta_5)=0
\end{aligned} \tag{2}$$

where m is the modulation index defined as

$$m=\pi V_1/(4V_{dc})\,. \tag{3}$$

Then using the active harmonic elimination method, the additional switchings required to eliminate the 17th, 19th, 23rd, 25th, 29th and 31st harmonics are:

$$N_{sw}\le\sum_{n\in\{17,19,23,25,29,31\}} n \tag{4}$$

where n is the harmonic number. For this example, the upper bound of the additional switching number is 144 by (4).

If a harmonic is near zero, and the control resolution is lower than that required to eliminate the harmonic, then switching will not occur. Another situation is for very short duration pulses or overlap — if a switching off time immediately follows a switching on time, then the switch will just stay off until next switching on occurs. Similarly, if a switching on time immediately follows a switching off time, then the switch will stay on until next switching off occurs. These situations will effectively lead to a slightly lower switching number

B. Method II:

To decrease the required switching number, the high order harmonics (31st, 29th, 23rd, and 19th) are cancelled by the fundamental frequency method, and the 5th, 7th, 11th, 13th, 17th, and 25th harmonics are eliminated by the active harmonic elimination method. Here, the 25th harmonic cannot be chosen to be eliminated by the fundamental frequency method because the active harmonic elimination method will generate a new 25th harmonic when it is used to eliminate the 5th harmonic [15].

Similar to method I, to get the switching angles for the fundamental frequency method, the following equations need to be solved.

$$\begin{aligned}
&\cos(\theta_1)+\cos(\theta_2)+\cos(\theta_3)+\cos(\theta_4)+\cos(\theta_5)=m\\
&\cos(19\theta_1)+\cos(19\theta_2)+\cos(19\theta_3)+\cos(19\theta_4)+\cos(19\theta_5)=0\\
&\cos(23\theta_1)+\cos(23\theta_2)+\cos(23\theta_3)+\cos(23\theta_4)+\cos(23\theta_5)=0\\
&\cos(29\theta_1)+\cos(29\theta_2)+\cos(29\theta_3)+\cos(29\theta_4)+\cos(29\theta_5)=0\\
&\cos(31\theta_1)+\cos(31\theta_2)+\cos(31\theta_3)+\cos(31\theta_4)+\cos(31\theta_5)=0
\end{aligned} \tag{5}$$

where the modulation index m is still given by (3).

Similar to method I, in this method, the additional switchings required to eliminate the 5th, 7th, 11th, 13th, 17th, and 25th harmonics are:

$$N_{sw}\le\sum_{n\in\{5,7,11,13,17,25\}} n\,. \tag{6}$$

The upper bound of the additional switching number is 78 by (6) due to the reasons mentioned in method I. It is about one half of method I (144). In theory, method II can decrease the required switching number when implementing the active harmonic elimination method.

To solve (2) and (5), a numerical method must be used, such as the resultant method or Newton's method. The advantage of the resultant method is that it can find all the solutions for the equation. Therefore, here the resultant method is employed to find the solutions of (2) when they exist. To utilize the resultant method, the transcendental equations of (2) must be converted into polynomial equations, and the resultant method can be used to solve the equations [19].

The 11-level solutions vs. modulation index $m=\pi V_1/(4V_{dc})$ are shown in Fig. 2. Fig. 2 illustrates the range of the modulation indices for which (2) has a solution. The continuous index range is from 2.21 to 4.23. For some modulation indices, there are several solution sets.

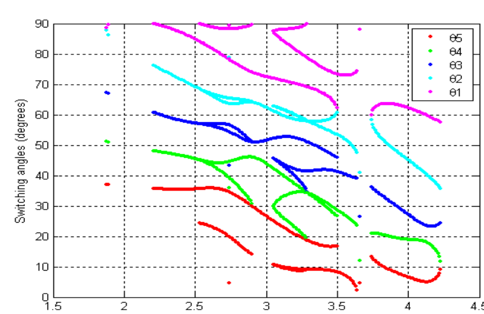

Fig. 2. Solutions of fundamental frequency switching angles of method I for 11-level multilevel converter.

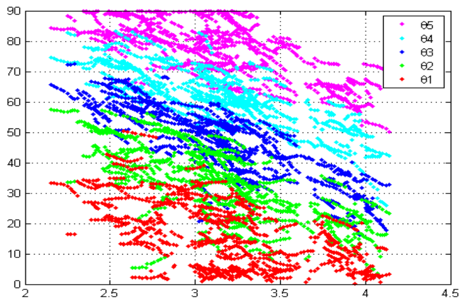

Fig. 3. Solutions of fundamental frequency switching angles of method II for an 11-level multilevel converter.

As the order of the harmonics increase, the degrees of the polynomials in the harmonic equations are large and one reaches the limitations of the capability of contemporary computer algebra software tools (e.g., Mathematic or Maple) to solve the system of polynomial equations by using elimination theory [20]. It is difficult to solve (5) by the resultant method for this reason. To conquer this problem, the fundamental frequency switching angle computation of (5) is solved by the Newton Climbing method although the Newton Climbing method cannot find all the solutions for the equations [21]. The initial guess is proposed from the solutions of (2).

The Newton iterative method for the fundamental frequency switching computation is:

$$x_{n+1} = x_n - J^{-1} f \tag{7}$$

where x_{n+1} is the new value, and x_n is the old value. J is the Jacobian matrix for the transcendental equations, and f is the set of transcendental functions.

$$f = \begin{bmatrix} \sum_{n=1}^{5} \cos(\theta_n) \\ \sum_{n=1}^{5} \cos(19\theta_n) \\ \sum_{n=1}^{5} \cos(23\theta_n) \\ \sum_{n=1}^{5} \cos(29\theta_n) \\ \sum_{n=1}^{5} \cos(31\theta_n) \end{bmatrix} \tag{8}$$

The Jacobian matrix is

$$J = -\begin{bmatrix} \sin(\theta_1) & \sin(\theta_2) & \sin(\theta_3) & \sin(\theta_4) & \sin(\theta_5) \\ \sin(19\theta_1) & \sin(19\theta_2) & \sin(19\theta_3) & \sin(19\theta_4) & \sin(19\theta_5) \\ \sin(23\theta_1) & \sin(23\theta_2) & \sin(23\theta_3) & \sin(23\theta_4) & \sin(23\theta_5) \\ \sin(29\theta_1) & \sin(29\theta_2) & \sin(29\theta_3) & \sin(29\theta_4) & \sin(29\theta_5) \\ \sin(31\theta_1) & \sin(31\theta_2) & \sin(31\theta_3) & \sin(31\theta_4) & \sin(31\theta_5) \end{bmatrix} \tag{9}$$

As expected, most of the continuous solutions can be found by the proposed search method. The solutions of (5) vs. modulation index $m = \pi V_1/(4V_{dc})$ are shown in Fig. 3. The figure illustrates that the modulation index range is from 2.15 to 4.14 for the solutions, and the solutions are continuous.

From Fig. 2 and Fig. 3, it is obvious that for a specific modulation index, there are more than one solution set for (5). The higher the order of the harmonics that appear in the equation, the more solution sets the equation has.

III. THD AND ITS CORRESPONDING SWITCHING NUMBER IN A CYCLE

Fig. 4 shows the lowest THD for method I and method II for a range of modulation indices where THD is calculated as

$$THD = \frac{\sqrt{\sum_{5,7,11,13,\cdots}^{49} V_n^{\,2}}}{V_1}. \tag{10}$$

It can be seen that for some modulation indices, the THD of method II is higher than that of method I. For most of the range of the modulation index, the difference of THD between the two methods is very low.

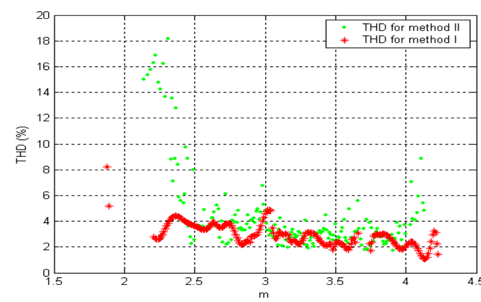

Fig. 4. Lowest THD for method I and method II.

The number of switchings in a cycle is shown in Fig. 5 corresponding to the lowest THD situation shown in Fig. 4. To relate the switching number in Fig. 5 to the average switching frequency of each switch in the inverter, an example is given. If the switching number is 144 in a cycle, this means that the effective switching frequency is 144 × 60 = 8,640 Hz and the actual switching frequency of each switch is 144 × 60/5 = 1728 Hz because the switchings are distributed among 5 H-bridges.

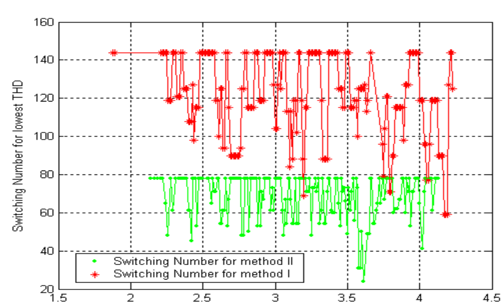

Fig. 5. Switching number in a cycle corresponding to the lowest THD for method I and method II.

As mentioned above, the upper bound of the switching number for method I is 144, and 78 for method II. It can be derived that to eliminate the same harmonics and achieve a similar THD, method II needs less switching number than that of method I. Therefore, method II has lower switching loss and higher efficiency.

IV. Experimental Results

To experimentally validate the proposed algorithm, a prototype three-phase 11-level cascaded H-bridge multilevel inverter has been built using 60 V, 70 A MOSFETs as the switching devices, which is shown in Fig. 6. A battery bank of 15 separate dc sources (SDCSs) of 36 V each feed the inverter (five SDCSs per phase). A real-time controller based on Altera FLEX 10K field programmable gate array (FPGA) is used to implement the algorithm with 8 µs control resolution. For convenience of operation, the FPGA controller was designed as a card to be plugged into a personal computer, which used a peripheral component interconnect (PCI) bus to communicate with the microcomputer. The FPGA controller board based on a PCI bus is shown in Fig. 7.

Fig. 6. 10 kW multilevel converter.

Fig. 7. FPGA controller for multilevel converter.

The m = 3.78 case was chosen for comparison between method I and method II to implement with the multilevel converter. Figs. 8 and 9 show the experimental phase voltage and line-line voltage for method I, and Fig. 10 shows the corresponding normalized FFT analysis of the line-line voltage. Fig. 11 and 12 show the experimental phase and line-line voltage for method II, and Fig. 13 shows the corresponding normalized FFT analysis for the line-line voltage.

From Fig. 10 and Fig. 13, it can be seen that the harmonics have been eliminated up to 31st for both method I and II. Their experimental THD are 3.06% and 3.52%, and this corresponds well with the theoretical computation of 3% and 2.75%. The switching number is 78 for method II, but 121 for method I.

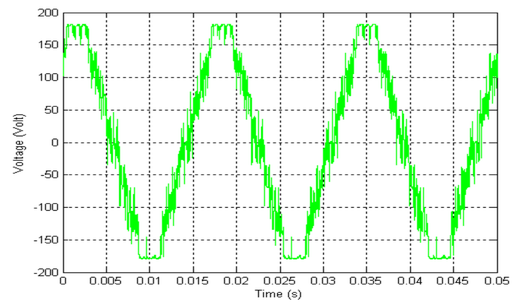

Fig. 8. Experimental multilevel phase voltage for method I (m=3.78).

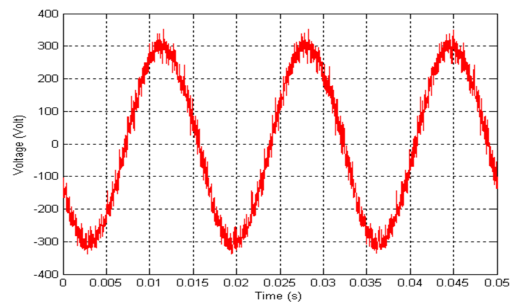

Fig. 9. Experimental multilevel line-line voltage for method I (m=3.78).

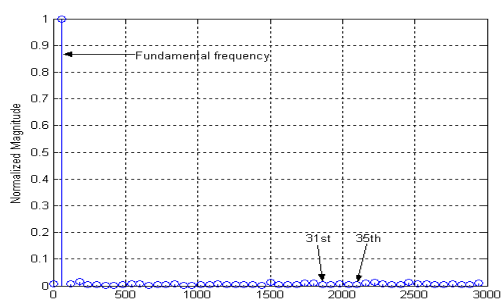

Fig. 10. Normalized FFT analysis of line-line voltage shown in Fig. 8 (THD=3.06%).

V. Conclusions

This paper proposed a new computed PWM method to eliminate high order harmonics by the fundamental frequency switching method and to eliminate low order harmonics by the active harmonic method for multilevel converter control. It can be derived from the computational results that this method can reduce the switching frequency and achieve similar THD when compared to the method proposed earlier [9].

The experiments validated that the proposed method can eliminate all the specified harmonics as expected, and the switching frequency is indeed lower.

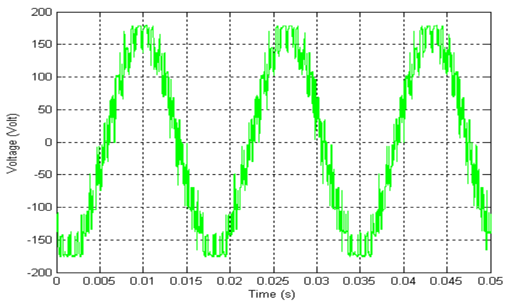

Fig. 11. Experimental multilevel phase voltage of method II (*m*=3.78).

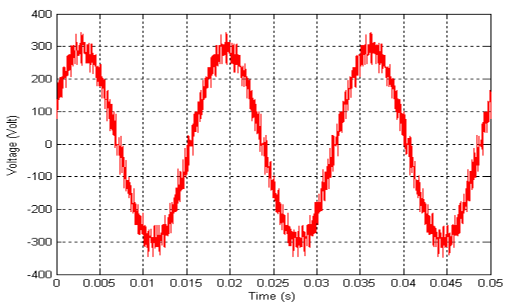

Fig. 12. Experimental multilevel line-line voltage of method II (*m*=3.78).

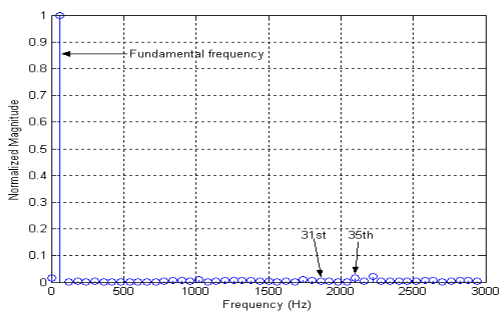

Fig. 13. Normalized FFT analysis of line-line voltage shown in Fig. 12 (THD=3.52%).

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REFERENCES

[1] L. M. Tolbert, F. Z. Peng, T. G. Habetler, "Multilevel converters for large electric drives," *IEEE Transactions on Industry Applications*, vol. 35, no. 1, Jan./Feb. 1999, pp. 36-44.

[2] J. S. Lai and F. Z. Peng, "Multilevel converters – A new breed of power converters," *IEEE Transactions on Industry Applications*, vol. 32, no.3, May./June 1996, pp. 509-517.

[3] J. Rodríguez, J. Lai, and F. Peng, "Multilevel inverters: a survey of topologies, controls and applications," *IEEE Transactions on Industry Applications*, vol. 49, no. 4, Aug. 2002, pp. 724-738.

[4] J. K. Steinke, "Control strategy for a three phase AC traction drive with a 3-level GTO PWM inverter," *IEEE PESC*, 1988, pp. 431-438.

[5] P. Hammond, "A new approach to enhance power quality for medium voltage ac drives," *IEEE Trans. Industry Applications*, vol. 33, Jan./Feb. 1997, pp. 202–208.

[6] W. A. Hill and C. D. Harbourt, "Performance of medium voltage multilevel inverters," *IEEE Industry Applications Society Annual Meeting*, October 1999, Phoenix, AZ, pp. 1186–1192.

[7] G. Carrara, S. Gardella, M. Marchesoni, R. Salutari, G. Sciutto, "A new multilevel PWM method: A theoretical analysis," *IEEE Trans. Power Electronics*, vol. 7, no. 3, July 1992, pp. 497-505.

[8] L. M. Tolbert, F. Z. Peng, T. G. Habetler, "Multilevel PWM methods at low modulation indices," *IEEE Trans. Power Electronics*, vol. 15, no. 4, July 2000, pp. 719-725.

[9] L. M. Tolbert, T. G. Habetler, "Novel multilevel inverter carrier-based PWM method," *IEEE Trans. Industry Applications*, vol. 35, no. 5, Sept./Oct. 1999, pp. 1098-1107.

[10] D. G. Holmes, "The significance of zero space vector placement for carrier based PWM schemes," *IEEE IAS Annual Meeting*, 1995, pp. 2451-2458.

[11] J. Vassallo, J. C. Clare, P. W. Wheeler, "A power-equalized harmonic-elimination scheme for utility-connected cascaded H-bridge multilevel converters," *IEEE Industrial Electronics Society Annual Conference*, 2-6 Nov. 2003, pp. 1185–1190.

[12] S. Sirisukprasert, J.-S. Lai, T.-H. Liu, "Optimum harmonic reduction with a wide range of modulation indexes for multilevel converters," *IEEE Trans. Ind. Electronics,* vol. 49, no. 4, Aug. 2002, pp. 875-881.

[13] T. Kato, "Sequential homotopy-based computation of multiple solutions for selected harmonic elimination in PWM inverters," *IEEE Trans. Circuits and Systems I*, vol. 46, no. 5, May 1999, pp. 586-593.

[14] P. C. Loh, D. G. Holmes, T. A. Lipo, "Implementation and control of distributed PWM cascaded multilevel inverters with minimum harmonic distortion and common-mode voltages," *IEEE Trans. on Power Electronics*, vol. 20, no. 1, Jan. 2005, pp. 90-99.

[15] Z. Du, L. M. Tolbert, J. N. Chiasson, "Harmonic elimination for multilevel converter with programmed PWM method," *IEEE Industry Applications Society Annual Meeting,* October 3-7, 2004, Seattle, Washington, pp. 2210-2215.

[16] H. S. Patel and R. G. Hoft, "Generalized harmonic elimination and voltage control in thyristor inverters: Part I –harmonic elimination," *IEEE Trans. Industry Applications*, vol. 9, May/June 1973, pp. 310-317.

[17] H. S. Patel and R. G. Hoft, "Generalized harmonic elimination and voltage control in thyristor inverters: Part II –voltage control technique," *IEEE Trans. Ind. Applications*, vol. 10, Sept./Oct. 1974, pp. 666-673.

[18] P. N. Enjeti, P. D. Ziogas, J. F. Lindsay, "Programmed PWM techniques to eliminate harmonics: A critical evaluation" *IEEE Transactions on Industry Applications*, vol. 26, no. 2, March/April. 1990. pp. 302 – 316.

[19] J. N. Chiasson, L. M. Tolbert, K. J. McKenzie, Z. Du, "Control of a multilevel converter using resultant theory," *IEEE Transactions on Control System Theory*, vol. 11, no. 3, May 2003, pp. 345-354.

[20] J. N. Chiasson, L. M. Tolbert, K. J. McKenzie, Z. Du, "A new approach to solving the harmonic elimination equations for a multilevel converter," *IEEE Industry Applications Society Annual Meeting*, October 12-16, 2003, Salt Lake City, Utah, pp. 640-645.

[21] Z. Du, L. M. Tolbert, J. N. Chiasson, "Modulation extension control for multilevel converters using triplen harmonic injection with low switching frequency," *IEEE Applied Power Electronics Conference,* March 6-10, 2004, Austin, Texas, pp. 419-423.