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Speed Sensorless Identification of the Rotor Time Constant in Induction Machines

Mengwei Li Campbell, John Chiasson, Marc Bodson, and Leon M. Tolbert

***Abstract*—A method is proposed to estimate the rotor time constant T_R of an induction motor without measurements of the rotor speed/position. The method consists of solving for the roots of a polynomial equation in T_R whose coefficients depend only on the stator currents, stator voltages, and their derivatives. Experimental results are presented.**

***Index Terms*—Induction motor, parameter identification, rotor time constant.**

I. Introduction

Induction motors are very attractive in many applications owing to their simple structure, low cost, and robust construction. Field-oriented control is now used to obtain high performance drive of the induction motor because it gives control characteristics similar to separately excited dc motors. Implementation of a (rotor-flux) field-oriented controller requires knowledge of the rotor speed and the rotor time constant T_R to estimate the rotor flux linkages. There has been considerable work done in the last several years to implement a field-oriented controller without the use of a speed sensor [1]–[6]. However, many of these methods still require the value of T_R, which can change with time due to ohmic heating. That is, to be able to update the value of T_R to the controller as it changes is valuable. The work presented here uses an algebraic approach to identify the rotor time constant T_R without the motor speed information. It is most closely related to the ideas described in [7]–[12]. Specifically, it is shown that T_R satisfies a polynomial equation whose coefficients are functions of the stator currents, the stator voltages, and their derivatives. A zero of this polynomial is the value of T_R. It is further shown that T_R is not identifiable under steady-state operation because the system is not sufficiently excited.

The note is organized as follows. Section II introduces a space vector model of the induction motor. Section III uses this model to develop a algebraic equation that T_R must satisfy. Section IV shows that in steady state, T_R is not identifiable by either the proposed algebraic method or a standard linear least-squares method. Section V presents the experimental results, while Section VI gives the conclusions and future work. A preliminary version of this work appeared in [13].

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M. Li Campbell was with the Electrical and Computer Engineering Department, The University of Tennessee, Knoxville, TN 37996 USA. She is now with the Advanced Technology Center, General Motors Corporation, Torrance, CA 90505 USA (e-mail: mengwei.campbell@gm.com).

J. Chiasson is with the Electrical and Computer Engineering Department, Bosie State University, Boise ID 83725 USA (e-mail: johnchiasson@boisestate.edu).

M. Bodson is with the Electrical and Computer Engineering Department, The University of Utah, Salt Lake City, UT 84112 USA (e-mail: bodson@ee.utah.edu).

L. M. Tolbert is with the Electrical and Computer Engineering Department, The University of Tennessee, Knoxville, TN 37996. He is also with the Oak Ridge National Laboratory, Oak Ridge, TN 37831 (e-mail: tolbert@utk.edu; tolbertlm@ornl.gov).

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II. Mathematical Model of Induction Motor

The starting point of the analysis is a space vector model of the induction motor given by (see, e.g., [14, p. 568])

$$\frac{d}{dt}\underline{i}_S = \frac{\beta}{T_R}(1 - jn_P\omega T_R)\underline{\psi}_R - \gamma\underline{i}_S + \frac{1}{\sigma L_S}\underline{u}_S \tag{1}$$

$$\frac{d}{dt}\underline{\psi}_R = -\frac{1}{T_R}(1 - jn_P\omega T_R)\underline{\psi}_R + \frac{M}{T_R}\underline{i}_S \tag{2}$$

$$\frac{d\omega}{dt} = \frac{n_P M}{JL_R}\mathrm{Im}\left\{\underline{i}_S\underline{\psi}_R^*\right\} - \frac{\tau_L}{J} \tag{3}$$

where $\underline{i}_S \triangleq i_{Sa} + ji_{Sb}$, $\underline{\psi}_R \triangleq \psi_{Ra} + j\psi_{Rb}$, and $\underline{u}_S \triangleq u_{Sa} + ju_{Sb}$. Here, θ is the position of the rotor, $\omega = d\theta/dt$ is the rotor speed, n_p is the number of pole pairs, i_{Sa}, i_{Sb} are the (two-phase equivalent) stator currents, ψ_{Ra}, ψ_{Rb} are the (two-phase equivalent) rotor flux linkages, R_S, R_R are the stator and rotor resistances, respectively, M is the mutual inductance, L_S and L_R are the stator and rotor inductances, respectively, J is the moment of inertia of the rotor, and τ_L is the load torque. The symbols $T_R = L_R/R_R$, $\sigma = 1 - (M^2/L_SL_R)$, $\beta = M/\sigma L_SL_R$, $\gamma = (R_S/\sigma L_S) + (\beta M/T_R)$ have been used to simplify the expressions. T_R is referred to as the rotor time constant, while σ is called the total leakage factor.

III. Algebraic Approach to T_R Estimation

The idea of the approach is to solve (1) and (2) for T_R. However, (1) and (2) are only four equations while there are six unknowns, namely ψ_{Ra}, ψ_{Rb}, $d\psi_{Ra}/dt$, $d\psi_{Rb}/dt$, ω, and T_R. Equation (3) is not used because it introduces the additional unknown τ_L. To find two more independent equations, (1) is differentiated to obtain

$$\frac{d^2}{dt^2}\underline{i}_S = \frac{\beta}{T_R}(1 - jn_P\omega T_R)\frac{d}{dt}\underline{\psi}_R - jn_P\beta\underline{\psi}_R\frac{d\omega}{dt} - \gamma\frac{d}{dt}\underline{i}_S + \frac{1}{\sigma L_S}\frac{d}{dt}\underline{u}_S. \tag{4}$$

Using the (complex-valued) (1) and (2), one can solve for $\underline{\psi}_R$ and $(d/dt)\underline{\psi}_R$ in terms of ω, $\underline{i}_S$ and $\underline{u}_S$ and substitute the resulting expressions into (4) to obtain

$$\begin{aligned}\frac{d^2}{dt^2}\underline{i}_S = &-\frac{1}{T_R}(1 - jn_P\omega T_R)\left(\frac{d}{dt}\underline{i}_S + \gamma\underline{i}_S - \frac{1}{\sigma L_S}\underline{u}_S\right)\\ &+\frac{\beta M}{T_R^2}(1 - jn_P\omega T_R)\underline{i}_S - \gamma\frac{d}{dt}\underline{i}_S + \frac{1}{\sigma L_S}\frac{d}{dt}\underline{u}_S\\ &-\frac{jn_PT_R}{1 - jn_P\omega T_R}\left(\frac{d}{dt}\underline{i}_S + \gamma\underline{i}_S - \frac{1}{\sigma L_S}\underline{u}_S\right)\frac{d\omega}{dt}.\end{aligned} \tag{5}$$

Solving (5) for $d\omega/dt$ gives

$$\frac{d\omega}{dt} = -\frac{(1-jn_P\omega T_R)^2}{jn_P T_R^2} + \frac{1-jn_P\omega T_R}{jn_P T_R} \times \frac{\frac{\beta M}{T_R^2}(1-jn_P\omega T_R)\underline{i}_S - \gamma\frac{d}{dt}\underline{i}_S + \frac{1}{\sigma L_S}\frac{d}{dt}\underline{u}_S - \frac{d^2}{dt^2}\underline{i}_S}{\frac{d}{dt}\underline{i}_S + \gamma\underline{i}_S - \frac{1}{\sigma L_S}\underline{u}_S}. \quad (6)$$

The left-hand side of (6) is real, so the right-hand side must also be real. Note by (1) that $d\underline{i}_S/dt + \gamma\underline{i}_S - \underline{u}_S/(\sigma L_S) = (\beta/T_R)(1 - jn_P\omega T_R)\underline{\psi}_R$ so that the right-hand side of (6) is singular if and only if $|\underline{\psi}_R| = 0$. Other than at startup, $|\underline{\psi}_R| \neq 0$ in normal operation of the motor. Separating the right-hand side of (6) into its real and imaginary parts, the real part has the form

$$\frac{d\omega}{dt} = a_2(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})\omega^2 + a_1(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})\omega + a_0(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}). \quad (7)$$

The expressions for $a_2(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})$, $a_1(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})$, and $a_0(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})$ are lengthy in terms of $u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}$, and their derivatives as well as of the machine parameters including T_R. As a consequence, they are not explicitly presented here. Appendix VII-B gives their steady-state expressions.

On the other hand, the imaginary part of the right-hand side of (6) must be zero. In fact, the imaginary part of (6) is a second degree polynomial equation in ω of the form

$$q(\omega) \triangleq q_2(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})\omega^2 + q_1(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})\omega + q_0(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \quad (8)$$

and, if ω is the speed of the motor, then $q(\omega) = 0$. The q_i are functions of u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}, and their derivatives as well as of the machine parameters including T_R. The expressions for $q_2(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})$, $q_1(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})$, and $q_0(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})$ are also lengthy and not explicitly presented here. (Their steady-state expressions are given in Appendix VII-A.) If the speed was measured, then (8) would be equal to zero and could then be solved for T_R. However, in the problem being considered, ω is not known. To eliminate ω, $q(\omega)$ in (8) is differentiated to obtain

$$\frac{d}{dt}q(\omega) = (2q_2\omega + q_1)\frac{d\omega}{dt} + \dot{q}_2\omega^2 + \dot{q}_1\omega + \dot{q}_0 \quad (9)$$

where $dq(\omega)/dt \equiv 0$ if ω is equal to the motor speed. Next, $d\omega/dt$ in (9) is replaced by the right-hand side of (7) so that (9) may be written as

$$\frac{dq(\omega)}{dt} = g(\omega) \quad (10)$$

where $g(\omega)$ is the third-order polynomial equation in ω (with time-varying coefficients) given by

$$g(\omega) \triangleq 2q_2a_2\omega^3 + (2q_2a_1 + q_1a_2 + \dot{q}_2)\omega^2 + (2q_2a_0 + q_1a_1 + \dot{q}_1)\omega + q_1a_0 + \dot{q}_0$$

for which the speed of the motor is one of its roots. Dividing[1] $g(\omega)$ in (10) by $q(\omega)$ in (8), $g(\omega)$ may be rewritten as ($q_2 \neq 0$ if ω and the stator electrical frequency ω_S are nonzero. See [6], [15])

$$g(\omega) = \frac{1}{q_2}\big((2q_2a_2\omega + 2q_2a_1 - q_1a_2 + \dot{q}_2)q(\omega) + r_1(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})\omega + r_0(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})\big) \quad (11)$$

$$r_1(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \triangleq 2q_2^2a_0 - q_2q_1a_1 + q_2\dot{q}_1 - 2q_2q_0a_2 + q_1^2a_2 - q_1\dot{q}_2 \quad (12)$$

$$r_0(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \triangleq q_2q_1a_0 + q_2\dot{q}_0 - 2q_2q_0a_1 + q_0q_1a_2 - q_0\dot{q}_2. \quad (13)$$

If ω is equal to the speed of the motor, then both $g(\omega) = 0$ and $q(\omega) = 0$, and one obtains

$$r(\omega) \triangleq r_1(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})\omega + r_0(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) = 0. \quad (14)$$

This is now a first-order polynomial equation in ω which uniquely determines the motor speed ω as long as r_1 (the coefficient of ω) is nonzero. (It is shown in Appendix VII-C that $r_1 \neq 0$ in steady state if $q_2 \neq 0$.) Solving for the motor speed ω using (14), one obtains

$$\omega = -r_0/r_1. \quad (15)$$

Next, replace ω in (8) by the expression in (15) to obtain

$$q_2r_0^2 - q_1r_0r_1 + q_0r_1^2 \equiv 0. \quad (16)$$

The expressions for q_i, r_i are in terms of motor parameters (including T_R) as well as the stator currents, voltages, and their derivatives. Expanding the expressions for q_0, q_1, q_2, r_0, and r_1, one obtains a twelfth-order polynomial equation in T_R, which can be written as

$$\sum_{i=0}^{12} C_i(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})T_R^i = 0. \quad (17)$$

Solving (17) gives T_R. The coefficients $C_i(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})$ of (17) contain third-order derivatives of the stator currents and second-order derivatives of the stator voltages making noise a concern. For short time intervals in which T_R does not vary, (17) must hold identically with T_R constant. In order to average out the effect of noise on the C_i, (17) is integrated over a time interval $[t_1, t_2]$ to obtain

$$\sum_{i=0}^{12}\left(\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} C_i(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})dt\right)T_R^i = 0. \quad (18)$$

The measured variables appear into the coefficients of (17) in a nonlinear manner, so that it would be difficult to quantify exactly how much noise is filtered out. However, assuming a sufficient frequency separation between the noise and the signal, one would expect that such filtering would help and the experimental results presented below bear this out.

[1]Given the polynomials $g(\omega), q(\omega)$ in ω with $\deg\{g(\omega)\} = n_g, \deg\{q(\omega)\} = n_q$, the Euclidean division algorithm ensures that there are polynomials $\gamma(\omega), r(\omega)$ such that $g(\omega) = \gamma(\omega)q(\omega) + r(\omega)$ and $\deg\{r(\omega)\} \leq \deg\{q(\omega)\} - 1 = n_q - 1$. Consequently if, for example, ω_0 is a zero of both $g(\omega)$ and $q(\omega)$, then it must also be a zero of $r(\omega)$.

There are 12 solutions satisfying (18). However, simulation results have always given 10 conjugate solutions. The remaining two solutions include the correct value of T_R while the other one was either negative or close to zero. The method is to compute the coefficients $(1/(t_2 - t_1))\int_{t_1}^{t_2} C_i dt$ and then compute the roots of (18). Among the positive real roots is the correct value of T_R. Experimental results using this method are presented in Section V.

Remark: The expression (14) was used by the authors in [6], [20] (assuming T_R is known) as a technique to estimate the speed of an induction motor for speed sensorless field-oriented control.

IV. Identifiability of T_R in Steady State

The goal of this section is to show that T_R is not identifiable with the machine in steady-state because it is not sufficiently excited. We show this explicitly for the method proposed here and then show it explicitly for a linear least-squares formulation. The terminology "steady state" means the machine is running at constant speed *and* the voltages/currents are in steady state.

A. Algebraic Approach

The polynomial (18) is now considered with the machine in steady-state so that, in particular, the speed is constant. That is, $u_{Sa} + ju_{Sb} = \underline{U}_S e^{j\omega_S t}$ and $i_{Sa} + ji_{Sb} = \underline{I}_S e^{j\omega_S t}$ are substituted into (8) and (14) where ω_S is the electrical frequency. In steady state, the motor speed in (15) becomes (see Appendix VII-C and [15])

$$\omega = -\frac{r_0}{r_1} = \frac{\omega_S(1-S)}{n_p} \tag{19}$$

where $S \triangleq (\omega_S - n_p\omega)/\omega_S$ is the normalized slip. Substituting the steady-state expressions for q_2, q_1, and q_0 from Appendix VII-A as well as the expression (19) for ω into (8), one obtains

$$\begin{aligned} & q_2\omega^2 + q_1\omega + q_0 \\ &\quad = \frac{n_p^2 T_R^2 |\underline{I}_S|^4 \omega_S^2 L_S (1-\sigma)^2 (1-S)}{\sigma\,(1+S^2\omega_S^2 T_R^2)} \left(\frac{\omega_S(1-S)}{n_p}\right)^2 \\ &\qquad + \frac{n_p\omega_S |\underline{I}_S|^4 L_S (1-\sigma)^2 \left(1-\omega_S^2 T_R^2 (1-S)^2\right)}{\sigma\,(1+S^2\omega_S^2 T_R^2)} \\ &\qquad \times \left(\frac{\omega_S(1-S)}{n_p}\right) - \frac{|\underline{I}_S|^4 \omega_S^2 L_S (1-\sigma)^2 (1-S)}{\sigma\,(1+S^2\omega_S^2 T_R^2)} \\ &\quad \equiv 0. \end{aligned}$$

That is, in steady state (8) and (14) hold independent of the value of T_R and thus so does (17) making T_R unidentifiable in steady state by this method.

B. Linear Least-Squares Approach

Vélez-Reyes *et al.* [3], [4] have used least-squares methods for simultaneous parameter and speed identification in induction machines. In the approach used herein, $d\omega/dt$ is taken to be zero so that a linear (in the parameters) regressor model can be obtained. Specifically, consider the mathematical model of the induction motor in (5). With $d\omega/dt = 0$ this equation reduces to

$$\begin{aligned} \frac{d^2}{dt^2}\underline{i}_S = & -\frac{1}{T_R}(1 - jn_P\omega T_R)\left(\frac{d}{dt}\underline{i}_S + \gamma\underline{i}_S - \frac{1}{\sigma L_S}\underline{u}_S\right) \\ & + \frac{\beta M}{T_R^2}(1 - jn_P\omega T_R)\underline{i}_S - \gamma\frac{d}{dt}\underline{i}_S + \frac{1}{\sigma L_S}\frac{d}{dt}\underline{u}_S \end{aligned} \tag{20}$$

where $\underline{i}_S = i_{Sa} + ji_{Sb}$ and $\underline{u}_S = u_{Sa} + ju_{Sb}$. Decomposing (20) into its real and imaginary parts gives

$$\begin{aligned} \frac{d^2 i_{Sa}}{dt} = & \frac{1}{T_R}\left(-\frac{di_{Sa}}{dt} - \frac{R_S}{\sigma L_S}i_{Sa} + \frac{1}{\sigma L_S}u_{Sa}\right) + n_p\omega \\ & \times\left(-\frac{di_{Sb}}{dt} - \frac{R_S}{\sigma L_S}i_{Sb} + \frac{1}{\sigma L_S}u_{Sb}\right) - \gamma\frac{di_{Sa}}{dt} + \frac{1}{\sigma L_S}\frac{du_{Sa}}{dt} \end{aligned} \tag{21}$$

and

$$\begin{aligned} \frac{d^2 i_{Sb}}{dt} = & \frac{1}{T_R}\left(-\frac{di_{Sb}}{dt} - \frac{R_S}{\sigma L_S}i_{Sb} + \frac{1}{\sigma L_S}u_{Sb}\right) - n_p\omega \\ & \times\left(-\frac{di_{Sa}}{dt} - \frac{R_S}{\sigma L_S}i_{Sa} + \frac{1}{\sigma L_S}u_{Sa}\right) - \gamma\frac{di_{Sb}}{dt} + \frac{1}{\sigma L_S}\frac{du_{Sb}}{dt}. \end{aligned} \tag{22}$$

The goal here is to estimate T_R without knowledge of ω. So, it is now assumed the motor parameters are all known except for T_R. The set of (21) and (22) may then be rewritten in regressor form as

$$y(t) = W(t)K \tag{23}$$

where $K \triangleq [1/T_R\ n_p\omega]^T \in \mathbb{R}^2$, and $y \in \mathbb{R}^2$, $W \in \mathbb{R}^{2\times 2}$ are given by

$$\begin{aligned} y(t) &\triangleq \begin{bmatrix} \frac{du_{Sa}}{dt} - \sigma L_S\frac{d^2 i_{Sa}}{dt} - R_S\frac{di_{Sa}}{dt} \\ \frac{du_{Sb}}{dt} - \sigma L_S\frac{d^2 i_{Sb}}{dt} - R_S\frac{di_{Sb}}{dt} \end{bmatrix} \\ W(t) &\triangleq \begin{bmatrix} L_S\frac{di_{Sa}}{dt} - u_{Sa} + R_S i_{Sa} & \sigma L_S\frac{di_{Sb}}{dt} - u_{Sb} + R_S i_{Sb} \\ L_S\frac{di_{Sb}}{dt} - u_{Sb} + R_S i_{Sb} & -\sigma L_S\frac{di_{Sa}}{dt} + u_{Sa} - R_S i_{Sa} \end{bmatrix} \end{aligned}$$

The regressor system (23) is linear in the parameters. The standard linear least-squares approach is to let (i.e., collect data at) $t = 0, T, 2T, \ldots, NT$, multiply (23) on the left by $W^T(nT)$, sum $W^T(nT)y(nT) = W^T(nT)W(nT)K$ from $t = 0$ to $t = NT$, and finally compute the solution to

$$R_W K = R_{YW} \tag{24}$$

where

$$R_W \triangleq \sum_{n=0}^{N} W^T(nT)W(nT) \quad R_{YW} \triangleq \sum_{n=0}^{N} W^T(nT)y(nT).$$

A unique solution to (24) exists if and only if R_W is invertible. However, R_W is never invertible in steady state as is now shown.[2] To proceed, define

$$D(t) = \begin{bmatrix} i_{Sb}(t) & -i_{Sa}(t) \\ i_{Sa}(t) & i_{Sb}(t) \end{bmatrix}.$$

In steady state where $u_{Sa} + ju_{Sb} = \underline{U}_S e^{j\omega_S t}$ and $i_{Sa} + ji_{Sb} = \underline{I}_S e^{j\omega_S t}$, $\det(D(t)) = i_{Sa}^2(t) + i_{Sb}^2(t) = |\underline{I}_S|^2$, $D(t)^T D(t) = |\underline{I}_S|^2 I_{2\times 2}$. Multiply both sides of (23) on the left by $D(t)$ to obtain

$$D(t)y(t) = D(t)W(t)K$$

or

$$\begin{aligned} & \begin{bmatrix} R_S\omega_S|\underline{I}_S|^2 - \omega_S P \\ \sigma L_S\omega_S^2|\underline{I}_S|^2 - \omega_S Q \end{bmatrix} \\ &\quad = \begin{bmatrix} -\omega_S L_S|\underline{I}_S|^2 + Q & R_S|\underline{I}_S|^2 - P \\ R_S|\underline{I}_S|^2 - P & \sigma L_S\omega_S|\underline{I}_S|^2 - Q \end{bmatrix} K \end{aligned} \tag{25}$$

[2] In [4], the machine is run at constant speed, but not in steady state.

where $P \triangleq u_{Sa}i_{Sa} + u_{Sb}i_{Sb}$ and $Q \triangleq u_{Sb}i_{Sa} - u_{Sa}i_{Sb}$ are the real and reactive powers, respectively, whose steady-state expressions are given by (30) and (31) in the Appendix. Using (30) and (31) to replace P and Q in (25), one obtains

$$\begin{aligned}\bar{D} &\triangleq D(t)W(t)\\ &= -\frac{|\underline{I}_S|^2(1-\sigma)\omega_S L_S}{1+S^2\omega_S^2T_R^2}\begin{bmatrix} S^2\omega_S^2T_R^2 & S\omega_S T_R\\ S\omega_S T_R & 1\end{bmatrix} \qquad (26)\\ \bar{Y} &\triangleq D(t)y(t)\\ &= -\omega_S\frac{|\underline{I}_S|^2(1-\sigma)\omega_S L_S}{1+S^2\omega_S^2T_R^2}\begin{bmatrix} S\omega_S T_R\\ 1\end{bmatrix}. \qquad (27)\end{aligned}$$

That is, in steady state, $\bar{D} \triangleq D(t)W(t) \in \mathbb{R}^{2\times 2}$ and $\bar{Y} \triangleq D(t)y(t) \in \mathbb{R}^2$ are *constant* matrices. Further, it is easily seen that the determinant of $\bar{D} \triangleq D(t)W(t)$ is zero. Also,

$$\begin{aligned}R_{DW} &\triangleq \sum_{n=1}^{N}(D(nT)W(nT))^T(D(nT)W(nT))\\ &= |\underline{I}_S|^2\sum_{n=1}^{N}W^T(nT)W(nT) = |\underline{I}_S|^2R_W.\end{aligned}$$

R_{DW} is singular as $D(t)W(t)$ is constant and singular. It then follows that R_W is also singular using steady-state data. Furthermore

$$\begin{aligned}R_{DWY} &\triangleq \sum_{n=1}^{N}(D(nT)W(nT))^T(D(nT)y(nT))\\ &= |\underline{I}_S|^2\sum_{n=1}^{N}W^T(nT)y(nT) = |\underline{I}_S|^2R_{YW}.\end{aligned}$$

Thus, R_W and R_{YW} are given by

$$\begin{aligned}R_W &= R_{DW}/|\underline{I}_S|^2 = N\bar{D}^T\bar{D}/|\underline{I}_S|^2\\ &= \frac{N|\underline{I}_S|^2(1-\sigma)^2\omega_S^2L_S^2}{1+S^2\omega_S^2T_R^2}\begin{bmatrix} S^2\omega_S^2T_R^2 & S\omega_S T_R\\ S\omega_S T_R & 1\end{bmatrix} \qquad (28)\\ R_{YW} &= R_{DWY}/|\underline{I}_S|^2 = N\bar{D}^T\bar{Y}/|\underline{I}_S|^2\\ &= \omega_S\frac{N|\underline{I}_S|^2(1-\sigma)^2\omega_S^2L_S^2}{1+S^2\omega_S^2T_R^2}\begin{bmatrix} S\omega_S T_R\\ 1\end{bmatrix} \qquad (29)\end{aligned}$$

where again $\bar{D}$ and $\bar{Y}$ are from (26) and (27), respectively.

By inspection of (28) and (29), $K = [0\ \omega_S]^T$ is one solution to (24). The null space of R_W is generated by $[-1/T_R\ S\omega_S]^T$ so that all possible solutions are given by $[0\ \omega_S]^T + \alpha[-1/T_R\ S\omega_S]^T$ for some $\alpha \in \mathbb{R}$. In summary, solving (24) using steady-state data leads to an infinite set of solutions so that T_R is not identifiable using the linear regressor (23) with steady-state data.

Remarks: There are a few ways to avoid the singularity problem in a real-time control application. For example, a small perturbation could be added to the speed reference. This type of technique has often been used for the adaptive control of insufficiently excited systems. A more interesting approach, however, would be to vary the flux reference while keeping the torque reference constant. The speed of the motor would not vary, but the voltages and currents would no longer be in sinusoidal steady-state, so that the speed and the rotor time constant would be identifiable. In [4], a linear regressor was obtained by assuming constant speed, but the data collected in [4] was *not* in sinusoidal steady state (see [4, Figs. 7.1a and 7.1b]). In the identification method given in [16], the speed is assumed constant, but it requires the flux magnitude be perturbed by a small amplitude sinusoidal signal so it is also not in sinusoidal steady state.

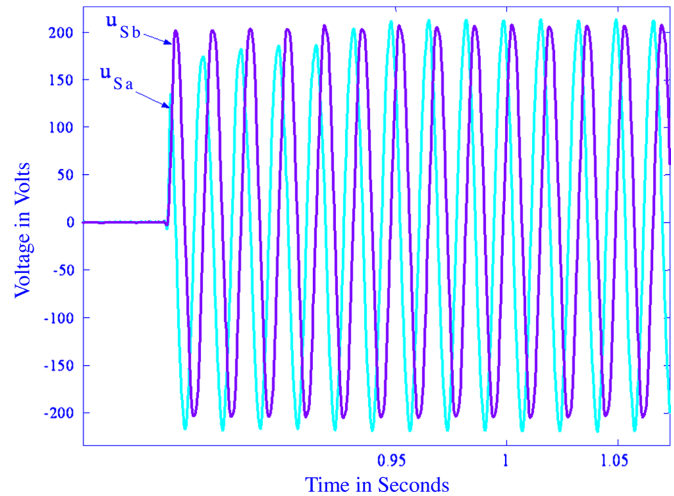

Fig. 1. Sampled two-phase equivalent voltages $\boldsymbol{u}_{Sa}, \boldsymbol{u}_{Sb}$.

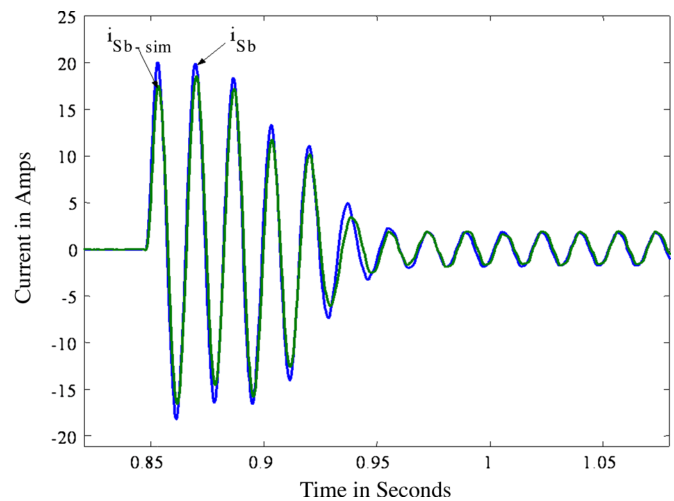

Fig. 2. Sampled phase ***b*** current $\boldsymbol{i}_{Sb}$ and its simulated response $\boldsymbol{i}_{Sb-sim}$.

V. Experimental Results

To demonstrate the viability of the speed sensorless estimator (18) for T_R, experiments were performed. A three-phase, 0.5 hp, 1735 rpm $(n_p = 2\ \text{pole}-\text{pair})$ induction motor was driven by an ALLEN-BRADLEY PWM inverter to obtain the data. Given a speed command to the inverter, it produces PWM voltages to drive the induction motor to the commanded speed. Here a step speed command was chosen to bring the motor from standstill up to the rated speed of 188 rad/s. The stator currents and voltages were sampled at 10 kHz so that the sample period is $T_s = 0.0001$ s. The real-time computing system RTLAB from OPAL-RT with a fully integrated hardware and software system was used to collect data [17]. Filtered differentiation (using digital filters) was used for the derivatives of the voltages and currents. Specifically, the signals were filtered with a third-order Butterworth filter whose cutoff frequency was 100 Hz. The voltages and currents were put through a 3-2 transformation to obtain the two-phase equivalent voltages u_{Sa}, u_{Sb}, which are plotted in Fig. 1 and with the corresponding two-phase equivalent currents i_{Sa}, i_{Sb} shown in Fig. 2.

Using the data $\{u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}\}$ collected between 0.84 and 0.91 sec ($T_b \triangleq 0.91 - 0.84 = 0.07$ sec is the batch data collection period), which includes the time the motor accelerates, the quantities du_{Sa}/dt, du_{Sb}/dt, di_{Sa}/dt, di_{Sb}/dt, d^2i_{Sa}/dt^2, d^2i_{Sb}/dt^2, d^3i_{Sa}/dt^3,

$d^3 i_{Sb}/dt^3$ are calculated and used to evaluate the coefficients C_i, $i = 1, 2, \cdots, 12$ in (18). Solving (18), one obtains the 12 solutions

$$\begin{aligned}
T_{R1} &= +0.1064\\
T_{R2} &= -0.0186\\
T_{R3} &= -0.0576 + j0.0593\\
T_{R4} &= -0.0576 - j0.0593\\
T_{R5} &= -0.0037 + j0.0166\\
T_{R6} &= -0.0037 - j0.0166\\
T_{R7} &= -0.0072 + j0.0103\\
T_{R8} &= -0.0072 - j0.0103\\
T_{R9} &= +0.0125 + j0.0077\\
T_{R10} &= +0.0125 - j0.0077\\
T_{R11} &= +0.0065 + j0.0018\\
T_{R12} &= +0.0065 - j0.0018.
\end{aligned}$$

T_R must be a real positive number, so $T_R = 0.1064$ is the only possible choice. This value compares favorably with the "cold" value of $T_R = 0.11$ obtained using the method of Wang *et al.* [18], [21] which required a speed sensor.

To illustrate the identified T_R, a simulation of the induction motor model was carried out using the measured voltages as input. The simulation's output [stator currents computed according to (1) and (2)] are used to compare with the measured (stator currents) outputs. Fig. 2 shows the sampled two-phase equivalent current i_{Sb} and its simulated response i_{Sb-sim}. (The phase a current i_{Sa} is similar, but shifted by $\pi/(2n_p)$.) The resulting phase b current i_{Sb-sim} from the simulation corresponds well with the actual measured current i_{Sb}. Note that in (1) the parameter $\gamma = (R_S/\sigma L_S) + (\beta M/T_R)$ also depends on T_R.

VI. Conclusion and Future Work

This note presented a algebraic approach to the estimation of the rotor time constant of an induction motor without using a speed sensor. The experimental results demonstrated the practical viability of this method. Though the method is not applicable in steady state, neither is a standard linear least-squares approach. Future work includes studying an on-line implementation of the estimation algorithm and using such an online estimate in a speed sensorless field-oriented controller.

Appendix
Steady-State Expressions

In the following, ω_S denotes the stator frequency and S denotes the normalized slip defined by $S \triangleq (\omega_S - n_p\omega)/\omega_S$. With $u_{Sa} + ju_{Sb} = \underline{U}_S e^{j\omega_S t}$ and $i_{Sa} + ji_{Sb} = \underline{I}_S e^{j\omega_S t}$, it is shown in [19] that under steady-state conditions, the complex phasors $\underline{U}_S$ and $\underline{I}_S$ are related by $(S_p \triangleq R_R/\sigma\omega_S L_R = 1/\sigma\omega_S T_R)$

$$\begin{aligned}
\underline{I}_S &= \frac{\underline{U}_S}{R_S + j\omega_S L_S\left(\left(1 + j\frac{S}{S_p}\right)/\left(1 + j\frac{S}{\sigma S_p}\right)\right)}\\
&= \frac{\underline{U}_S}{\left(R_S + \frac{(1-\sigma)S\omega_S^2 L_S T_R}{1+S^2\omega_S^2 T_R^2}\right) + j\frac{\omega_S L_S\left(1+\sigma S^2\omega_S^2 T_R^2\right)}{1+S^2\omega_S^2 T_R^2}}
\end{aligned}$$

and straightforward calculations (see [6], [15], and [20]) give

$$\begin{aligned}
P &\triangleq u_{Sa}i_{Sa} + u_{Sb}i_{Sb} = R_e\left(\underline{U}_S\underline{I}_S^*\right)\\
&= |\underline{I}_S|^2\left(R_S + \frac{(1-\sigma)S\omega_S^2 L_S T_R}{1+S^2\omega_S^2 T_R^2}\right) \qquad (30)\\
Q &\triangleq u_{Sb}i_{Sa} - u_{Sa}i_{Sb} = I_m\left(\underline{U}_S\underline{I}_S^*\right)\\
&= |\underline{I}_S|^2\frac{\omega_S L_S\left(1+\sigma S^2\omega_S^2 T_R^2\right)}{1+S^2\omega_S^2 T_R^2}. \qquad (31)
\end{aligned}$$

A. Steady-State Expressions for q_2, q_1, and q_0

The steady-state expressions for q_2, q_1, and q_0 are (see [6], [15], and [20])

$$q_2 = n_p^2 T_R^2 |\underline{I}_S|^4 \frac{\omega_S^2 L_S(1-\sigma)^2(1-S)}{\sigma\left(1+S^2\omega_S^2 T_R^2\right)} \qquad (32)$$

$$q_1 = n_p\omega_S|\underline{I}_S|^4\frac{L_S(1-\sigma)^2\left(1-\omega_S^2T_R^2(1-S)^2\right)}{\sigma\left(1+S^2\omega_S^2T_R^2\right)} \qquad (33)$$

$$q_0 = -|\underline{I}_S|^4\frac{\omega_S^2 L_S(1-\sigma)^2(1-S)}{\sigma\left(1+S^2\omega_S^2T_R^2\right)}. \qquad (34)$$

With $\omega \neq 0$ (equivalent to $S \neq 1$), it is seen that $q_2 \neq 0$. Conversely, $q_2 = 0$ if and only if $S = 1$ (i.e., $\omega = 0$). Also, if $\omega = 0$, then $S = 1$ and $q_1 \neq 0$.

B. Steady-State Expressions for a_2, a_1, a_0

The steady-state expressions for a_2, a_1, and a_0 are (see [6], [15], and [20])

$$a_2 = -n_p^2|\underline{I}_S|^4\frac{\omega_S(1-\sigma)^2}{\sigma^2\left(1+S^2\omega_S^2T_R^2\right)}\frac{1}{\mathrm{den}} \qquad (35)$$

$$a_1 = n_p|\underline{I}_S|^4\frac{2\omega_S^2(1-\sigma)^2(1-S)}{\sigma^2\left(1+S^2\omega_S^2T_R^2\right)}\frac{1}{\mathrm{den}} \qquad (36)$$

$$a_0 = -|\underline{I}_S|^4\frac{\omega_S^3(1-\sigma)^2(1-S)^2}{\sigma^2\left(1+S^2\omega_S^2T_R^2\right)}\frac{1}{\mathrm{den}} \qquad (37)$$

$$\mathrm{den} \triangleq n_pT_R|\underline{I}_S|^4\left(\left(\frac{(1-\sigma)}{\sigma T_R}\frac{1+S^2\omega_S^2T_R^2 - S\omega_S^2T_R^2}{1+S^2\omega_S^2T_R^2}\right)^2 + \left(\frac{(1-\sigma)}{\sigma}\frac{\omega_S}{1+S^2\omega_S^2T_R^2}\right)^2\right). \qquad (38)$$

Recall from Section III [following (6)] that $\mathrm{den} = 0$ if and only if $|\underline{\psi}_R| = 0$.

C. Steady-State Expressions for r_1 and r_0

It is now shown that the steady-state value of r_1 in (12) is nonzero. Substituting the steady-state values of q_2, q_1, q_0, a_2, a_1, and a_0 (noting that $\dot{q}_1 \equiv 0$ and $\dot{q}_2 \equiv 0$ in steady state) into (12) gives

$$\begin{aligned}
r_1 &= -|\underline{I}_S|^6\left(\frac{1}{1+S^2\omega_S^2T_R^2}\right)^3\frac{n_p^4(1-\sigma)^6L_S^2}{\sigma^4}\omega_S^3\\
&\quad\times\left(1+T_R^2\omega_S^2(1-S)^2\right)^2/\mathrm{den}\\
r_0 &= |\underline{I}_S|^6\left(\frac{1}{1+S^2\omega_S^2T_R^2}\right)^3\frac{n_p^3(1-\sigma)^6L_S^2}{\sigma^4}\omega_S^4(1-S)\\
&\quad\times\left(1+\omega_S^2T_R^2\times(1-S)^2\right)^2/\mathrm{den}
\end{aligned}$$

where den is given by (38). It is then seen that $r_1 \neq 0$ in steady state.

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Comments on "Optimizing Simultaneously Over the Numerator and Denominator Polynomials in the Youla-Kucera Parameterization"

Fikret A. Aliev and Vladimir B. Larin

***Abstract*—It is noted that the parameterization of the set of stabilizing regulators was first presented in a monograph by Larin V.B., Naumenko K.I., and Suntsev V.N.**

These comment were prompted by the recent note [1] which, in its historical survey of parameterization of feedback systems, has overlooked reference [2]. We use this opportunity to re-iterate the fact that [2] was the first known publication that presented the parameterization of the set of stabilizing regulators, definitely before [3], as also acknowledged in [4] (see, for instance, the comment to [4, ref [29]]). It appears that the Youla–Bongiorno parameterization was rediscovered a couple of years later, but most probably without any knowledge of [2]. A discussion on parameterization can also be found in [5].

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F. A. Aliev is with the Institute of Applied Mathematics, BSU, AZ 1148, Baku, Azerbaijan (e-mail: f_aliev@yahoo.com).
V. B. Larin is with the Department of Dynamics of Complex Systems, the Institute of Mechanics NAS of Ukraine, 03057 Kiev, Ukraine (e-mail: vblarin@hotmail.com).
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Author's Reply

Vladimir Kučera

We would like to thank F.A. Aliev and V.B. Larin for their comments on [7], namely for bringing to our attention apparently the first publication [1] that presented a parametrization of all stabilizing controllers for a given plant.

The parametrization is obtained in [1] in the context of solving a linear-quadratic control problem with stability, in the frequency domain, applying the Wiener–Hopf approach. The free parameter represents a function to be varied in order to minimize the cost while assuring stability of the closed-loop system for any plant, stable or unstable, minimum phase or nonminimum phase. The exposition of the subject is elegant and instructive, showing why the parameter should be a linear combination of specific closed-loop transfer functions.

The setting of the best-known publication [2] on the parametrization result is the same; just the construction of the free parameter is slightly different, making full use of polynomial matrix fractions.

Reference [3] approaches the feedback system stability directly, in an algebraic manner, without any appeal to an optimization problem, to show that the set of stabilizing controllers for a given plant corresponds to the solution set of a Bézout equation. Since the solution set can be parametrized, the explicit controller parametrization immediately follows [4].

The algebraic nature of the parametrization result was further emphasized in [5]. A survey of research directions advanced by this fundamental result is presented in [6].

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