

# Reduced Switching-Frequency Active Harmonic Elimination for Multilevel Converters

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**Abstract**—This paper presents a reduced switching-frequency active-harmonic-elimination method (RAHEM) to eliminate any number of specific order harmonics of multilevel converters. First, resultant theory is applied to transcendental equations to eliminate low-order harmonics and to determine switching angles for a fundamental frequency-switching scheme. Next, based on the number of harmonics to be eliminated, Newton climbing method is applied to transcendental equations to eliminate high-order harmonics and to determine switching angles for the fundamental frequency-switching scheme. Third, the magnitudes and phases of the residual lower order harmonics are computed, generated, and subtracted from the original voltage waveform to eliminate these low-order harmonics. Compared to the active-harmonic-elimination method (AHM), which generates square waves to cancel high-order harmonics, RAHEM has lower switching frequency. The simulation results show that the method can effectively eliminate all the specific harmonics, and a low total harmonic distortion (THD) near sine wave is produced. An experimental 11-level H-bridge multilevel converter with a field-programmable gate-array controller is employed to experimentally validate the method. The experimental results show that RAHEM does effectively eliminate any number of specific harmonics, and the output voltage waveform has low switching frequency and low THD.

**Index Terms**—Field-programmable gate-array (FPGA) controller, multilevel converter, reduced switching-frequency active harmonic elimination.

## I. INTRODUCTION

MULTILEVEL converters have received more and more attention because of their capability of high-voltage operation, high efficiency, and low electromagnetic interference [1]–[3]. The desired output of a cascaded multilevel converter is synthesized by several sources of dc voltages. With an increasing number of dc-voltage sources, the converter voltage output waveform approaches a nearly sinusoidal waveform while using a fundamental frequency-switching scheme. This

results in low switching losses, and because of several dc sources, the switches experience a lower  $dV/dt$ . As a result, the multilevel-converter technology is a promising technology for high-power electronic devices for utility applications [4], [5] such as flexible ac transmission devices. For these applications, the output voltage of the converters must meet maximum voltage and current total-harmonic-distortion (THD) limitations such as those specified in IEEE 519 [6]. Therefore, a method must be used to limit the harmonics produced by the converters.

Generally, different pulsewidth-modulation (PWM) methods such as sinusoidal-triangle PWM and space-vector PWM combined with different control technologies such as feedforward control are widely used [5]–[17]. But, they do not completely eliminate any number of high-order harmonics of the output voltage [18]–[23], and selective-harmonic-elimination method cannot guarantee THD required by applications [24]–[32]. To address the problem of having high-order harmonics at low-modulation indexes, the active-harmonic-elimination method (AHM) has been proposed [33], [34]. AHM uses a fundamental frequency-switching scheme in which the switching angles are determined using elimination theory [24], [25] to eliminate low-order harmonics. Then, the specifically chosen higher harmonics (e.g., the odd nontriplen harmonics) are eliminated by using an additional switching angle (one for each higher harmonic) to generate the negative of the harmonic to cancel it. But, AHM described in [33] and [34] has a disadvantage in that it uses a high switching frequency to eliminate higher order harmonics. A special case to use a low switching frequency to eliminate some harmonics is discussed in [35].

Due to the disadvantage of high switching frequency of AHM, a new reduced switching-frequency active-harmonic-elimination method (RAHEM) is proposed to eliminate any specific number of harmonics. First, resultant theory is applied to transcendental equations to eliminate low-order harmonics and to determine switching angles for the fundamental frequency-switching scheme (e.g., the 5th, 7th, 11th, and 13th). Next, based on the number of harmonics to be eliminated, Newton climbing method is applied to transcendental equations to eliminate high-order harmonics (the odd nontriplen harmonics, such as the 19th, 23rd, 25th, and 29th in the experiments) and to determine switching angles for the fundamental frequency-switching scheme. Third, the magnitudes and phases of the residual lower order harmonics are computed, generated, and subtracted from the original voltage waveform to eliminate these low-order harmonics. The experimental results show that

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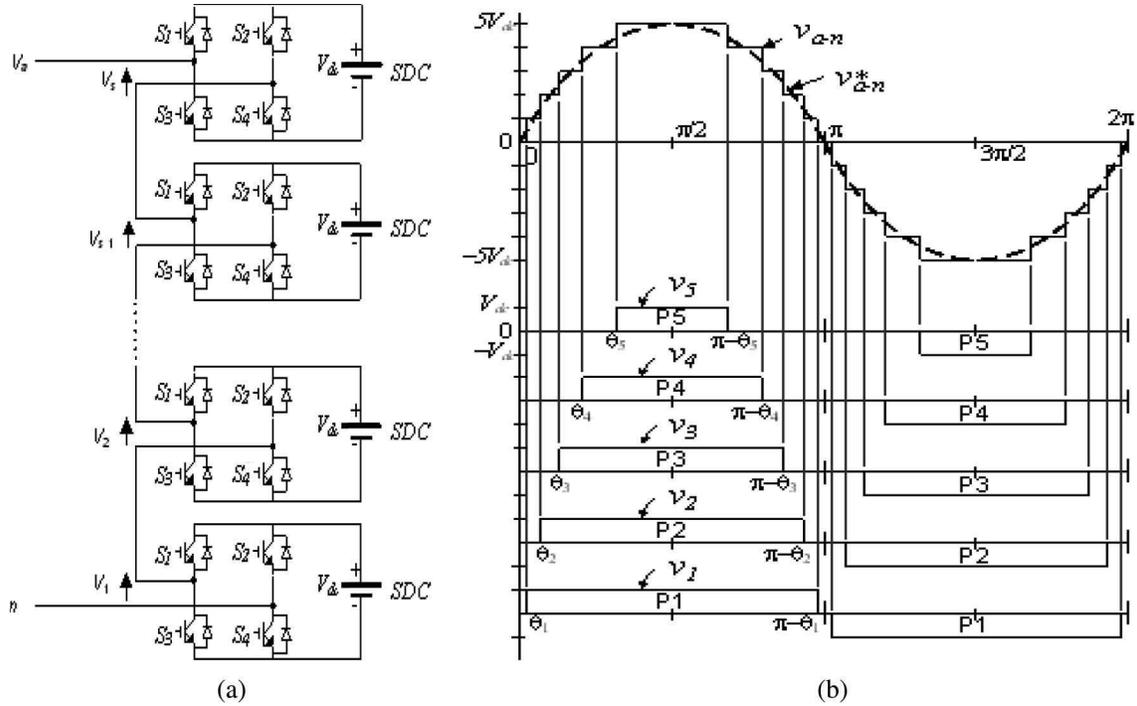


Fig. 1. (a) Topology of single-phase cascaded H-bridge multilevel converter. (b) Output waveform of multilevel converter using the fundamental frequency-switching scheme.

the method can effectively eliminate all the specific harmonics, and a low THD near sine wave is produced. Compared to AHEM in [33] and [34], RAHEM that is proposed in this paper has lower switching frequency and retains all the other advantages of AHEM.

## II. HARMONIC ELIMINATION FOR MULTILEVEL CONVERTER

### A. Switching-Angle Determination for Low-Order Harmonic Elimination

A cascaded H-bridge multilevel converter uses several dc sources to synthesize a sinusoidal waveform. Fig. 1(a) shows the topology of a single-phase cascaded H-bridge multilevel converter. The control of the multilevel converter is to choose a series of switching angles to synthesize a desired sinusoidal voltage waveform. The 11-level multilevel-converter output voltage waveform generated by the fundamental frequency-switching scheme is shown in Fig. 1(b). In Fig. 1(b),  $P_1, P_2, \dots, P_5$  are conduction periods of different H-bridges.

If the separate dc-source (SDCS) voltages for all the H-bridges are equal, which is defined as  $V_{dc}$  here, the Fourier series expansion of the output voltage waveform shown in Fig. 1(b) is

$$V(\omega t) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_{dc}}{n\pi} (\cos(n\theta_1) + \cos(n\theta_2) + \cos(n\theta_3) + \dots + \cos(n\theta_s)) \sin(n\omega t) \quad (1)$$

where  $s$  is the number of dc sources in a cascaded H-bridge multilevel converter. Ideally, given a desired fundamental

voltage  $V_1$ , one wants to determine the switching angles  $\theta_1, \dots, \theta_s$  so that  $V(\omega t) = V_1 \sin(\omega t)$ , and specific higher harmonics of  $V(n\omega t)$  are equal to zero. For a three-phase application, the triplen harmonics in each phase need not be cancelled as they automatically cancel in the line-to-line voltages. For example, in the case of  $s = 5$  dc sources, usually, the low-order 5th, 7th, 11th, and 13th harmonics can be cancelled.

The switching angles can be found by solving the following:

$$\begin{aligned} \cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4) \\ + \cos(\theta_5) &= m \\ \cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3) + \cos(5\theta_4) \\ + \cos(5\theta_5) &= 0 \\ \cos(7\theta_1) + \cos(7\theta_2) + \cos(7\theta_3) + \cos(7\theta_4) \\ + \cos(7\theta_5) &= 0 \\ \cos(11\theta_1) + \cos(11\theta_2) + \cos(11\theta_3) + \cos(11\theta_4) \\ + \cos(11\theta_5) &= 0 \\ \cos(13\theta_1) + \cos(13\theta_2) + \cos(13\theta_3) + \cos(13\theta_4) \\ + \cos(13\theta_5) &= 0 \end{aligned} \quad (2)$$

where the modulation index  $m$  is defined as  $m = \pi V_1 / (4V_{dc})$ .

These transcendental equations characterizing the harmonic content are converted into polynomial equations, and the resultant method is employed to find all their solutions when they exist [29], [30]. The 11-level solutions are shown in Fig. 2(a). The higher order harmonic voltages  $V_n$  are computed by (1),

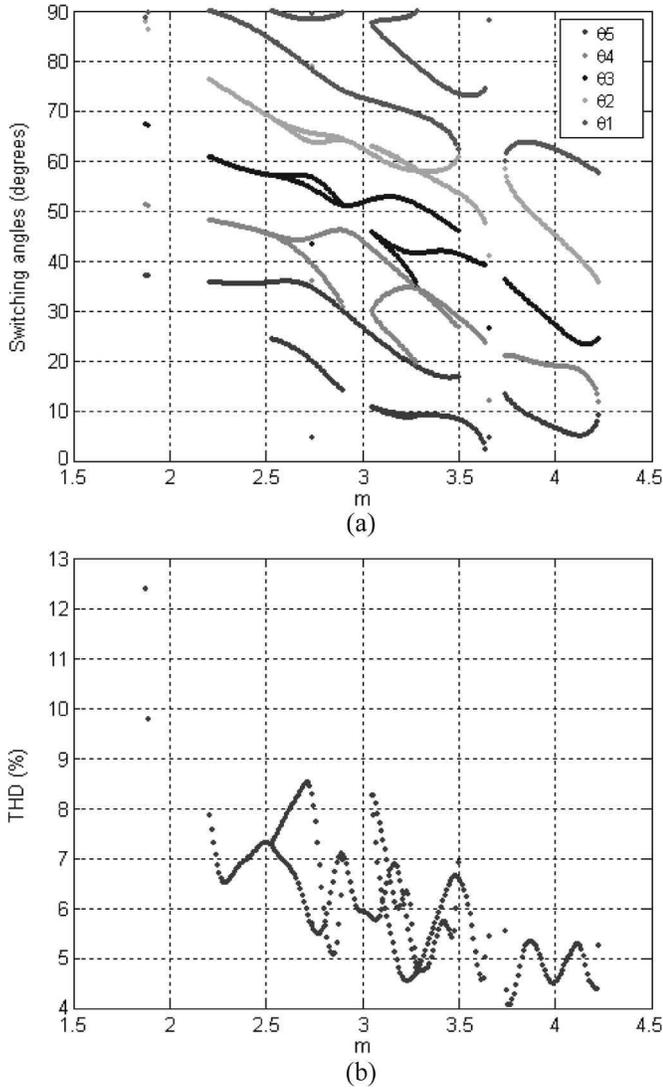


Fig. 2. (a) Solutions for switching angles versus  $m$ . (b) Corresponding THD versus  $m$ .

and the THD for the corresponding solution computed by (3) is shown in Fig. 2(b).

$$THD = \sqrt{\sum_{n=5,7,11,\dots}^{49} V_n^2 / V_1}. \quad (3)$$

**B. Newton Climbing Method for High-Order Harmonic Elimination**

If the fundamental frequency-switching scheme is used to eliminate any high-order harmonics (in this 11-level case, any other four  $h_1, h_2, h_3, h_4$  high-order harmonics such as 19th, 23rd, 29th, and 31st) instead of low-order harmonics (such as 5th, 7th, 11th, and 13th), then the transcendental equations that need to be solved are

$$\begin{aligned} &\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4) \\ &\quad + \cos(\theta_5) = m \\ &\cos(h_1\theta_1) + \cos(h_1\theta_2) + \cos(h_1\theta_3) + \cos(h_1\theta_4) \\ &\quad + \cos(h_1\theta_5) = 0 \end{aligned}$$

$$\begin{aligned} &\cos(h_2\theta_1) + \cos(h_2\theta_2) + \cos(h_2\theta_3) + \cos(h_2\theta_4) \\ &\quad + \cos(h_2\theta_5) = 0 \\ &\cos(h_3\theta_1) + \cos(h_3\theta_2) + \cos(h_3\theta_3) + \cos(h_3\theta_4) \\ &\quad + \cos(h_3\theta_5) = 0 \\ &\cos(h_4\theta_1) + \cos(h_4\theta_2) + \cos(h_4\theta_3) + \cos(h_4\theta_4) \\ &\quad + \cos(h_4\theta_5) = 0. \end{aligned} \quad (4)$$

As the order of the harmonics increase, the degrees of the polynomials in the harmonic equations dramatically increase and one reaches the limitations of the capability of contemporary computer algebra software tools (e.g., Mathematica or Maple) to solve the system of polynomial equations by using elimination theory [30]. It is difficult to solve (4) by the resultant method for this reason. To conquer this problem, the fundamental frequency-switching angle computation of (4) is solved by the Newton climbing method whose initial guess is obtained from the solutions of (2).

The Newton iterative method for (4) computation is

$$x_{n+1} = x_n - J^{-1}f \quad (5)$$

where  $x_{n+1}$  is the new value and  $x_n$  is the old value.  $J$  is the Jacobian matrix for the transcendental equations, and  $f$  is the set of transcendental functions.

$$f = \begin{bmatrix} \sum_{n=1}^5 \cos(\theta_n) \\ \sum_{n=1}^5 \cos(h_1\theta_n) \\ \sum_{n=1}^5 \cos(h_2\theta_n) \\ \sum_{n=1}^5 \cos(h_3\theta_n) \\ \sum_{n=1}^5 \cos(h_4\theta_n) \end{bmatrix}. \quad (6)$$

The Jacobian matrix is expressed in (7), shown at the bottom of the next page.

By using the proposed Newton climbing method, the solution for (4) can be found.

**C. Low-Order Harmonic Elimination**

The voltage content in (1) has the following four parts: fundamental frequency voltage, triplen harmonic voltages, low-order harmonic voltages (such as 5th, 7th, 11th, 13th, and 17th), and high-order harmonic voltages (such as 19th, 23rd, 29th, 31st, and above). Assuming that the application is a balanced three-phase system, the triplen harmonics need not be eliminated, because these harmonics cancel in the line-line voltage automatically. As part of the high-order harmonics have been eliminated by the fundamental frequency-switching scheme, here, a quasi-square wave with a precalculated fundamental frequency and magnitude (determined by duty ratio) equal to those of the harmonic that needs to be eliminated is subtracted. In Fig. 3, an example of fifth harmonic elimination is shown. The results will be compared to AHEM.

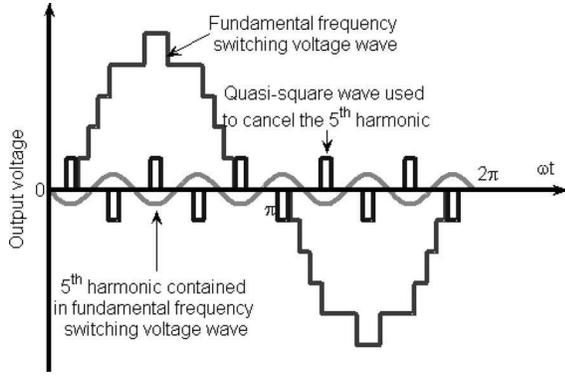


Fig. 3. Fifth harmonic elimination.

To eliminate the low-order nontriplen harmonics, a square wave is generated (one for each of these harmonics) whose fundamental is equal to the opposite of the harmonic that is to be eliminated, as it is done in AHM [33], [34]. For example, the seventh harmonic content is

$$V_7(t) = \frac{4V_{dc}}{7\pi} [\cos(7\theta_1) + \cos(7\theta_2) + \dots + \cos(7\theta_s)] \sin(7\omega t). \quad (8)$$

To eliminate the seventh harmonic (let  $h = 7$ ), a square wave whose Fourier series expansion is

$$V_{k_1}(t) = - \sum_{q=1,3,5,7,\dots} \frac{4V_{dc}}{q\pi} [\cos(qh\theta_1) + \cos(qh\theta_2) + \dots + \cos(qh\theta_s)] \sin(qh\omega t) \quad (9)$$

is generated. The  $q = 1$  and  $h = 7$  term of (9) cancels the seventh harmonic of (8). Here, as the waveform injected into the converter is a square wave, the square wave contains not only the fundamental frequency content which is used to cancel the specified harmonic but also contains higher order harmonics (such as fifth, seventh, . . . , etc.) of the fundamental frequency of the injected square wave. Because the fundamental frequency of the injected square wave is the frequency of the harmonic that will be cancelled in the output voltage, the orders of actual harmonics generated by the injected square wave are products of the harmonic order of the injected square wave and these nontriplen numbers. One example to cancel the fifth harmonic is shown in Fig. 3.

In Fig. 3, the harmonic to be cancelled is a sinusoidal waveform, and the generated waveform to cancel it is a square waveform. The difference between the harmonic to be cancelled and the square wave is higher order harmonics. For example, in Fig. 3, the next harmonic of concern that is produced by (9) is at  $5 \times 7 = 35$ . This harmonic and higher ones ( $7 \times 11$ , etc.)

are easy to filter using a low-pass filter. Repeating the earlier procedure, the 5th, 11th, . . . , 25th harmonics can all be eliminated. The net effect of this method is to remove the low-order harmonics at the expense of increasing the switching frequency when new harmonics are eliminated. This method is referred to as RAHEM, because in this method, a square wave is used to cancel low-order harmonics instead of high-order harmonics. As low-order harmonic elimination needs lower number of switchings than that of high-order harmonic elimination, the total switching frequency for RAHEM will be lower than that of AHM.

By using RAHEM, any specific number of harmonics can be eliminated. Here, the cases that will eliminate harmonics up to 17th, 25th, and 31st are discussed.

### III. REDUCED SWITCHING-FREQUENCY ACTIVE HARMONIC ELIMINATION FOR 11-LEVEL MULTILEVEL CONVERTER

#### A. Harmonic Elimination Up to 17th

For the harmonic-elimination requirements, the fundamental frequency-switching scheme will be used to eliminate the 7th, 11th, 13th, and 17th harmonics, and negative square waves will be used to eliminate the fifth harmonic. The equation of the fundamental frequency-switching scheme can be

$$\begin{aligned} & \cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4) \\ & + \cos(\theta_5) = m \\ & \cos(7\theta_1) + \cos(7\theta_2) + \cos(7\theta_3) + \cos(7\theta_4) \\ & + \cos(7\theta_5) = 0 \\ & \cos(11\theta_1) + \cos(11\theta_2) + \cos(11\theta_3) + \cos(11\theta_4) \\ & + \cos(11\theta_5) = 0 \\ & \cos(13\theta_1) + \cos(13\theta_2) + \cos(13\theta_3) + \cos(13\theta_4) \\ & + \cos(13\theta_5) = 0 \\ & \cos(17\theta_1) + \cos(17\theta_2) + \cos(17\theta_3) + \cos(17\theta_4) \\ & + \cos(17\theta_5) = 0. \end{aligned} \quad (10)$$

Fig. 4 shows the switching-angle solutions for (10) to eliminate the 7th, 11th, 13th, and 17th harmonics. It is shown in the figure that, for some modulation-index ranges, there are several solution sets; and for some modulation-index ranges, there is only one solution set. This is similar to the solution of the fundamental frequency-switching scheme to eliminate the 5th, 7th, 11th, and 13th harmonics. Fig. 5(a) shows the lowest THD for RAHEM and AHM, and Fig. 5(b) shows

$$J = \begin{bmatrix} \sin(\theta_1) & \sin(\theta_2) & \sin(\theta_3) & \sin(\theta_4) & \sin(\theta_5) \\ \sin(h_1\theta_1) & \sin(h_1\theta_2) & \sin(h_1\theta_3) & \sin(h_1\theta_4) & \sin(h_1\theta_5) \\ \sin(h_2\theta_1) & \sin(h_2\theta_2) & \sin(h_2\theta_3) & \sin(h_2\theta_4) & \sin(h_2\theta_5) \\ \sin(h_3\theta_1) & \sin(h_3\theta_2) & \sin(h_3\theta_3) & \sin(h_3\theta_4) & \sin(h_3\theta_5) \\ \sin(h_4\theta_1) & \sin(h_4\theta_2) & \sin(h_4\theta_3) & \sin(h_4\theta_4) & \sin(h_4\theta_5) \end{bmatrix} \quad (7)$$

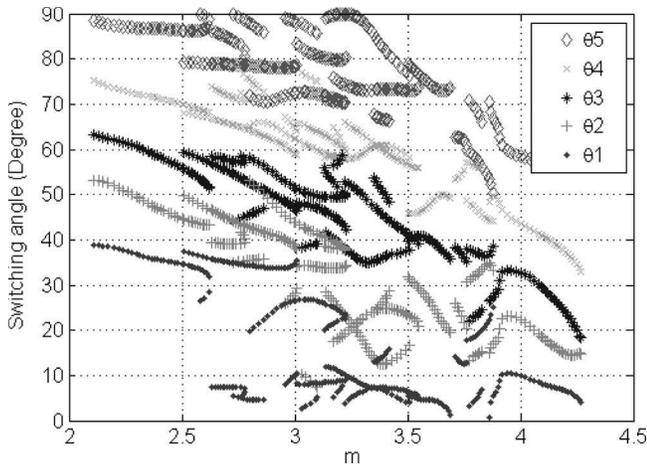


Fig. 4. Switching angles for harmonic elimination up to 17th.

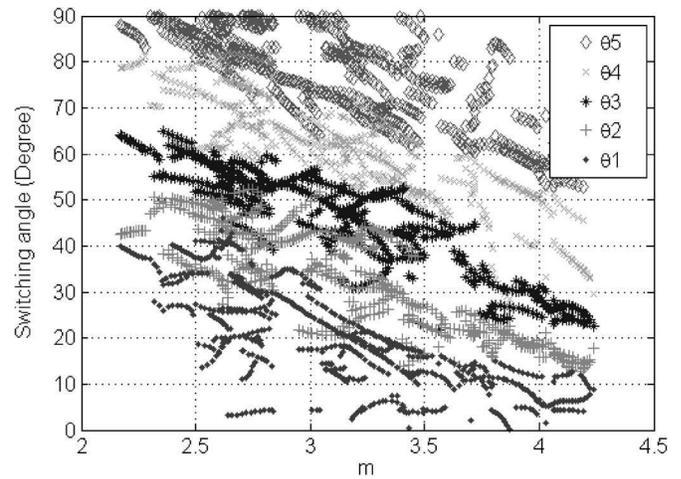


Fig. 6. Switching angles for harmonic elimination up to 25th.

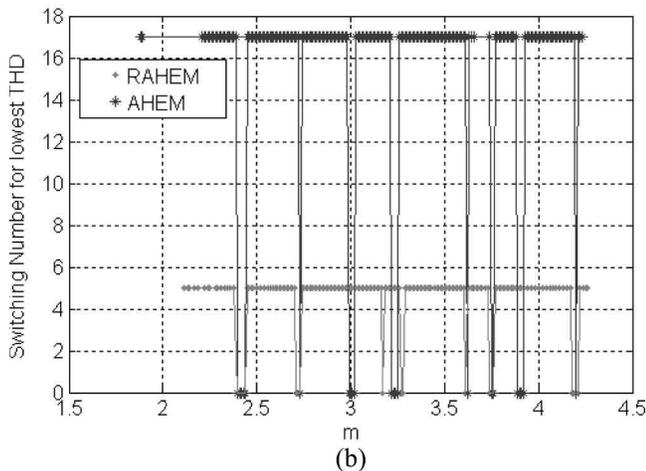
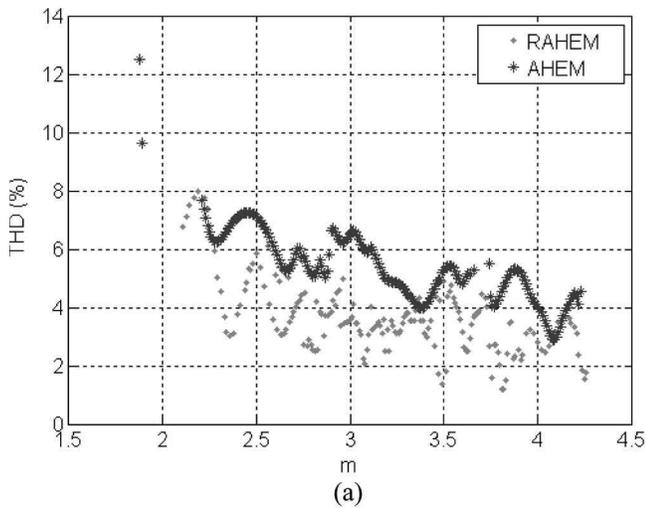


Fig. 5. Harmonic elimination up to 17th. (a) Lowest THD for RAHEM and AHEM. (b) Switching number in a cycle corresponding to the lowest THD for RAHEM and AHEM.

the switching numbers corresponding to the lowest THDs for RAHEM and AHEM. From the figure, it is shown that the lowest THD of RAHEM is lower than that of AHEM for most of the modulation-index range. The upper bound switching number for RAHEM is only 5, and it is 17 for AHEM.

### B. Harmonic Elimination Up to 25th

For the harmonic-elimination requirements, the fundamental frequency-switching scheme will be used to eliminate the 5th, 19th, 23rd, and 25th harmonics, and negative square waves will be used to eliminate the 7th, 11th, 13th, and 17th harmonics. Here, the 5th harmonic needs to be eliminated by the fundamental frequency-switching scheme instead of the 17th harmonic, because if negative square waves are used to eliminate the 5th harmonic, it will generate a new 25th harmonic. Therefore, the 5th and 25th harmonics must be tied together for elimination. They need to be eliminated by a fundamental switching scheme or negative square waves. Here, the fundamental frequency-switching scheme is used to eliminate both the 5th and 25th harmonics, and the equation of the fundamental frequency-switching scheme can be

$$\begin{aligned}
 &\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4) \\
 &\quad + \cos(\theta_5) = m \\
 &\cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3) + \cos(5\theta_4) \\
 &\quad + \cos(5\theta_5) = 0 \\
 &\cos(19\theta_1) + \cos(19\theta_2) + \cos(19\theta_3) + \cos(19\theta_4) \\
 &\quad + \cos(19\theta_5) = 0 \\
 &\cos(23\theta_1) + \cos(23\theta_2) + \cos(23\theta_3) + \cos(23\theta_4) \\
 &\quad + \cos(23\theta_5) = 0 \\
 &\cos(25\theta_1) + \cos(25\theta_2) + \cos(25\theta_3) + \cos(25\theta_4) \\
 &\quad + \cos(25\theta_5) = 0.
 \end{aligned} \tag{11}$$

Fig. 6 shows the switching-angle solutions for (11) to eliminate the 5th, 19th, 23rd, and 25th harmonics. Again, the solution distribution is similar to that of the fundamental frequency-switching scheme to eliminate the 5th, 7th, 11th, and 13th harmonics.

Fig. 7(a) shows the lowest THD for RAHEM and AHEM, and Fig. 7(b) shows the switching numbers corresponding to the lowest THDs for RAHEM and AHEM. From the figure,

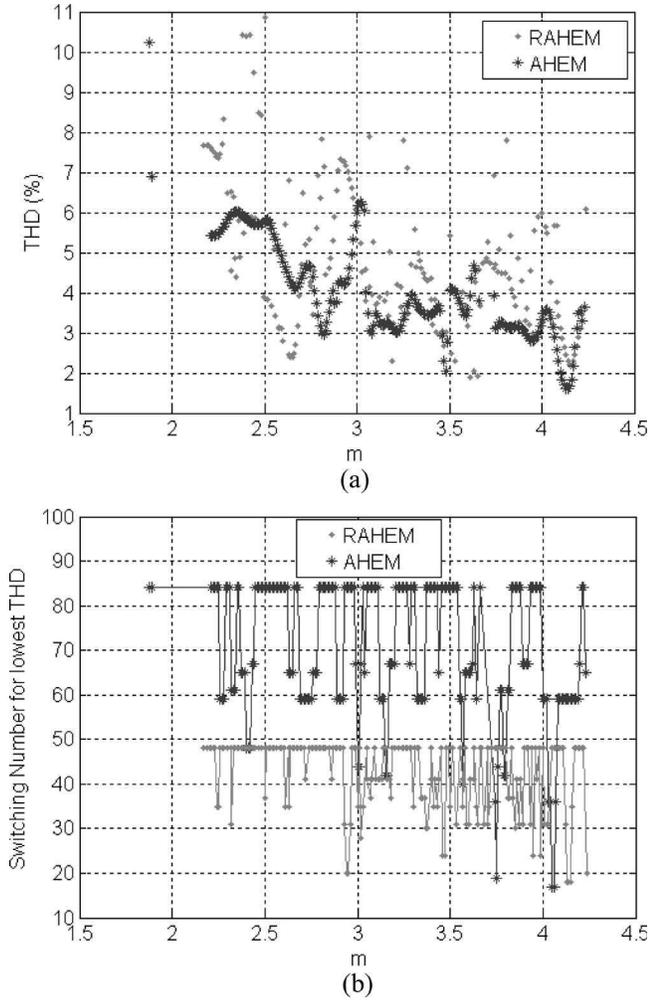


Fig. 7. Harmonic elimination up to 25th. (a) Lowest THD for RAHEM and AHEM. (b) Switching number in a cycle corresponding to the lowest THD for RAHEM and AHEM.

it is shown that the lowest THD of RAHEM is a little higher than that of AHEM for most of the modulation-index range. The upper bound switching number for RAHEM is 48, and it is 84 for AHEM.

### C. Harmonic Elimination Up to 31st

The fundamental frequency-switching scheme is used to eliminate the 19th, 23rd, 29th, and 31st harmonics, and negative square waves are used to eliminate the 5th, 7th, 11th, 13th, 17th, and 25th harmonics. The equation of the fundamental frequency-switching scheme can be

$$\begin{aligned}
 &\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4) \\
 &\quad + \cos(\theta_5) = m \\
 &\cos(19\theta_1) + \cos(19\theta_2) + \cos(19\theta_3) + \cos(19\theta_4) \\
 &\quad + \cos(19\theta_5) = 0 \\
 &\cos(23\theta_1) + \cos(23\theta_2) + \cos(23\theta_3) + \cos(23\theta_4) \\
 &\quad + \cos(23\theta_5) = 0
 \end{aligned}$$

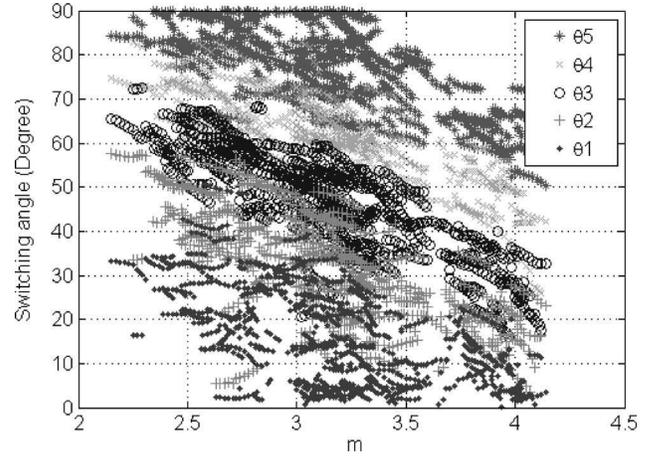


Fig. 8. Switching angles for harmonic elimination up to 31st.

$$\begin{aligned}
 &\cos(29\theta_1) + \cos(29\theta_2) + \cos(29\theta_3) + \cos(29\theta_4) \\
 &\quad + \cos(29\theta_5) = 0 \\
 &\cos(31\theta_1) + \cos(31\theta_2) + \cos(31\theta_3) + \cos(31\theta_4) \\
 &\quad + \cos(31\theta_5) = 0.
 \end{aligned} \tag{12}$$

The switching-angle solution is shown in Fig. 8. Fig. 9(a) shows the lowest THD for RAHEM and AHEM, and Fig. 9(b) shows the switching numbers corresponding to the lowest THDs for RAHEM and AHEM. From the figure, it is shown that the lowest THD of RAHEM is a little higher than that of AHEM for most of the modulation-index range. But, the upper bound switching number for RAHEM is 78, and it is 144 for AHEM.

From the cases of an 11-level multilevel converter to eliminate harmonics up to 17th, 19th, 23rd, 25th, 29th, and 31st, it can be concluded that the lowest THDs for RAHEM and AHEM for all the cases are similar for much of the modulation-index range. However, the switching numbers for RAHEM are much lower than that of AHEM. Usually, the switching numbers of RAHEM are only half of that of the corresponding AHEM.

For practical applications, the lookup table should be as small as possible to achieve high dynamic performance. In the proposed method, the size of the lookup table can be computed as  $(m_{\max}/0.01) \times 2 \times L$  bytes (here,  $L$  is the number of H-bridges, and 0.01 is the modulation-index control resolution). For example, for an 11-level multilevel inverter, there are five H-bridges for each phase ( $L = 5$ ). Therefore, the lookup table size is around 5000 B, and a very small memory chip can hold all the switching-angle data. Therefore, such a small lookup table will be very helpful for the system to achieve high dynamic transient performance. Because low-order voltage harmonics have been removed by harmonic elimination, the system's dynamic performance will be comparable to other modulation strategies, which have much higher switching frequencies.

Another issue for the cascaded H-bridge multilevel inverter is uneven-load power sharing among different dc sources. In the proposed method, this can be fixed by rotating the switching angles among all the H-bridges every half cycle or every

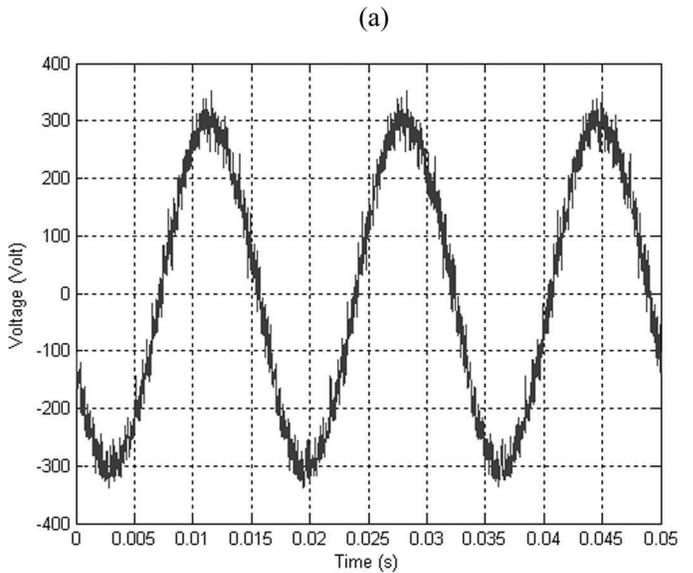
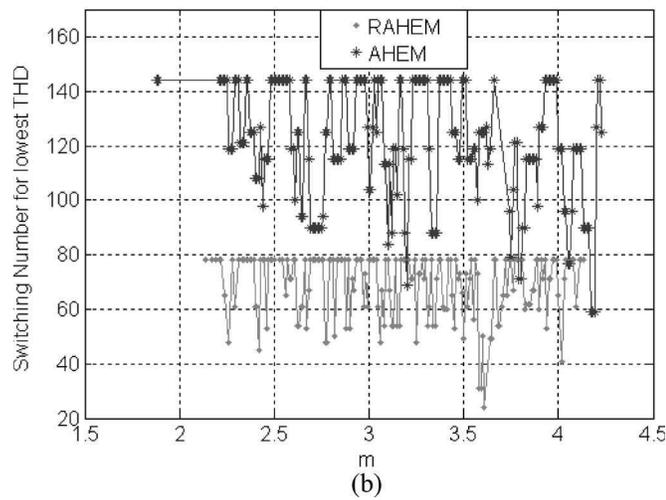
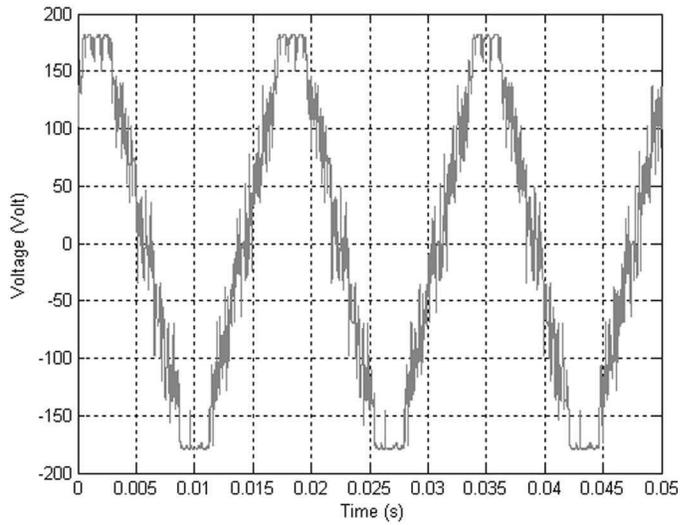
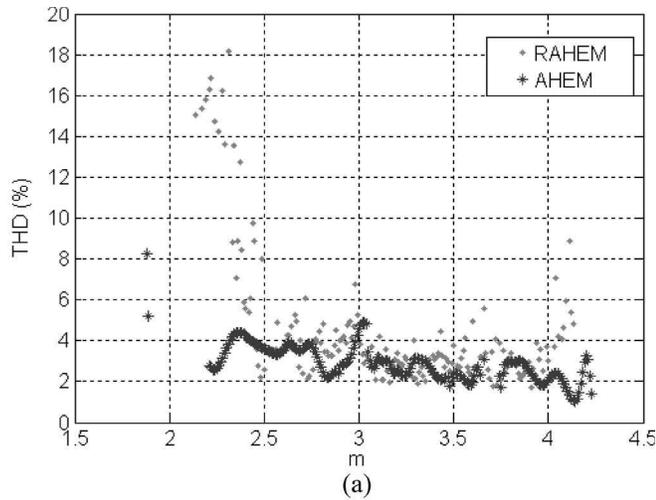


Fig. 9. Harmonic elimination up to 31st. (a) Lowest THD for RAHEM and AHEM. (b) Switching number in a cycle corresponding to the lowest THD for RAHEM and AHEM.

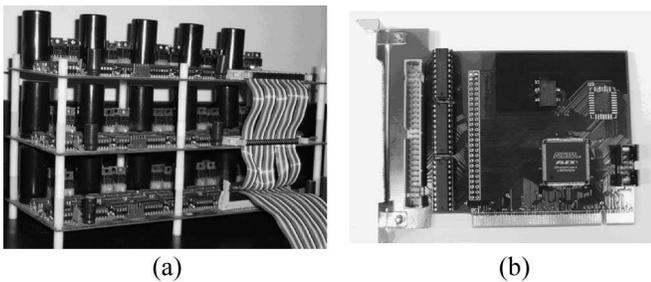


Fig. 10. (a) 10-kW multilevel converter. (b) FPGA controller for multilevel converter.

cycle. It is simple and effective to balance uneven load among different dc sources [3].

RAHEM can be used for most any multilevel-converter-based power-electronics application. One promising application is for cascaded H-bridge multilevel-converter-based static Var compensation (STATCOM). This scheme can easily meet IEEE 519 [36] harmonic standards for grid connection and reduce the filter cost. Therefore, the whole system performance can be increased.

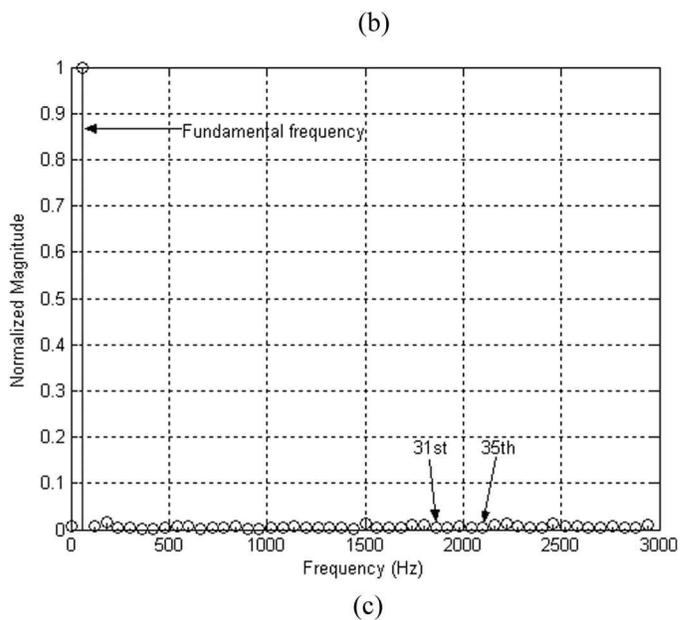


Fig. 11. (a) Experimental multilevel-converter phase voltage for AHEM to eliminate harmonics up to 31st ( $m = 3.78$ ). (b) Line-line voltage. (c) Normalized FFT analysis of line-line voltage shown in (b) (THD = 3.06%).

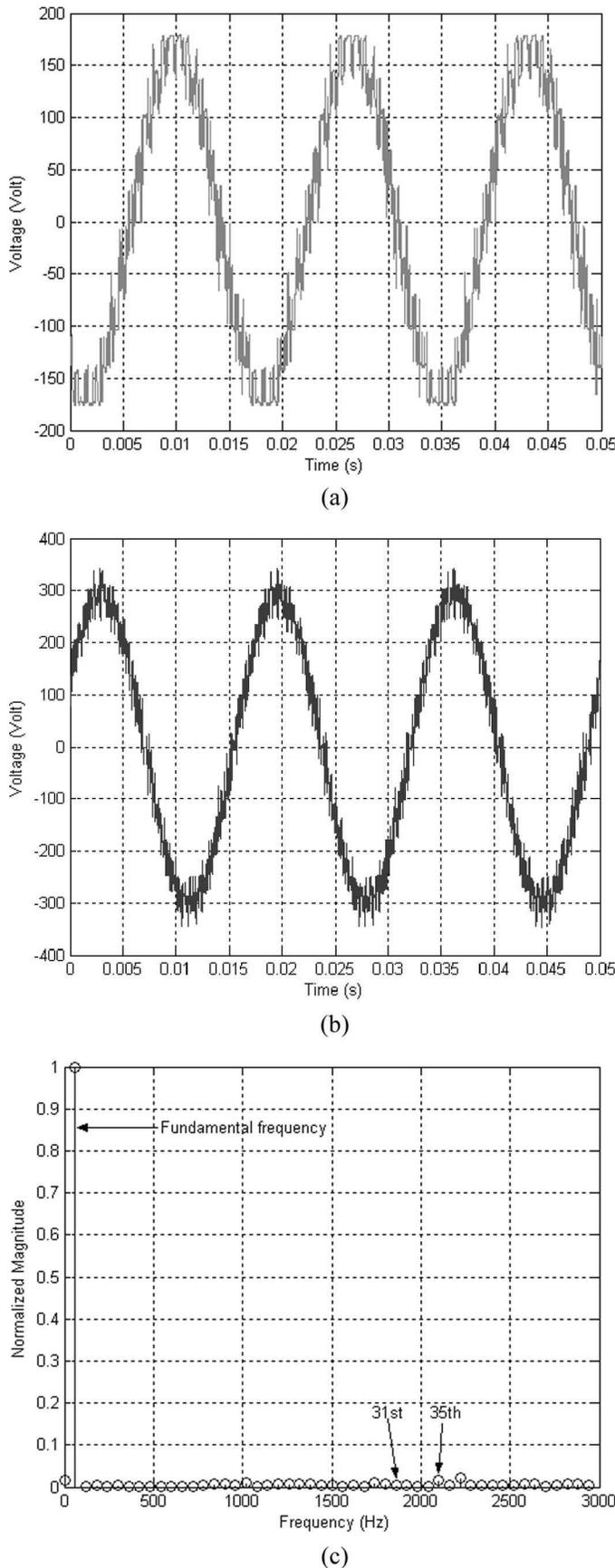


Fig. 12. (a) Experimental multilevel phase voltage of RAHEM to eliminate harmonics up to 31st ( $m = 3.78$ ). (b) Line-line voltage. (c) Normalized FFT analysis of line-line voltage shown in (b) (THD = 3.52%).

#### IV. EXPERIMENTAL VERIFICATION

To experimentally validate the proposed algorithm, a prototype three-phase 11-level cascaded H-bridge multilevel inverter has been built using 60-V 70-A MOSFETs as the switching devices, which is shown in Fig. 10(a). A battery bank of 15 SDCSs of 36 V each feed the inverter (five SDCSs per phase). A real-time controller based on Altera FLEX 10-K field-programmable gate array (FPGA) is used to implement the algorithm with 8- $\mu$ s control resolution. For convenience of operation, the FPGA controller was designed as a card to be plugged into a personal computer, which used a peripheral-component-interconnect (PCI) bus to communicate with the microcomputer. The FPGA controller board based on a PCI bus is shown in Fig. 10(b).

The  $m = 3.78$  and harmonic elimination up to 31st case was chosen for comparison between RAHEM and AHM to implement with the multilevel converter. Fig. 11 shows the experimental phase voltage and line-line voltage for AHM, and Fig. 11(c) shows the corresponding normalized fast Fourier transform (FFT) analysis of the line-line voltage. Fig. 12 shows the experimental phase and line-line voltage for RAHEM, and Fig. 12(c) shows the corresponding normalized FFT analysis for the line-line voltage.

From Figs. 11 and 12, it is shown that the harmonics have been eliminated up to 31st for both AHM and RAHEM. Their experimental THD are 3.06% and 3.52%, and this corresponds well with the theoretical computation of 3% and 2.75%. The switching number is 78 for RAHEM but 121 for AHM.

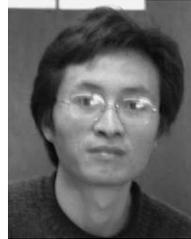
#### V. CONCLUSION

A RAHEM has been proposed and developed to eliminate any number of specific harmonics for multilevel converters. It can be derived from the computational results that this method can reduce the switching frequency and achieve similar THD to AHM. The experiments validated that the proposed method can eliminate all the specified harmonics, and the switching frequency is dramatically decreased.

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