Desk Copy

ECE 300
Spring Semester, 2004
HW Set #6

March 2, 2004
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Name Green
Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. <u>Be sure to show how you got your answers.</u>** Each problem counts 10 points.

6.57 easy problem: (c) $v_1(t) = [k_1e^{-t}\cos 2t + k_2e^{-t}\sin 2t]$ V u(t)

6.61 $v(t) = [10e^{-4t}\cos 2t - 40e^{-t}\sin 2t\,]$ V u(t)

6.63 $v(t) = [12 - 13.75e^{-1127t} + 1.75e^{8873t}]$ V u(t)

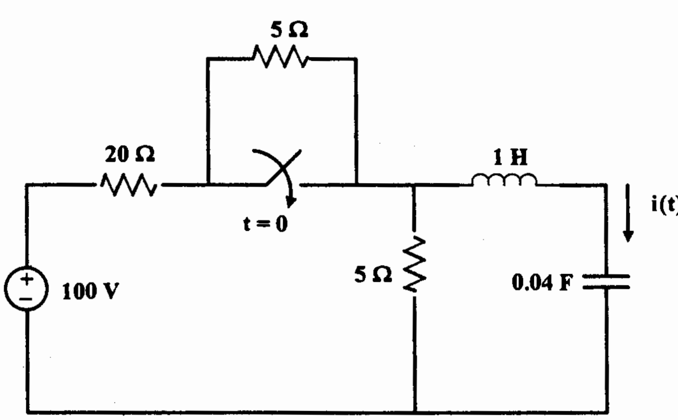
6.64 $v_o(t) = [16.67[e^{-200000\,t} - e^{-800000\,t}]]$ V u(t)

6.67 $I(t) = (4.57e^{-8t} - 0.57e^{-t})$ A u(t)

6.71 Characteristic equation: $s^2 + 4x10^6s + 3x10^{12} = 0$

$v_o(t) = 19.2[e^{-1000000\,t} - e^{-3000000\,t}]$ V u(t)

6x1 Find i(t) for the circuit below. Answer: $i(t) = 0.727e^{-2t}\sin(4.58t)$ A u(t)



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6.57 V(t) of a network is given as

$$\frac{d^2 V_1(t)}{dt^2} + 2\frac{dV_1(t)}{dt} + 5V_1(t) = 0$$

(a) Characteristic equation:

$$s^2 + 2s + 5 = 0$$

(b) the circuit natural frequencies.

(only one natural frequency)

Compare:

$$s^2 + 2s + 5 = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

ω_n is the natural frequency.

$$\omega_n^2 = 5$$

$$\boxed{\omega_n = \sqrt{5} = 2.24 \text{ rad/sec}}$$

$$\xi = \frac{2}{2\omega_n} = 0.45$$

(c) the expression for $V_1(t)$.

Since the equation does not have a forcing function we need only $x_c(t)$. First write the root of the C.E.

$$s = -1 \pm j2$$

$$V_1(t) = e^{-\xi\omega_n t}\left(A\cos(\omega_n\sqrt{1-\xi^2})t + B\sin(\omega_n\sqrt{1-\xi^2})t\right)$$

$$V_1(t) = e^{-t}\left[A\cos(2t) + B\sin(2t)\right] \; V \; u(t)$$

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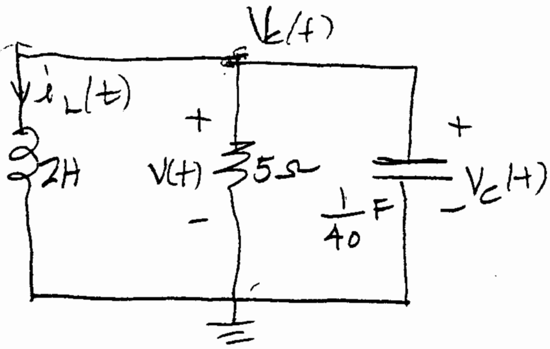
6.61 For the following circuit $i_L(0^-) = 1$ A, $V_c(0^-) = 10$ V. Determine $v(t)$.



$$\frac{V_c(t)}{R} + C\frac{dV_c}{dt} + i_L(t) = 0 \qquad (1)$$

$$\frac{dV_c(0^+)}{dt} = -\frac{i_L(0^+)}{C} - \frac{V_c(0^+)}{RC} = -40 - 80 = -120$$

so: $V_c(0^+) = 10$ V, $\dot{V}_c(0^+) = -120$ V/sec

with

$$i_L = \frac{1}{L}\int_0^t V_c(t)\,dt + i_L(0^+)$$

substituting into (1)

$$\frac{V_c(t)}{R} + C\frac{dV_c}{dt} + \frac{1}{L}\int_0^t V_c(t)\,dt + 1 = 0 \qquad (2)$$

Take the derivative of (2)

$$C\frac{d^2V_c}{dt^2} + \frac{1}{R}\frac{dV_c}{dt} + \frac{V_c(t)}{L} = 0$$

$$\frac{d^2V_c}{dt^2} + \frac{1}{RC}\frac{dV_c}{dt} + \frac{V_c(t)}{L} = 0 \qquad (3)$$

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6.61 continued

Putting parameter values into (3) gives;

$$\frac{d^2 V_c(t)}{dt^2} + 8\frac{dV_c(t)}{dt} + 20V_c(t) = 0$$

compare with standard form;

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s^2 + 8s + 20 = 0$$

$$\omega_n = \sqrt{20} \doteq 4.47 \text{ rad/sec}$$

$$2\xi\omega_n = 8 \qquad \xi\omega_n = 4$$

$$\xi = \frac{4}{\omega_n} = \frac{4}{\sqrt{20}} = 0.894 \text{ (underdamped)}$$

$$V_c(t) = e^{-\xi\omega_n t}\left[K_1\cos(\omega_d t) + K_2\sin(\omega_d t)\right]$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{20}\left(\sqrt{1-0.894^2}\right.$$

$$\omega_d = 2.0 \text{ rad/sec}$$

$$V_c(t) = V(t) = e^{-4t}\left[K_1\cos(2t) + K_2\sin(2t)\right] \qquad (4)$$

$$V(0^+) = 10V = K_1$$

$$\frac{dV(0^+)}{dt} = -200 = e^{-4t}\left[-2K_1\sin(2t) + 2K_2\cos(2t)\right] \qquad (5)$$

$$-4e^{-4t}\left[K_1\cos(2t) + K_2\sin(2t)\right]$$

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6.61 continued

Evaluate (5) at $t=0$

$$-120 = 2K_2 - 4K_1 \qquad (6)$$

$$-2K_1 + K_2 = -60$$

and $K_1 = 10$

$$K_2 = -60 + 20 = -40$$

$$\boxed{V(t) = e^{-4t}\left[10\cos(2t) - 40\sin(2t)\right]}$$

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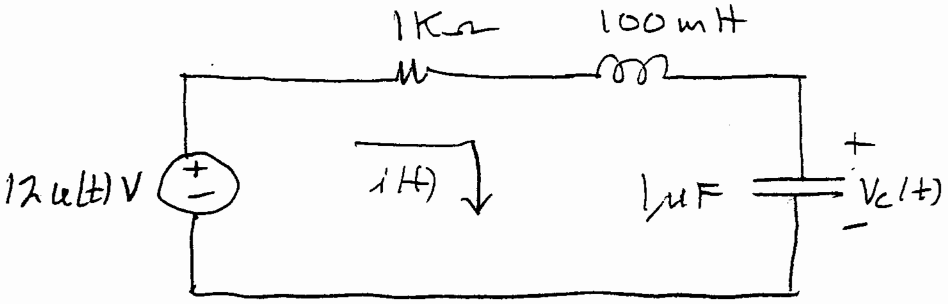
6.63. Find $V_c(t)$ for $t > 0$; $V_c(0^-) = 0 = V_c(0^+)$

$i_L(0^-) = i_L(0^+) = 0$



$$R\,i(t) + L\frac{di}{dt} + V_c(t) = 12 \qquad (1)$$

but

$$i(t) = C\frac{dV_c}{dt} \qquad (2)$$

(2) into (1);

$$RC\frac{dV_c}{dt} + LC\frac{d^2V_c}{dt^2} + V_c(t) = 0$$

$$\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{V_c(t)}{LC} = \frac{12}{LC} \qquad (3)$$

$$\frac{d^2V_c}{dt^2} + 10{,}000\frac{dV_c}{dt} + 10{,}000{,}000\,V_c(t) = 120{,}000{,}000$$

$V_c = V_f + V_n$

$V_f = 12\,V$

Char. Eq.

$$s^2 + 10{,}000s + 10{,}000{,}000 = 0$$

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6.63 continued

Roots of characteristic equation are:

$$\left.\begin{aligned} s_1 &= -1127 \\ s_2 &= -8873 \end{aligned}\right\} \text{overdamped}$$

$$v_c(t) = \left[12 + K_1 e^{-1127t} + K_2 e^{-8873t}\right] u(t)\ \text{V} \qquad (4)$$

$\boxed{V_c(0^+) = 0}$

and from (2), with $i(0^+) = 0 \rightarrow \boxed{\dot{V}_c(0^+) = 0}$

Back to (4), evaluated at $t = 0$:

$$0 = 12 + K_1 + K_2$$

$$\boxed{K_1 + K_2 = -12}$$

Take the derivative of (4)

$$\dot{V}_c(t) = -1127 K_1 e^{-t} - 8873 e^{-t} \Big|_{t=0}$$

$$0 = -1127 K_1 - 8873 K_2$$

$$\begin{bmatrix} 1 & 1 \\ -1127 & -8873 \end{bmatrix} \begin{bmatrix} \ \\ \ \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \end{bmatrix} \qquad \begin{aligned} K_1 &= -13.8 \\ K_2 &= 1.75 \end{aligned}$$

$\therefore$ Back to (4)

$$\boxed{V_c(t) = \left[12 - 13.8 e^{-1127t} + 1.75 e^{-8873t}\right] u(t)\ V}$$

QED

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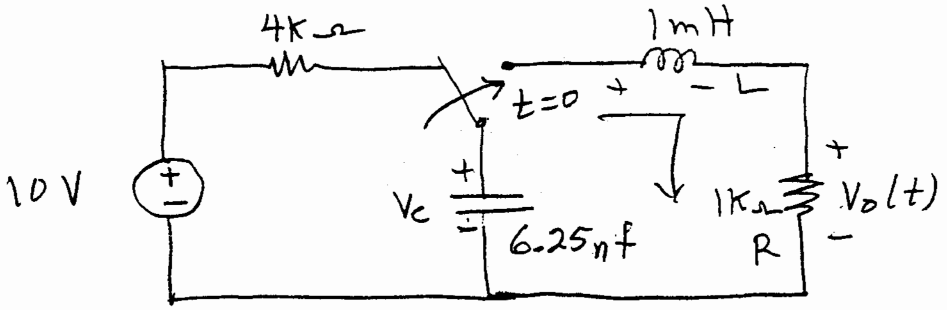
6.64 Find $V_o(t)$ for $t > 0$. Plot using MATLAB



$V_c(0^+) = 10\text{ V}, \quad i(0^+) = 0, \quad V_o(0) = 0$

We can write

$$-\frac{1}{C}\int_0^t i\,dt - 10 + L\frac{di}{dt} + Ri(t) = 0 \qquad (1)$$

Take the derivative of (1)

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i(t)}{C} = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i(t)}{LC} = 0 \qquad (2)$$

Since $V_o(t) = Ri(t)$ this means we can multiply (2) by R and have have $V_o(t)$

Thus,

$$\frac{d^2 V_o}{dt^2} + \frac{R}{L}\frac{dV_o}{dt} + \frac{V_o(t)}{LC} = 0 \qquad (3)$$

substituting in numbers gives

$$\ddot{V}_o(t) + 1\times10^6\,\dot{V}_o(t) + 16\times10^{10}\,V_o(t) = 0 \qquad (4)$$

$$s^2 + 1\times10^6 s + 16\times10^{10} = 0 \qquad (5)$$

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6.64 continued

Roots of the characteristic equation are;

$s_1 = -200{,}000$, $s_2 = -800{,}000$

$V_o(t) = K_1 e^{-200{,}000t} + K_2 e^{-800000t}$ (6)

Now $V_o(0^+) = 10$

and $\dot{V}_o(0^+)$ we need. $\rightarrow \dot{v}_o(0^+) = R\frac{di(0^+)}{dt}$

Go to Eq (1)

$$\frac{di(0^+)}{dt} = \frac{-R\cancel{i(0^+)}^{\,0} + 10}{L}$$

so $R\frac{di(0^+)}{dt} = \dot{V}_o(0^+) = +\frac{10R}{L} = +10{,}000{,}000$

From Eq (6) evaluated at $t=0$;

$\boxed{0 = K_1 + K_2}$ (7)

Take the derivative of Eq (6), evaluate at $t=0$,

$\dot{v}_o(0^+) = -200000K_1 - 800000K_2 = +10{,}000{,}000$

$\boxed{-2K_1 - 8K_2 = +100}$ (8)

From (7) & (8) $K_1 = 16.67$, $K_2 = -16.67$

$V_o(t) = 16.67[e^{-200000t} - 16.67e^{-800000t}]u(t)V$

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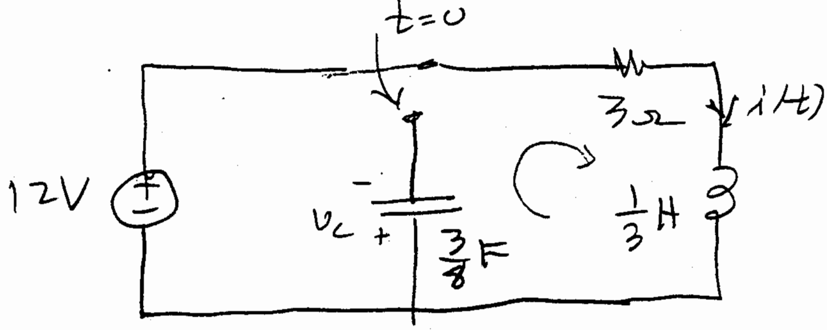
6.67 Find $i(t)$, switch closed for a long time



$$\frac{1}{\frac{1}{3}\times\frac{3}{8}} = 8$$

$$i(0^-) = \boxed{i(0^+) = 4A} \qquad (1)$$

$$\frac{1}{C}\int_0^t i\,dt + v_c(0) + R\,i(t) + L\frac{di}{dt} = 0 \qquad (2)$$

From (1)

$$\frac{di(0^+)}{dt} = -\frac{R}{L}\,i(0^+) = -\frac{3\times 10^3\times 4}{1/3} = -36 \qquad (3)$$

Taking the derivative of (2).

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i(t)}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i(t)}{LC} = 0 \qquad (4)$$

Putting in numbers

$$\frac{d^2i}{dt^2} + 9\frac{di}{dt} + 8\,i(t) = 0$$

$$s^2 + 9s + 8 = 0$$

$$(s+8)(s+1) = 0 \qquad (5)$$

overdamped

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6.67 continued

$$i(t) = K_1 e^{-8t} + K_2 e^{-t} \qquad (6)$$

$$i(0^+) = 4 = \left[K_1 e^{-8t} + K_2 e^{-2t}\right]\Big|_{t=0}$$

so $\boxed{K_1 + K_2 = 4}$

Take the Derivative of Eq (6)

$$\frac{di}{dt} = -8 K_1 e^{-8t} - K_2 e^{-t} \qquad (7)$$

Evaluate Eq (7) at $t=0$

$$-36 = -8K_1 - K_2 \qquad \text{From Eq (3)}$$

$$K_1 + K_2 = 4$$
$$-8K_1 - K_2 = -36$$

$K_1 = 4.57$

$K_2 = -0.57$

Back to Equation (6)

$$\boxed{i(t) = \left[4.57 e^{-8t} - 0.57 e^{-t}\right] u(t) \text{ A}}$$

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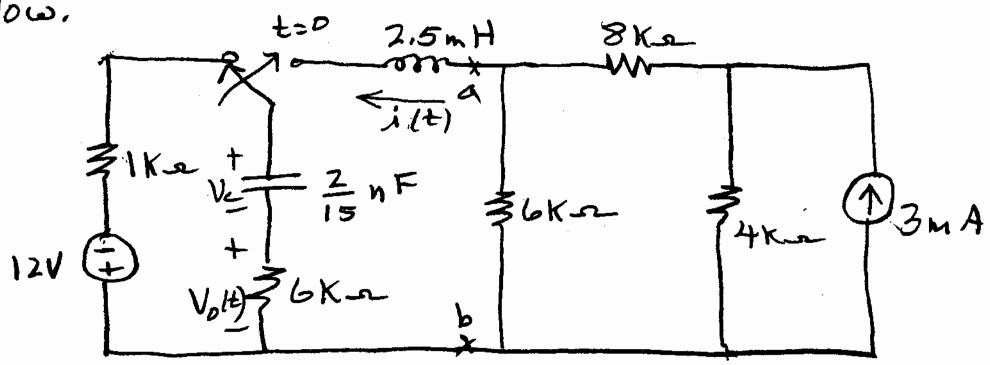
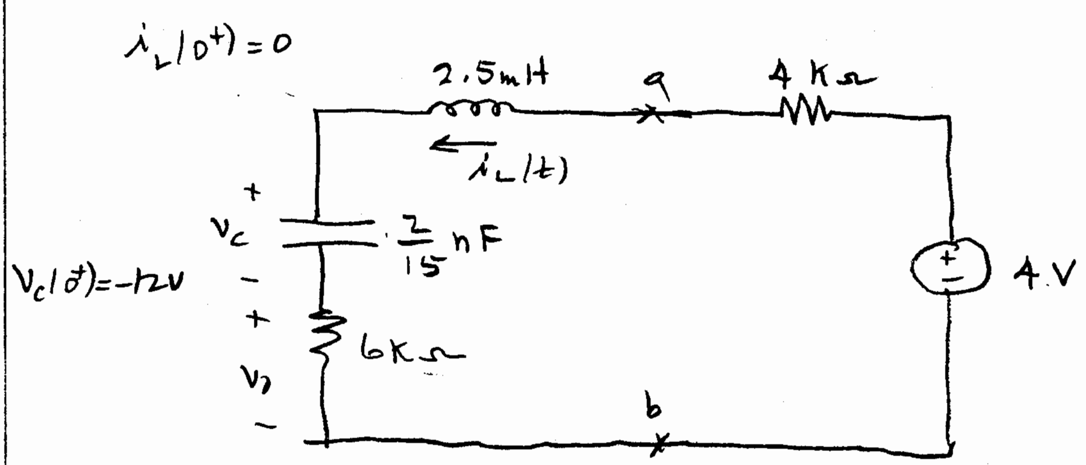
6.7 Find $V_o(t)$ for $t>0$ in the circuit below.



The capacitor is charged to $V_c = -12V$ prior to $t=0$. Just after $t=0$, that is, $t=0^+$ the capacitor voltage is $V_c(0^+) = -12\ V$.

The inductor current, $i(t)$, does not change instantaneously so $i(0^+)=0$.

Using Thevenin's Theorem, the circuit to the right of terminals a-b is reduced as shown in the following diagram.



Writing KVL around the circuit gives the following equation:

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6.7 cont.

$$R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int_0^t i(t)\,dt - 12 = 4 \qquad (1)$$

$R = 10\ k\Omega$

$L = 2.5\ mH$

$C = \frac{2}{15}\ nF$

Differentiating (1) gives,

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i(t)}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i(t)}{LC} = 0 \qquad (2)$$

OR

$$\frac{d^2i}{dt^2} + 4\times 10^6\frac{di}{dt} + \quad \times 10\ i(t) = 0 \qquad (3)$$

Characteristic equation is

$$s^2 + 4\times 10^6 s + 3\times 10^{12}\times \quad = 0 \qquad (4)$$

$$(s + 1\times 10^6)(s + 3\times 10^6) = 0$$

$$i(t) = K_1 e^{-1\times 10^6 t} + K_2 e^{-3\times 10^6 t} \qquad (5)$$

There is no steady state term as such.

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6.71 continued

We know $i(0^+) = 0$; from (1) we have

$$Ri(0^+) + L\frac{di(0^+)}{dt} = 16$$

OR

$$\frac{di(0^+)}{dt} = \frac{16}{L} = \frac{16}{2.5} \times 10^3 = 6.4 \times 10^3$$

$$V_0(0^+) = 0 \;; \quad V_0(t) = 6 \times 10^3 \times \frac{di(0^+)}{dt} = 38.4 \times 10^6 \qquad (6)$$

From (5)

$$i(t) = \left[k_1 e^{-1\times10^6 t} + k_2 e^{-3\times10^6 t}\right]$$

and (7)

$$V_0(t) = 6\times10^3 \, i(t)$$

$$V_0(t) = K_3 e^{-1\times10^6 t} + K_4 e^{-3\times10^6 t} \qquad (8)$$

$$V(0^+) = 0 = K_3 + K_4$$

$$\frac{dV(0^+)}{dt} = 38.4\times10^6 = -1\times10^6 K_3 - 3\times10^6 K_4$$

$$K_3 + K_4 = 0 \qquad K_1 = 19.2$$

$$-K_3 - 3K_3 = 38.4 \qquad K_2 = -19.2$$

$$\boxed{V_0(t) = 19.2\left[e^{-1\times10^6 t} - e^{-3\times10^6 t}\right] V, \; u(t)}$$

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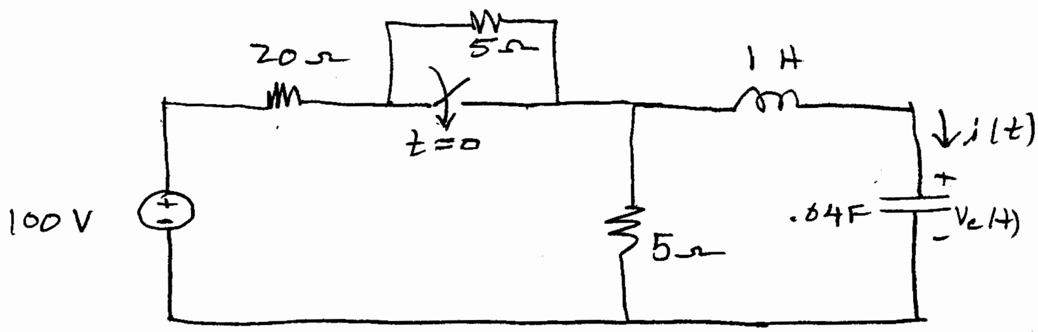
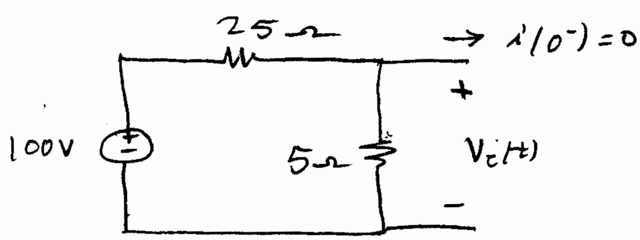
①

6x1 Find $i(t)$ for the circuit below.

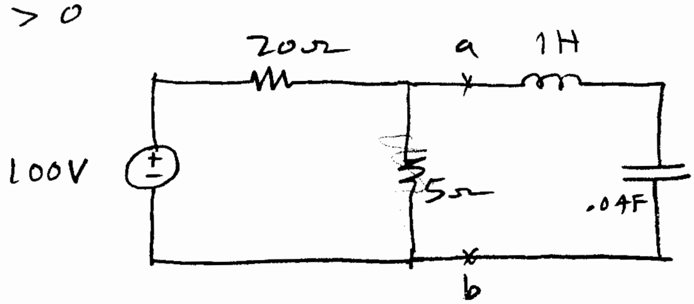


For $t < 0$

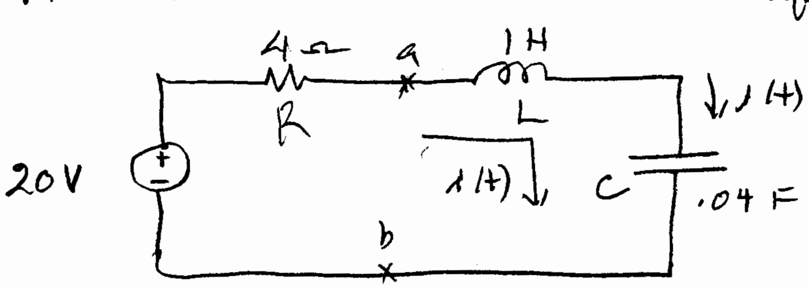


$$V_c(0^-) = \frac{100 \times 5}{30} = \frac{50}{3} V = 16.67 V \qquad (A)$$

$t > 0$



Find the Thevenin to the left of a-b



Write KVL:

$$R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int_0^t i(t) + V_c(0) = 20 \qquad (1)$$

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i(t)}{C} = 0 \qquad (2)$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i(t)}{LC} = 0 \qquad (3)$$

From (1), with $V(0^+) = 16.67$ (from (A))

$$\frac{di(0^+)}{dt} = \frac{20 - 16.67}{L = 1H} = 3.33 \qquad (4)$$

From (3), the char. Eq. is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \qquad (5)$$

putting in numbers;

$$s^2 + 4s + 25 = 0$$

$$(s+2+j\,4.58)(s+2-j4.58) = 0$$

$$i(t) = e^{-2t}\left[K_1\cos(4.58t) + K_2\sin(4.58t)\right] \qquad (6)$$

$$i(0) = 0 = K_1 + 0 \times K_2$$

$$\boxed{K_1 = 0}$$

$$\left.\frac{di}{dt}\right|_{t=0} = e^{-2t}\left[4.58K_2\cos(4.58t)\right] - 2e^{-2t}K_2\sin 4.58t$$

From (4)

$$3.33 = 4.58K_2 \qquad (7)$$

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6x1 continued

From (7)

$$K_2 = \frac{3.33}{4.58} = 0.727$$

$$\therefore \boxed{i(t) = 0.727\, e^{-2t} \sin(4.58t) \text{ A}}$$

QED