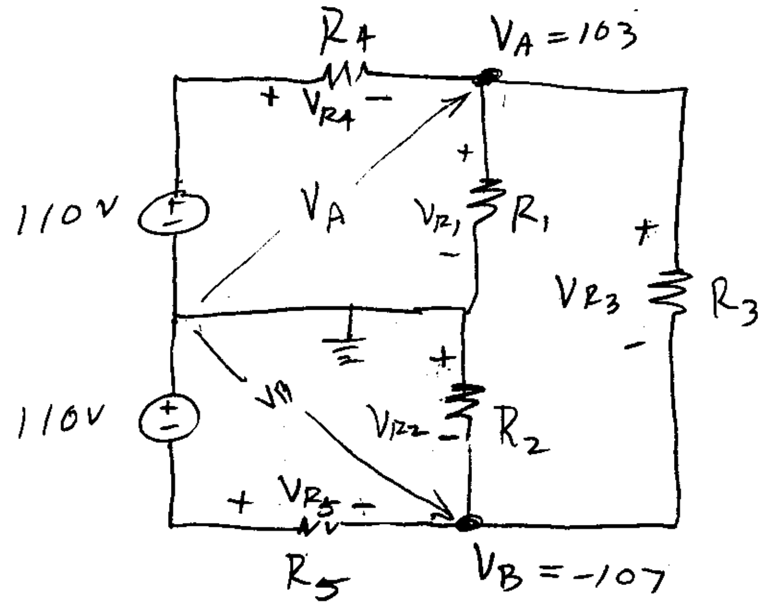
wlz

H.W. # 2
ECE301
Fall Semester 07

3.2 Determine the voltage across each resistor in the following circuit by using Nodal Analysis. Check by using mesh analysis.



$$\boxed{V_{R1} = V_A = 103\ V}$$

$$V_B + V_{R2} = 0$$

$$\boxed{V_{R2} = -V_B = +107\ V}$$

$$V_A - V_{R3} - V_B = 0$$

$$V_{R3} = V_A - V_B = 103 - (-107)$$

$$\boxed{V_{R3} = 210\ V}$$

3.2 cont.

$V_A + V_{R4} - 110 = 0$

$V_{R4} = 110 - V_A = 110 - 103 = 7\,V$

$$\boxed{V_{R4} = +7\,V}$$

$V_B + V_{R5} + 110 = 0$

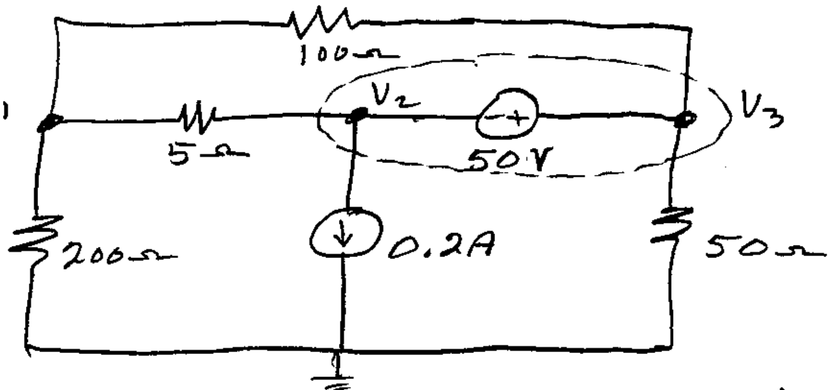
$V_{R5} = -V_B - 110$

$V_{R5} = 107 - 110$

$$\boxed{V_{R5} = -3\,V}$$

3,5

Use nodal analysis to find the current i as indicated in the following diagram.



Form a super node as shown.

At V_1:

$$\frac{V_1}{200} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{100} = 0$$

OR

$$V_1 + 40V_1 - 40V_2 + 2V_1 - 2V_3 = 0$$

$$\boxed{43V_1 - 40V_2 - 2V_3 = 0}$$

At the supernode:

$$\frac{V_2 - V_1}{5} + 0.2 + \frac{V_3 - V_1}{100} + \frac{V_3}{50} = 0$$

$$20V_2 - 20V_1 + V_3 - V_1 + 2V_3 = -20$$

$$\boxed{-21V_1 + 20V_2 + 3V_3 = -20}$$

3.5

$$V_2 + 50 - V_3 = 0$$

$$\boxed{0V_1 + V_2 - V_3 = -50}$$

$$\begin{bmatrix} 43 & -40 & -2 \\ -21 & 20 & 3 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -20 \\ -50 \end{bmatrix}$$

$V_1 = -45.23V$ $V_2 = -48.69V$ $V_3 = 1.31V$

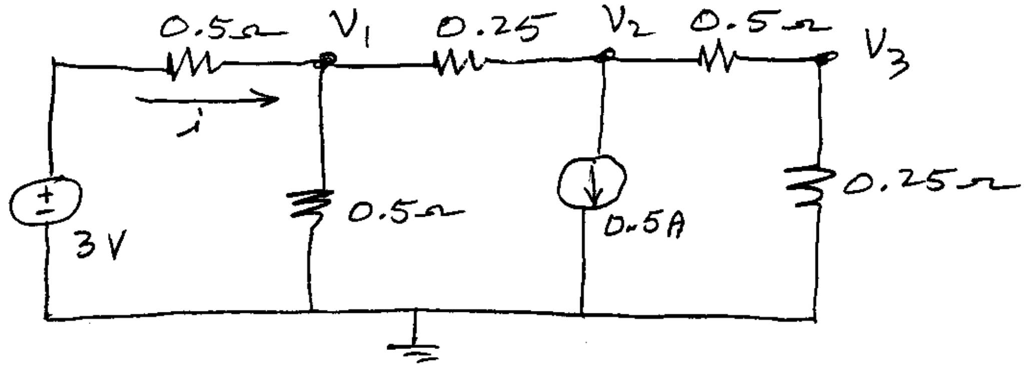
We have at V_2

$$\frac{V_2 - V_1}{5} + 0.2 + i = 0$$

$$i = -0.2 + \frac{V_1 - V_2}{5} = -0.2 + \frac{(-45.23 + 48.69)}{5}$$

$$i = .494A$$

3.7 For the circuit below, use nodal analysis to find the current i.



At V_1

$$\frac{V_1 - 3}{0.5} + \frac{V_1}{0.5} + \frac{V_1 - V_2}{0.25} = 0$$

$$V_1 - 3 + V_1 + 2V_1 - 2V_2 = 0$$

$$4V_1 - 2V_2 = 3$$

At V_2

$$\frac{V_2 - V_1}{.25} + \frac{V_2}{.75} + 0.5 = 0$$

$$3V_2 - 3V_1 + V_2 = -0.375$$

$$\boxed{-3V_1 + 4V_2 = -0.375}$$

You don't need node V_3.

3.2 continued

$$\begin{bmatrix} 4 & -2 \\ -3 & 4 \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{bmatrix} 3 \\ -.375 \end{bmatrix}$$

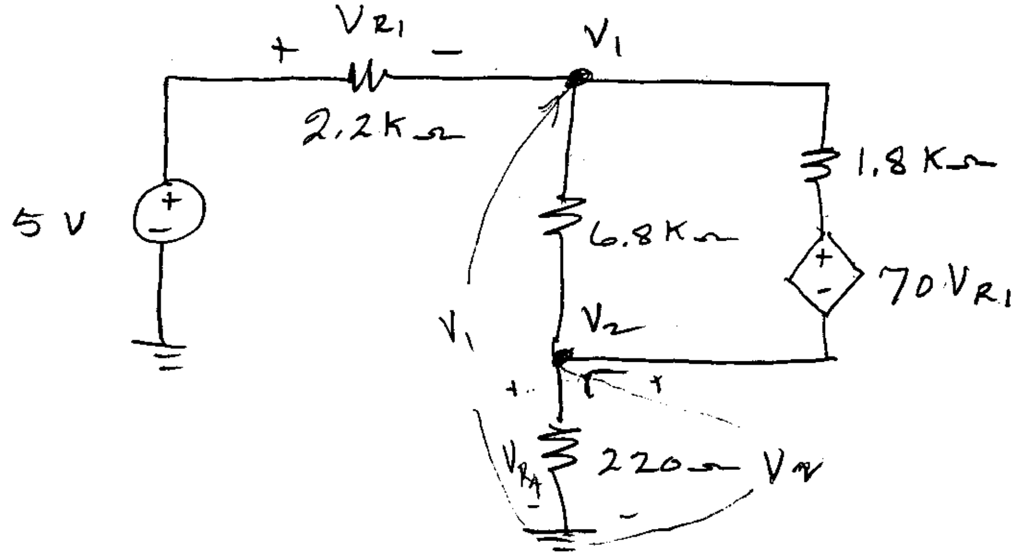
$V_1 = 1.125\ V \qquad V_2 = 0.75\ V$

$3 - 0.5\,i - V_1 = 0$

$i = 6 - 2V_1 = 6 - 2 \times 1.125$

$i_1 = 3.75\ A$

3.13 Given the circuit below.
Find the voltage across R_4.
Use nodal analysis.



At V_1

$$\frac{V_1 - 5}{2.2K} + \frac{V_1 - V_2}{6.8K} + \frac{V_1 - 70V_{R1} - V_2}{1.8K} = 0$$

Also

$$5 - V_{R1} - V_1 = 0$$

$$V_{R1} = 5 - V_1$$

$$3.09V_1 - 15.5 + V_1 - V_2 + 3.78V_1 - 264.4(5 - V_1) - 3.78V_2 = 0$$

$$\boxed{272.3V_1 - 4.78V_2 = 1337.5}$$

At V_2

$$\frac{V_2}{0.220K} + \frac{V_2 - V_1}{6.8K} + \frac{(V_2 + 70V_{R1} - V_1)}{1.8K} = 0$$

Again; $V_{R1} = 5.0 - V_1$

mult by 6.8K

$$30.91V_2 + V_2 - V_1 + 3.78V_2 + 264.4(5 - V_1) - 3.78V_1 = 0$$

$$\boxed{-269.2V_1 + 35.7V_2 = -1322}$$

$$\begin{bmatrix} 272.3 & -4.78 \\ -269.2 & 35.7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1337.5 \\ -1322 \end{bmatrix}$$

$V_1 = 4.91 \text{ V}$

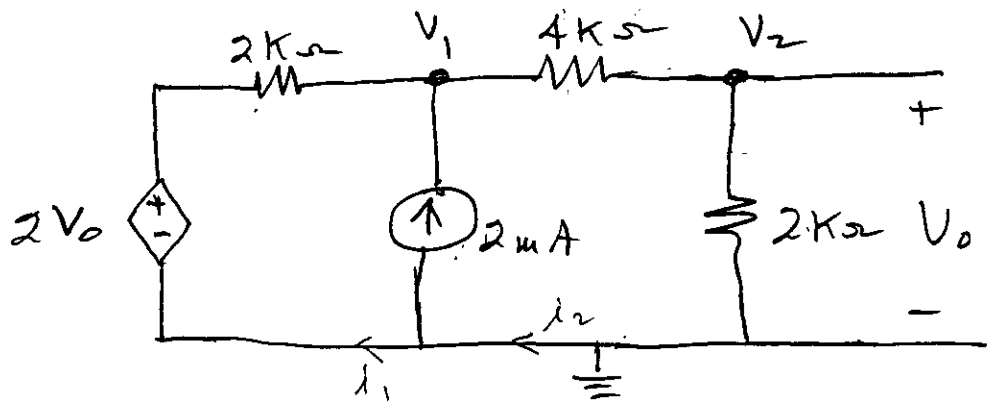
$V_2 = 0.00882 \text{ V}$

$V_2 = V_{R4} = .00882 \text{ V}$

Answer book gives 5.1 mV

3.X1

For the circuit below, use nodal analysis to find V_o.



At V_1:

$$\frac{V_1 - 2V_o}{2K} + \frac{V_1 - V_2}{4K} = 2\,mA$$

$$2V_1 - 4V_o + V_1 - V_2 = 8$$

but $V_o = V_2$

$$\boxed{3V_1 - 5V_2 = -8}$$

At V_2

$$\frac{V_2 - V_1}{4K} + \frac{V_2}{2K} = 0$$

$$V_2 - V_1 + 2V_2 = 0$$

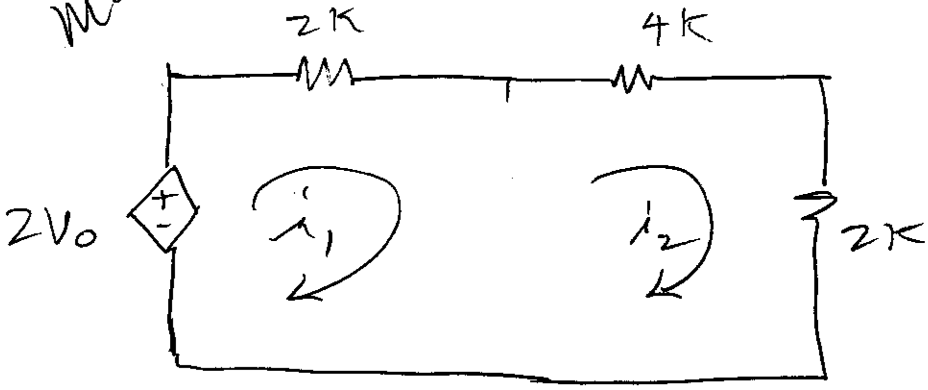
$$\boxed{-V_1 + 3V_2 = 0}$$

3.X1

$$\begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -8 \\ 0 \end{bmatrix}$$

$V_1 = -6\,V$ $\qquad$ $V_2 = -2\,V$

mesh



$$2K\,i_1 + 6K\,i_2 = 2V_o$$

$$V_o = 2K\,i_2$$

$$2K\,i_1 + 6K\,i_2 = 4K\,i_2$$

$$\cancel{2K}\,i_1 + \cancel{2K}\,i_2 = 0$$

$$i_1 + i_2 = 0$$

AND

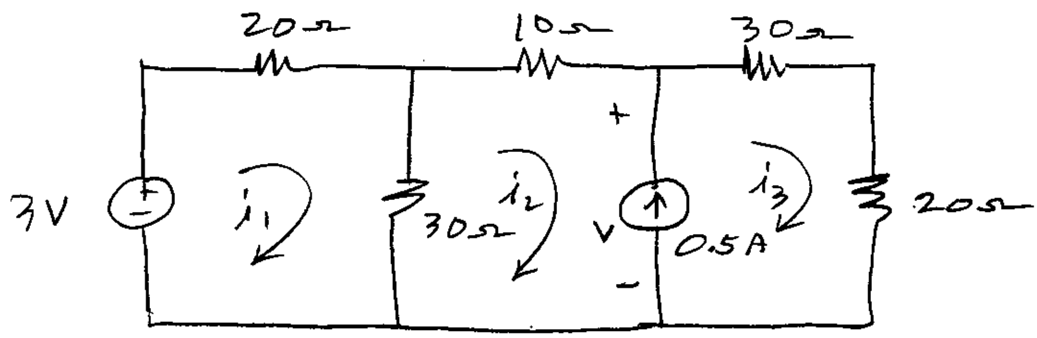
$$i_1 - i_2 = -2\times10^{-3}$$

$$i_2 = 1\ mA$$

$$V_o = 2\ V$$

3.15

Use mesh analysis to find the voltage V in the circuit below.



$$50 i_1 - 30 i_2 = 3$$

$$-30 i_1 + 40 i_2 + 50 i_3 = 0$$

$$-i_2 + i_3 = 0.5$$

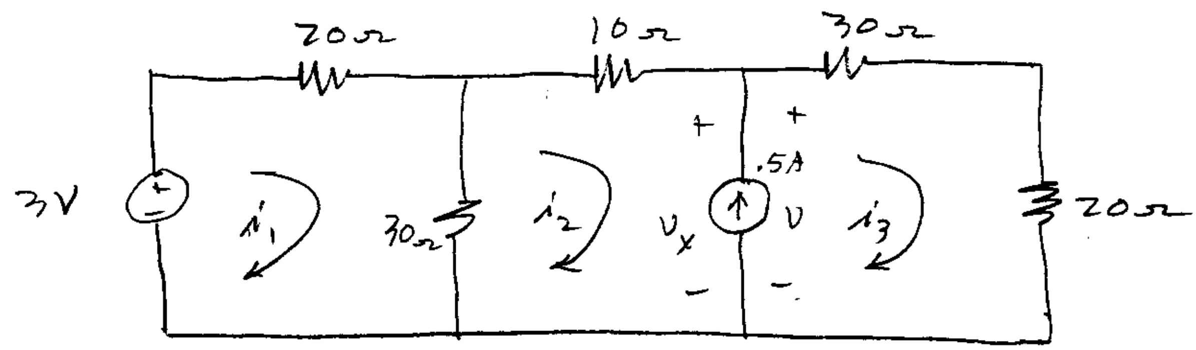
$$\begin{bmatrix} 50 & -30 & 0 \\ -30 & 40 & 50 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0.5 \end{bmatrix}$$

$i_1 = -0.13\ A$, $i_2 = -0.32\ A$, $i_3 = 0.178\ A$

$$\boxed{V = 50 i_3 = 8.89\ V}$$

3.15

Alternate solution

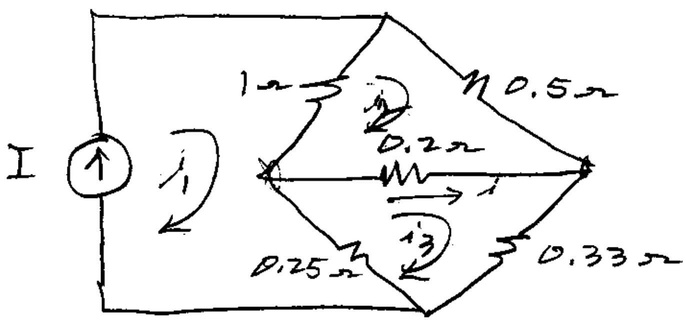


$$50i_1 - 30i_2 = 3$$
$$-30i_1 + 40i_2 + V_x = 0$$
$$50i_3 - V_x = 0$$
$$-i_2 + i_3 = 0.5\text{ A}$$

$$\begin{array}{cccc} i_1 & i_2 & i_3 & V_x \end{array}$$
$$\begin{bmatrix} 50 & -30 & 0 & 0 \\ -30 & 40 & 0 & 1 \\ 0 & 0 & 50 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ V_x \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0.5 \end{bmatrix}$$

$i_1 = -.13$ A, $i_2 = -0.32$ A, $i_3 = 0.178$ A, $V_x = 0.89$ V

3.19 Use mesh analysis to find the current i in the following circuit



Open the source, I.

$$-i_1 + 1.7i_2 - 0.2i_3 = 0$$

$$-.25i_1 - .2i_2 + 0.783i_3 = 0$$

$$i_1 = I$$

$$\begin{bmatrix} 1.7 & -.2 \\ -.2 & 0.783 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} I \\ .25I \end{bmatrix}$$

Assume $I = 1$, multiply solutions by I.

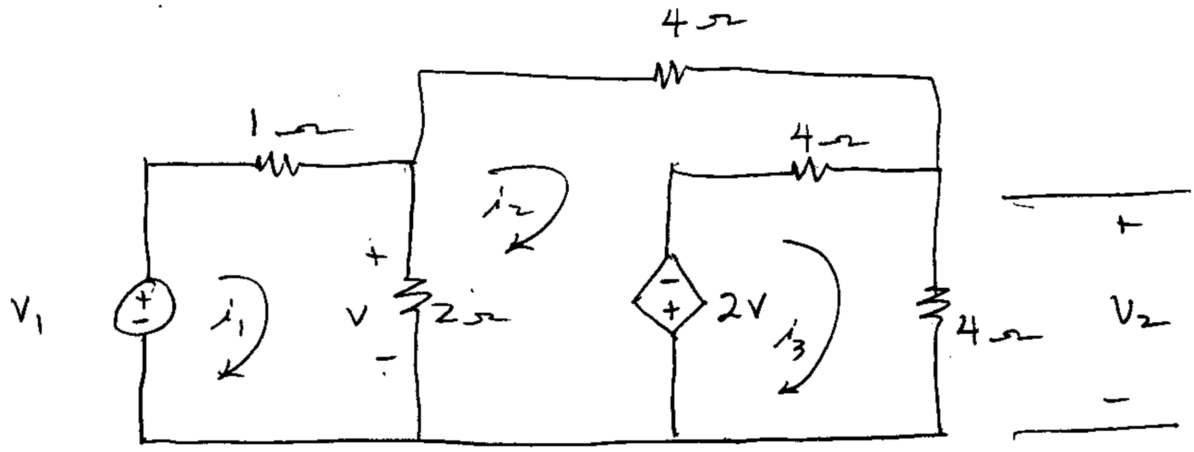
$$i_2 = 0.645\,I$$

$$i_3 = 0.484\,I$$

$$i = i_3 - i_2 = (0.484 - .645)I$$

$$i = -0.161\,I$$

3.20



mesh 1

$$\boxed{3i_1 - 2i_2 = V_1}$$

mesh 2 $-2i_1 + 10i_2 - 4i_3 - 2V = 0$ but $V = 2(i_1 - i_2)$

$$-4i_1 + 4i_2$$

so,

$$\boxed{-6i_1 + 14i_2 - 4i_3 = 0}$$

mesh 3

$-4i_2 + 8i_3 + 2V = 0$ but $V = 2(i_1 - i_2)$

$$4i_1 - 4i_2$$

so,

$$\boxed{4i_1 - 8i_2 + 8i_3 = 0}$$

$$\begin{array}{ccc} i_1 & i_2 & i_3 \\ \end{array}$$

$$\begin{bmatrix} 3 & -2 & 0 \\ -6 & 14 & -4 \\ 4 & -8 & 8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix}$$

3.20 cont.

$$i_3 = \frac{\begin{vmatrix} 3 & -2 & V_1 \\ -6 & 14 & 0 \\ 4 & -8 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 0 \\ -6 & 14 & -4 \\ 4 & -8 & 8 \end{vmatrix}} = \frac{V_1(48-56)}{176} = \frac{-8V_1}{176}$$

$$i_3 = \frac{-8}{176}V_1 = -.045455\,V_1$$

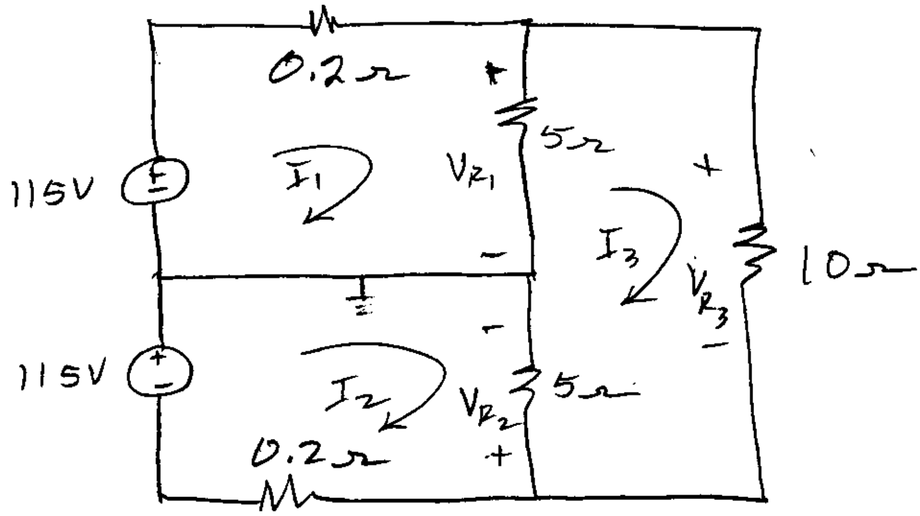
$$V_2 = 4\,i_3$$

$$V_2 = -.1818\,V_1$$

so

$$\boxed{\frac{V_2}{V_1} = -.1818}$$

3.28 You are given the following circuit.



$$5.2 I_1 - 5 I_3 = 115\text{ V}$$

$$5.2 I_2 - 5 I_3 = 115\text{ V}$$

$$-5 I_1 - 5 I_2 + 20 I_3 = 0$$

$$\begin{bmatrix} 5.2 & 0 & -5 \\ 0 & 5.2 & -5 \\ -5 & -5 & 20 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 115 \\ 115 \\ 0 \end{bmatrix}$$

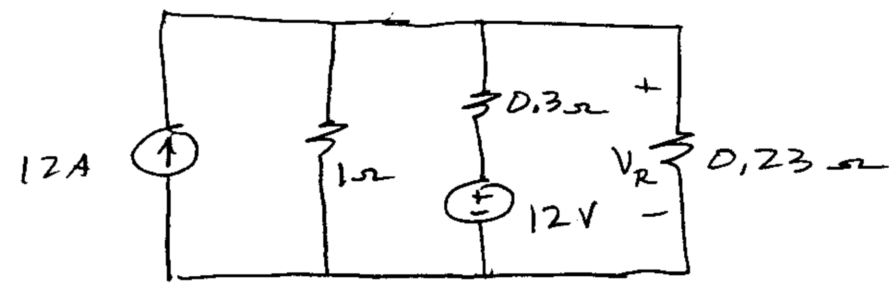
$I_1 = 42.6\text{ A}$, $I_2 = 42.6\text{ A}$, $I_3 = 21.3\text{ A}$

$V_{R_1} = 5(I_1 - I_3) = 106.5\text{ V}$ positive up

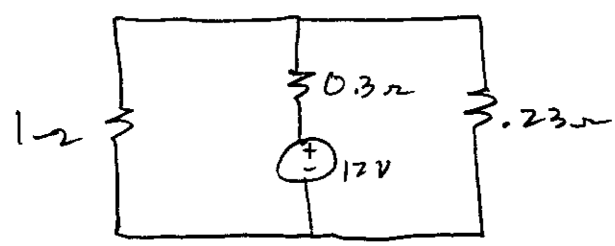
$V_{R2} = 5(I_3 - I_2) = -106.5\text{ V}$ positive down

$V_{R3} = 10\, I_3 = 213\text{ V}$ positive up

3.33 Determine the voltage across R(V_R) using superposition.

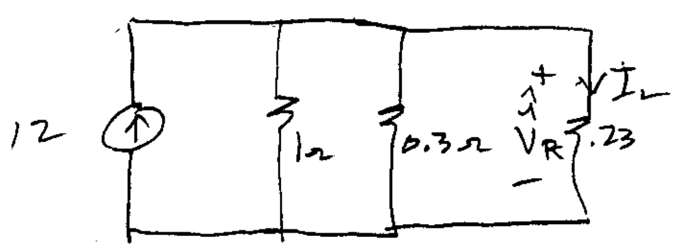


Deactivate the current source:



$$\hat{V}_{R_1} = \frac{12 \times \left(\frac{.23 \times 1}{1 + .23}\right)}{.3 + \left(\frac{.23 \times 1}{1 + .23}\right)} = 4.59\ V$$

Deactivate the voltage source



$1 \parallel .3 = 0.231\ \Omega$

$I_L \doteq 6\text{A}$

$\hat{\hat{V}}_R = 6 \times .23 = 1.38\ V$

$V_R = \hat{V}_R + \hat{\hat{V}}_R = 4.59 + 1.38 = 5.97\ V$

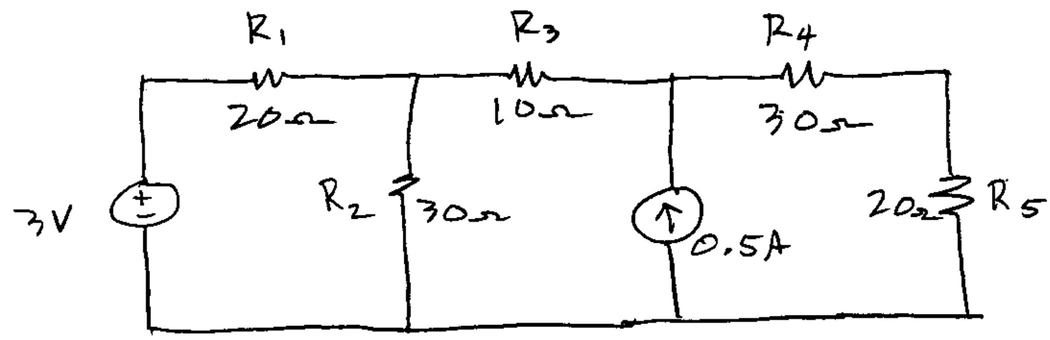
close to Answer book

3,48

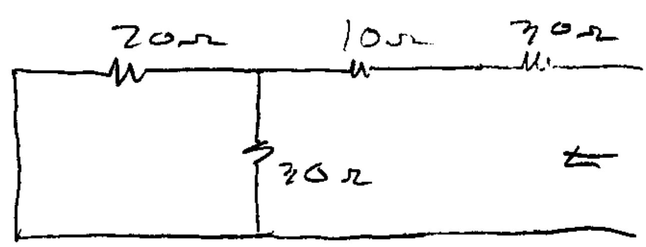
(a) Find the Thevenin resistance seen looking in terminals a-b

(b) Find the Thevenin voltage between a-b with the load resistor removed,

(c) Replace R_5 with a short and thereby find I_{Norton}



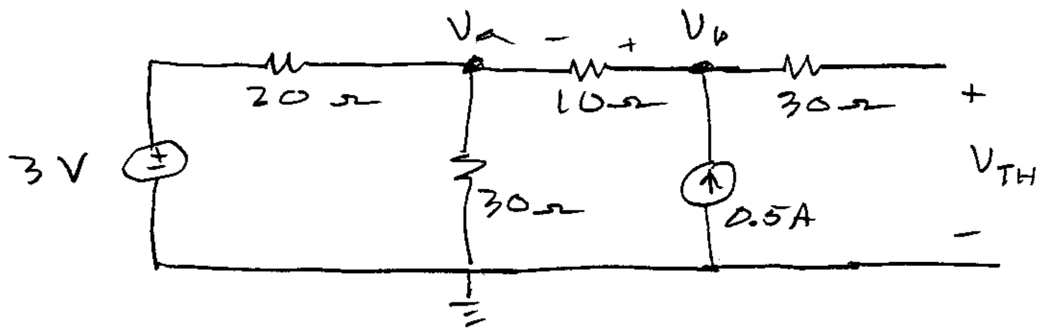
(a) Thevenin resistance.



$$R_{TH} = 40 + 20 \| 30 = 40 + 12 = 52\Omega$$

V_{TH} is the open-circuit voltage for the following ckt,

3.48 cont.



No current flows in the 30 Ω resistor since that portion of the circuit is open. Thus, no voltage drop. Thus $V_{TH} = V_b$

At V_a by nodal analysis,

$$\frac{V_a - 3}{20} + \frac{V_a}{30} = 0.5$$

$$3V_a - 9 + 2V_a = 30$$

$$5V_a = 39$$

$$V_a = 7.8\,V$$

Then

$$V_b - 10 \times 0.5 - V_a = 0$$

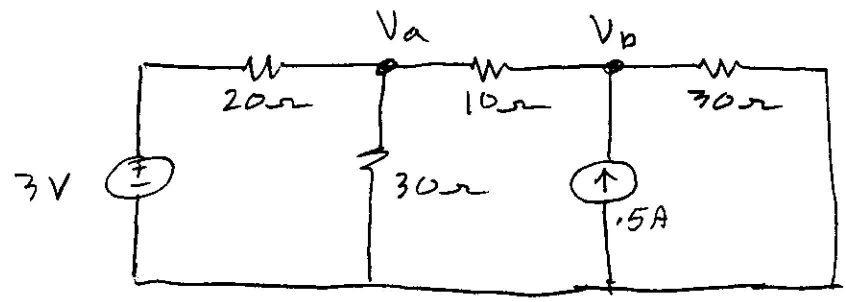
$$\boxed{V_b = 5 + 7.8 = 12.8\,V = V_{TH}}$$

$$I_{NORTON} = \frac{V_{TH}}{R_{TH}} = \frac{12.8}{52} = 0.246\,A$$

Can also find I_{NORTON} by direct definition.

3)

3.48



$$\frac{V_a - 3}{20} + \frac{V_a}{30} + \frac{V_a - V_b}{10} = 0$$

60

$$3V_a - 9 + 2V_a + 6V_a - 6V_b = 0$$

$$\boxed{11V_a - 6V_b = 9}$$

$$\frac{V_b - V_a}{10} + \frac{V_b}{30} = 0.5$$

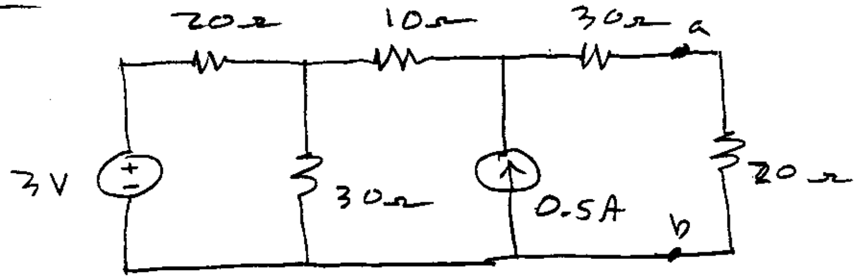
$$3V_b - 3V_a + V_b = 15$$

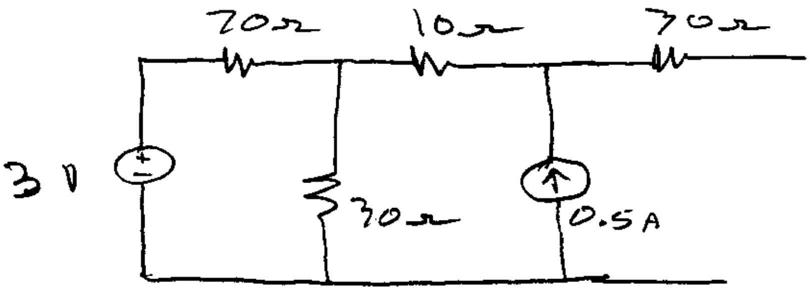
$$\boxed{-3V_a + 4V_b = 15}$$

$$V_b = 7.385 \text{ V}$$

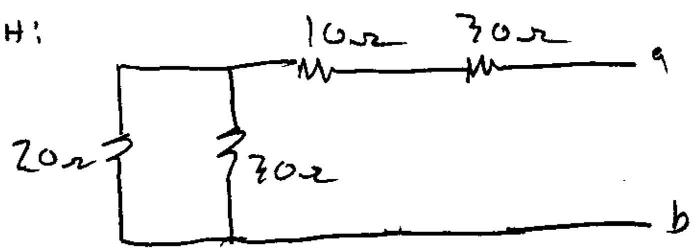
$$I_{NORTON} = \frac{7.385}{30} = 0.246 \text{ A}; \text{ As before}$$

3.48



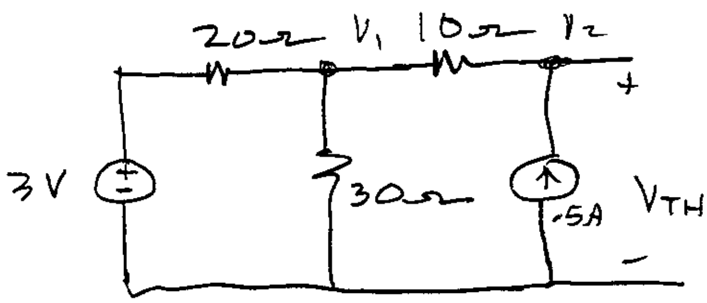


R_{TH}:



$20 \parallel 30 = 12\,\Omega$

$R_{TH} = 12 + 10 + 30 = 52\,\Omega$



$$\frac{V_1 - 3}{20} + \frac{V_1}{30} + \frac{V_1 - V_2}{10} = 0$$

$$3V_1 - 9 + 2V_1 + 6V_1 - 6V_2 = 0$$

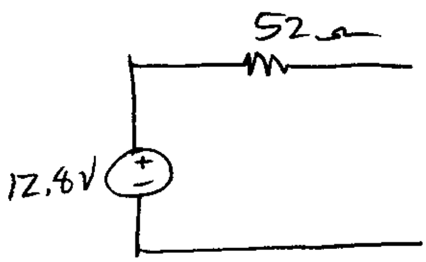
$$\boxed{11V_1 - 6V_2 = 9}$$

3.48 cont.

$$\frac{V_2 - V_1}{10} = 0.5$$

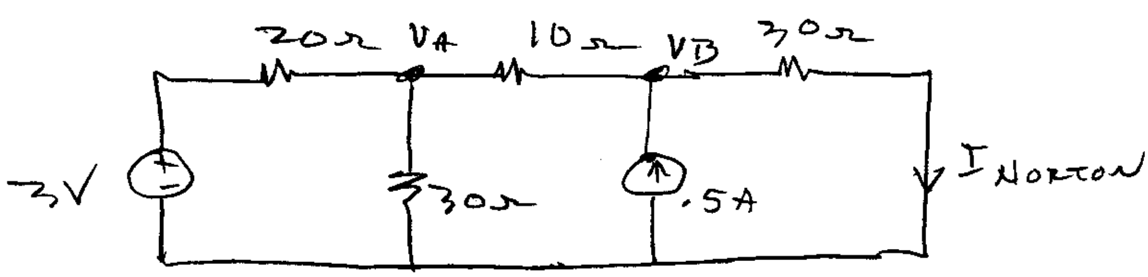
$$\boxed{-V_1 + V_2 = 5}$$

$V_2 = 12.8$ V



$$I_{NORTON} = \frac{12.8}{52} = 246 \text{ mA}$$

Also, find I_{NORTON} from ckt below



$$I_{NORTON} = \frac{V_B}{30}$$

$$\frac{V_A - 3}{20} + \frac{V_A}{30} + \frac{V_A - V_B}{10} = 0$$

$$\boxed{11V_1 - 6V_2 = 9}$$

3,48 mA,

$$\frac{V_B - V_A}{10} + \frac{V_B}{30} = 0.5$$

$$3V_B - 3V_A + V_B = 15$$

$$\boxed{-3V_A + 4V_B = 15}$$

$V_A = 4.85\ V$, $V_B = 7.385\ V$

$$I_{NORTON} = \frac{V_B}{30} = \frac{7.385}{30} = 0.246$$

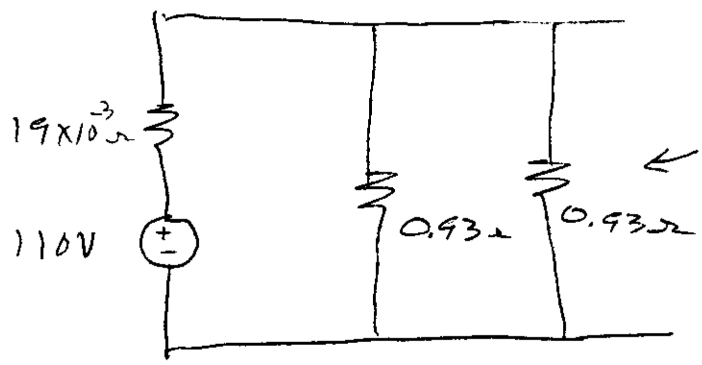
so

$$\boxed{I_{NORTON} = 246\ mA}$$

As before

3.51

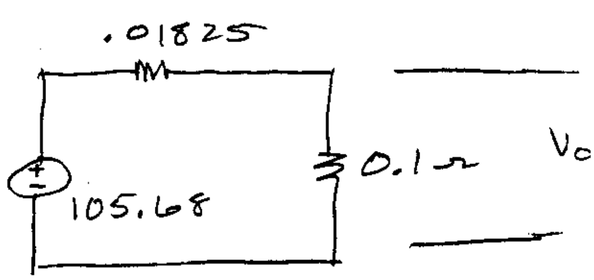
Find the Thevenin for the following circuit.



$$\frac{1}{R_{TH}} = \frac{1}{.019} + \frac{1}{.93} + \frac{1}{.93} = 54.785$$

$$R_{TH} = 0.01825\ \Omega$$

$$V_{TH} = \frac{110 \times 0.465}{0.465 + .019} = 105.68\ V$$



$$V_o = \frac{105.68 \times 0.1}{0.1 + .01825} = 89.37\ V$$

$$\Delta V = 89.37 - 105.68$$

$$\boxed{\Delta V = -16.3\ V}$$