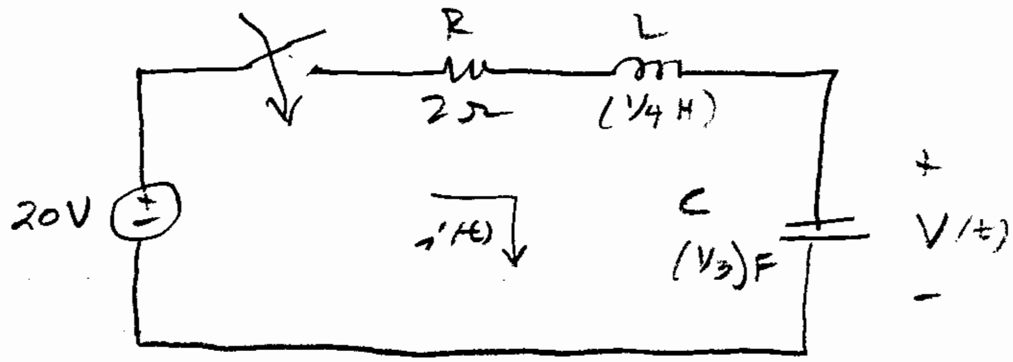
H.W. 6

(1)



(a)

$$Ri + L\frac{di}{dt} + V(t) = 20$$

$$i = C\frac{dv}{dt}$$

$$LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v(t) = 20$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{V(t)}{LC} = \frac{20}{LC}$$

(b) with numbers

$$\frac{d^2v}{dt^2} + 8\frac{dv}{dt} + 12V(t) = 240$$

$$s^2 + 8s + 12 = 0$$

$$(s+2)(s+6) = 0$$

(c) $$v(t) = K_s + K_1 e^{-2t} + K_2 e^{-6t}$$

$$K_s = 20$$

(1) cont.

$$V(t) = 20 + k_1 e^{-2t} + k_2 e^{-6t}$$

$V(0^+) = 0$, $i(0^+) = 0$

with $i(0^+) = 0$, $\frac{dV(0^+)}{dt} = 0$

$$V(0^+) = 0 = 20 + k_1 + k_2$$

$$\frac{dV(0^+)}{dt} = 0 = -2k_1 - 6k_2$$

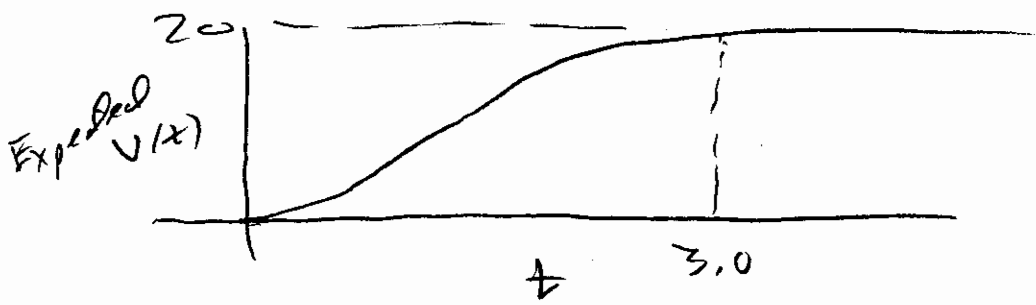
$$k_1 + k_2 = -20$$
$$k_1 + 3k_2 = 0$$

$$k_1 = -30, \quad k_2 = 10$$

$$\boxed{V(t) = 20 - 30e^{-2t} + 10e^{-6t}}$$

(d) MATLAB program:

- slowest time constant = 0.5, Run for $6 \times 0.5 = 3.0$ sec



1.3

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%  program for homework set 6 for ECE 301, Fall 2004,  Probalem 1

% Want to write a program that will plot

% v = 20  - 30*exp(-2*t) + 10*exp(-6*t)

% Written by W. Green, October 19, 2004,  Office computer

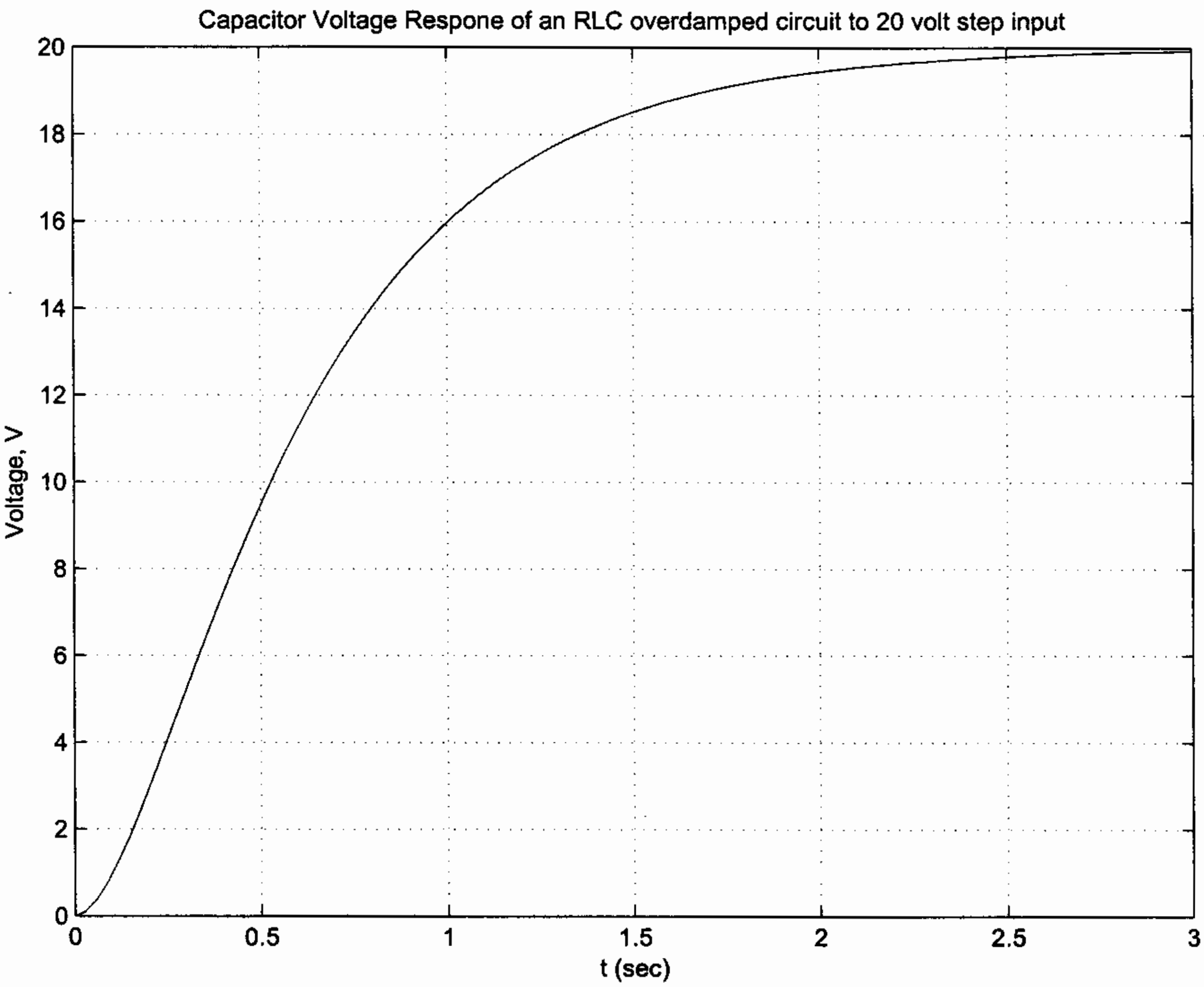
% program name rlc_overdamped.m

t = 0 : .03 : 3;

v = 20 - 30*exp(-2*t) + 10*exp(-6*t);

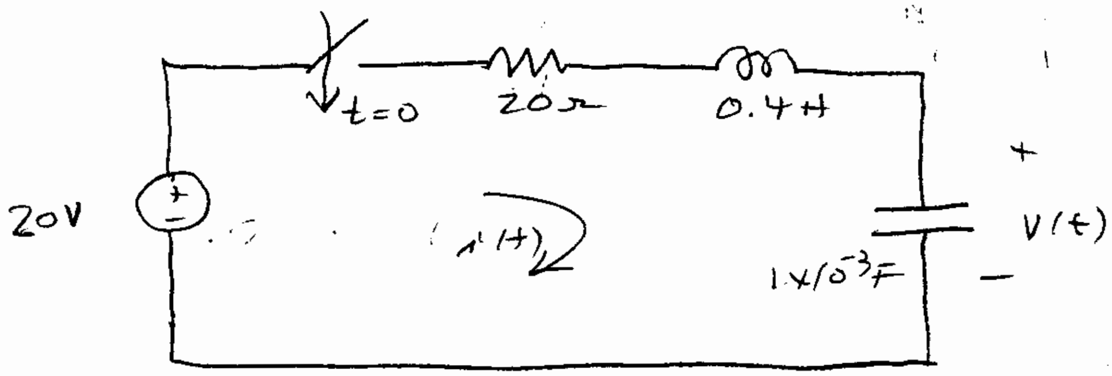
plot(t,v)
grid
ylabel('Voltage, V')
xlabel('t (sec)')
title('Capacitor Voltage Respone of an RLC overdamped circuit to 20 volt step input')
```

1.4


Capacitor Voltage Respone of an RLC overdamped circuit to 20 volt step input
Voltage, V
t (sec)
0
2
4
6
8
10
12
14
16
18
20
0
0.5
1
1.5
2
2.5
3

(2) Given the circuit

2.1



We have

$$Ri + L\frac{di}{dt} + v = 20 \qquad \text{Eq (1)}$$

$i = C\frac{dv}{dt}$

so

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v(t)}{LC} = \frac{20}{LC}$$

In standard form this would be

$$\frac{d^2v}{dt^2} + 2\xi\omega_n\frac{dv}{dt} + \omega_n^2 v(t) = \omega_n^2 K = \omega_n^2 20$$

This leads to (underdamped case)

$$v(t) = 20 + e^{-\xi\omega_n t}\left[K_1\cos(\omega_d t) + K_2\sin\omega_d t\right]$$

compared to

$$v(t) = A + e^{-Bt}\left[K_1\cos\omega_d t + K_2\sin\omega_d t\right]$$

(b) From

$$i(t) = C\frac{dV}{dt}$$

2.V

since

$$i(0^+) = 0, \quad \frac{dV(0^+)}{dt} = 0$$

(c) The characteristic equation, with numbers, is

$$s^2 + \frac{20}{.4}s + \frac{1}{0.4 \times 1 \times 10^{-3}} = 0$$

$$s^2 + 50s + 2500$$

$$\omega_n = \sqrt{2500} = 50$$

$$2\zeta\omega_n = 50$$

$$2\zeta \times 50 = 50$$

$$\zeta = 0.5$$

$$\zeta\omega_n = 0.5 \times 50 = 25$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 50\sqrt{1-.25}$$

$$\omega_d = 43.3$$

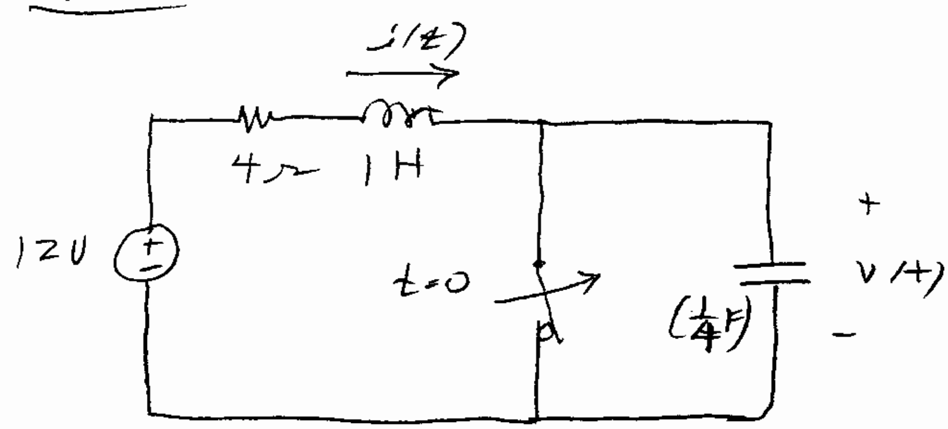
$\therefore$ Comparing coefficients

$A = 20,$

$B = \zeta\omega_n = 25$

$C = D = \omega_d = 43.3$

(3) <u>Given</u> 3.1



$t < 0$

$i(t) = 3\ A$

$v(t) = 0\ V$

$t > 0$ (series RLC

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v(t)}{LC} = \frac{12}{LC}$$

putting in numbers

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 4v(t) = 4 \times 12$$

$$s^2 + 4s + 4 = 0$$

$$(s+2)(s+2) = 0$$

repeated roots

$$v(t) = k_3 + (k_1 + k_2 t)e^{-2t}$$

$k_3 = 12$ (inspection)

3.2

$v(t) = 12 + (K_1 + K_2 t)e^{-2t}$

$v(0) = 0$ gives

$0 = 12 + K_1$

$K_1 = -12$

From $i(t) = C \frac{dv}{dt}$

evaluated at $t = 0^+$

$3 = \frac{1}{4} \frac{dv(0^+)}{dt}$

$\frac{dv(0^+)}{dt} = 12$

$\frac{dv}{dt} = -2K_1 e^{-2t} - 2K_2 t e^{-2t} + K_2 e^{-2t}$

evaluated at $t = 0$

$12 = -2K_1 + K_2 = +24 + K_2$

$12 = 24 + K_2$

$K_2 = -12$

$\therefore$ $\boxed{v(t) = 12 - 12e^{-2t} - 12te^{-2t}}$