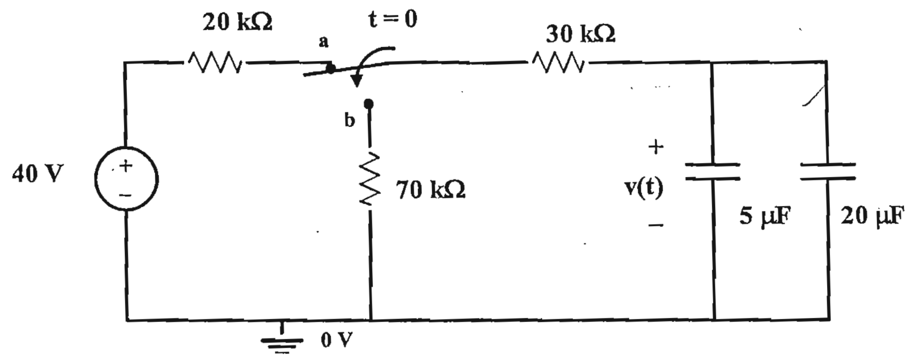
ECE 300
Test 3A
Fall Term '07

(1) Consider the circuit shown in Figure 1. The switch has been in position "a" for a very long time and is switched to position "b" at t = 0.

(a) Develop and solve the differential equation for v(t). Do not use the step-by-step method.
(b) Sketch the voltage v(t) vs t approximately to scale.



20%

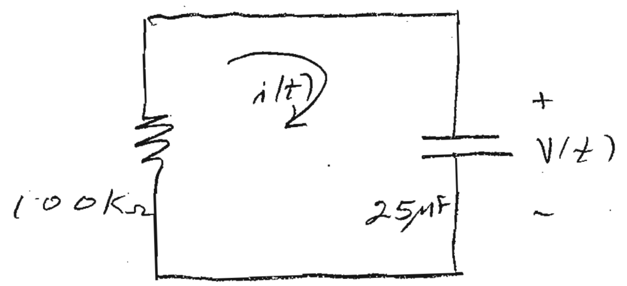
For $t < 0$

The capacitors charge to the source voltage of 40V.

$$V(0^-) = 40V$$

For $t > 0$

$$V(0^+) = V(0^-) = 40V$$



$$C_q = 5\mu F + 20\mu F$$
$$= 25\mu F$$

$$Ri(t) + V(t) = 0 \qquad \text{but } i = C\frac{dV}{dt}$$

$$RC\frac{dV}{dt} + V(t) = 0$$

$$\frac{dV(t)}{dt} + \frac{V(t)}{RC} = 0$$

The solution to the 1st order differential equation is of the form

$$V(t) = V_p(t) + V_c(t)$$

$V_p(t) = 0$ because there is no forcing function. We know that $V_c(t)$ is of the form

$$V_c(t) = Ke^{st}$$

$$s + \frac{1}{RC} = 0 \; ; \quad s = -\frac{1}{.1 \times 10^{6} \times 25 \times 10^{-6}}$$

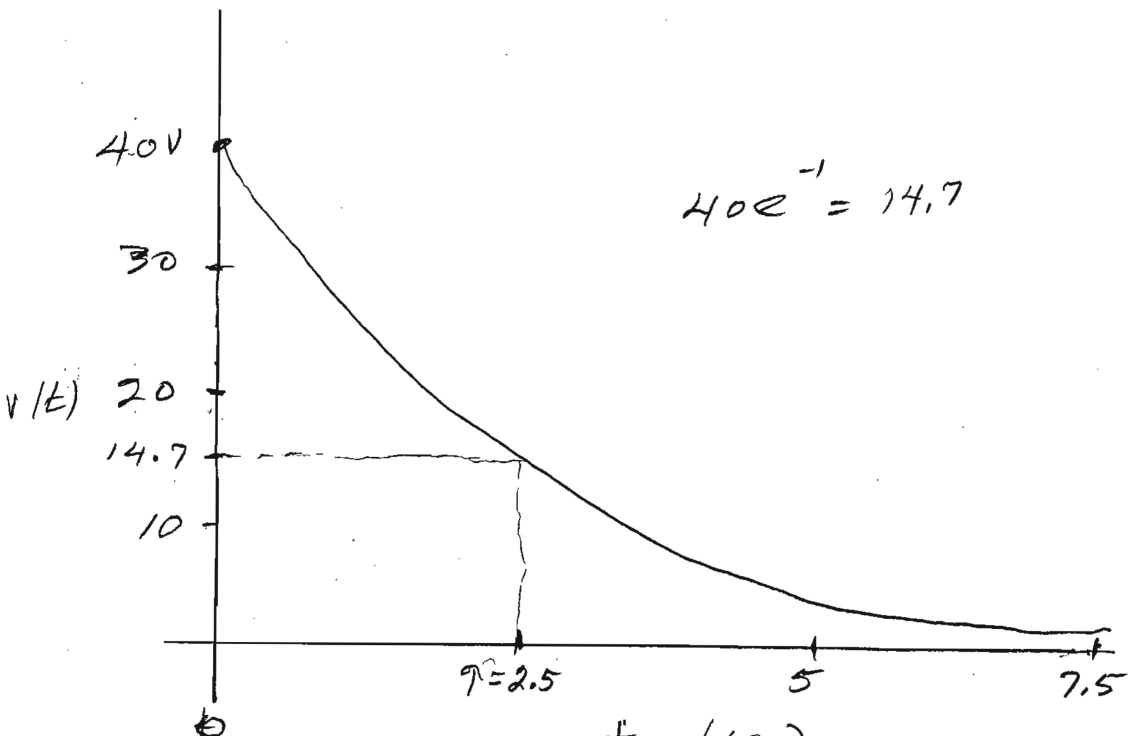
$$\tau = RC = 2.5 \text{ sec}$$

$$s = -0.4$$

$$v = V_c = Ke^{-0.4t} u(t) \text{ V}$$

We know that $V_c(0^+) = 40 = Ke^{0} = K$

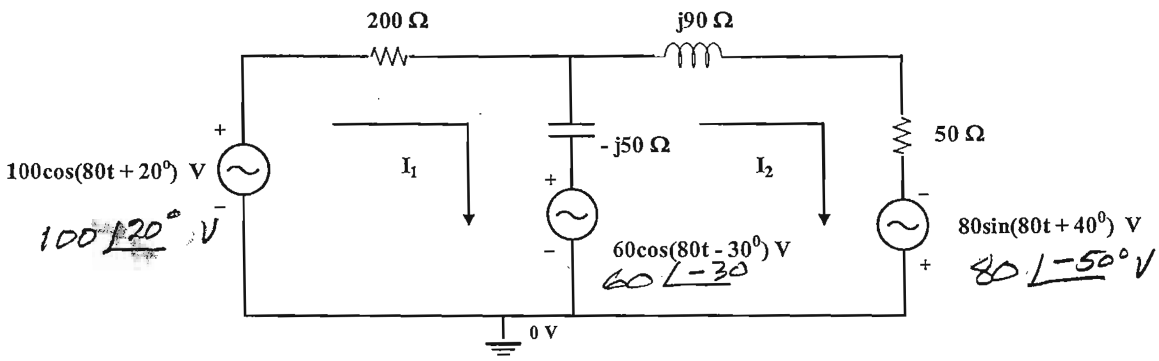
$$\therefore \boxed{v(t) = 40e^{-0.4t} u(t) \text{ V}}$$



$$40e^{-1} = 14.7$$

3A

(2) You are given the AC circuit of Figure 2. Use a cosine reference. Use mesh analysis to solve for the phasor currents $\mathbf{I_1}$ and $\mathbf{I_2}$. Give their values in polar form.



20%

$80 \sin(80t + 40^\circ) = 80\cos(80t - 50^\circ)$

As a phasor $80\angle{-50}$

mesh #1

$-100\angle 20 + 200\hat{I}_1 - j50\hat{I}_1 + j50\hat{I}_2 + 60\angle{-30} = 0$

$$\boxed{(200 - j50)\hat{I}_1 + j50\hat{I}_2 = 100\angle 20 - 60\angle{-30}}$$

mesh #2

$-60\angle{-30} - j50(\hat{I}_2 - \hat{I}_1) + j90 I_2 + 50 I_2 - 80\angle{-50}$

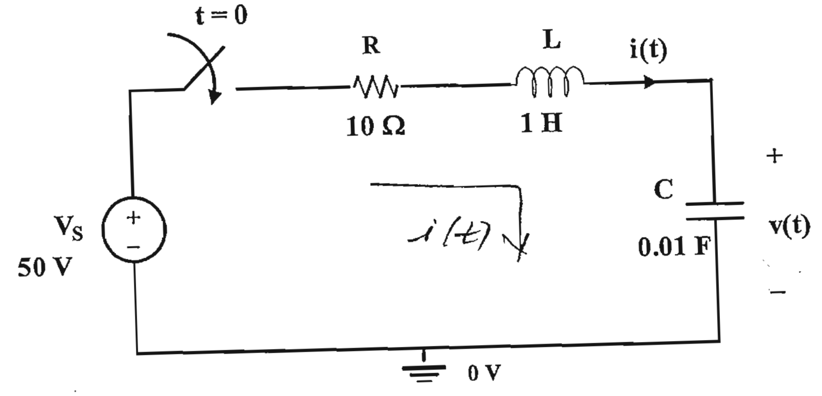
$$\boxed{j50 I_1 + (50 + j40) I_2 = 60\angle{-30} + 80\angle{-50}}$$

$$\begin{bmatrix} (200 - j50) & (0 + j50) \\ (0 + j50) & (50 + j40) \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 100\angle 20 - 60\angle{-30} \\ 60\angle{-30} + 80\angle{-50} \end{bmatrix}$$

$\boxed{\hat{I}_1 = 0.325\angle 162.4^\circ \text{ A}}$ $\boxed{\hat{I}_2 = 2.06\angle{-73.6^\circ} \text{ A}}$

3A.

(3) You are given the RLC circuit of Figure 3.



20%

(a) Develop the differential equation that can be used to solve for v(t). Show your steps.

(b) Give (determine) the values of $v(0^+)$ *and* $\frac{dv(0^+)}{dt}$.

(c) Determine which form below should be used to solve for v(t). Do not solve the equation. Explain your answer.

- $v(t) = 50 + (A_1 + A_2 t)e^{-5t}\ u(t),\ V$
- $v(t) = 50 + e^{-5t}\left[A_1 \cos(8.66t) + A_2 \sin(8.66t)\right]\ u(t),\ V$
- $v(t) = 50 + A_1 e^{-4t} + A_2 e^{-8t}\ u(t),\ V$

FOR $t > 0$

$$R i(t) + L\frac{di}{dt} + v(t) = V_s$$

but $i = C\frac{dv}{dt}$

$$RC\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + v = V_s$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v(t)}{LC} = \frac{V_s}{LC}$$

with numbers

$$\frac{d^2v}{dt^2} + 10\frac{dv}{dt} + 100\,v(t) = 50$$

3A

3A-3-2

Characteristic equation

$$s^2 + 10s + 100 = 0$$

$$(s + 5 + j8.66)(s + 5 - j8.66) = 0$$

So

$$v(t) = 50 + e^{-5t}\left[A_1 \cos 8.66t + A_2 \sin 8.66t\right]$$

Initial conditions

$$i(0^-) = 0 = i(0^+) = 0$$

$$v(0^-) = 0 = v(0^+) = 0$$

$$\boxed{\dot{v}(0^+) = 0}$$

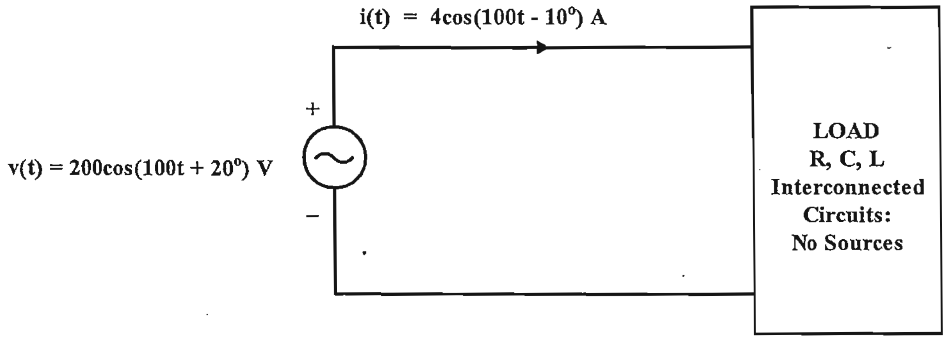
$$i(t) = C\frac{dv}{dt}$$

$$\frac{dv(0^+)}{dt} = \frac{i(0^+)}{C} = 0$$

$$\boxed{\frac{dv(0^+)}{dt} = 0}$$

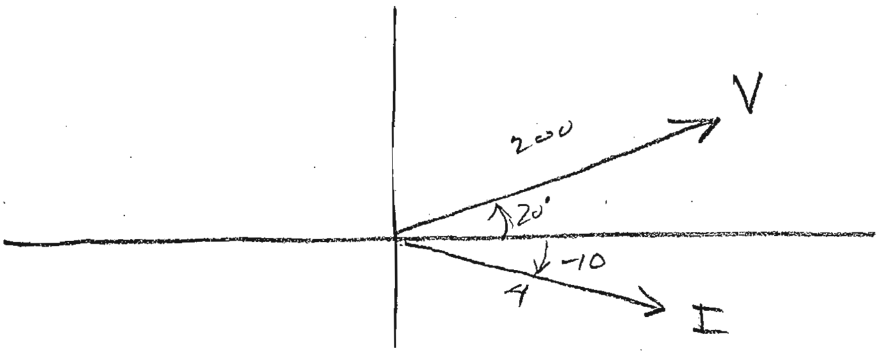
3A

(4) You are given the circuit configuration of Figure 4. Assume **V** is the phasor voltage for 200cos(100t + 20°) V and **I** is the phasor current for 4cos(100t – 10°) A
(a) Prepare the phasor diagram showing phasors **V** and **I**.
(b) Use inspection to determine if the load is inductive or capacitive. Explain your answer.



10%

1a) $\vec{V} = 200\angle 20^\circ\ V$ $\quad \vec{I} = 4\angle -10^\circ\ A$



(b) Since V leads I the load is inductive. ELI

Another View

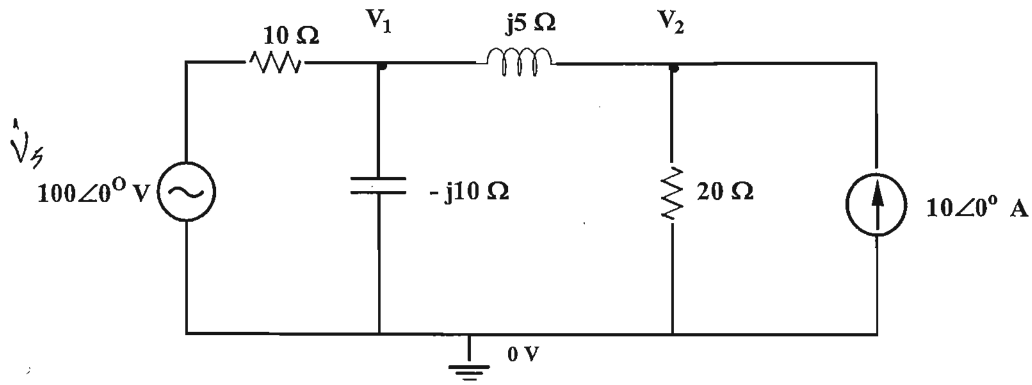
$$Z = \frac{V}{I} = \frac{200\angle 20}{4\angle -10} = 50\angle 30$$

Since Z has a positive j part the load is inductive

3A

(5) You are given the circuit shown in Figure 5. Assume a cosine reference.

(a) Use nodal analysis to find the phasor voltages $\mathbf{V_1}$ and $\mathbf{V_2}$ and express these voltages in polar form.
(b) Give the expression for $v_1(t)$.
(c) Prepare a phasor diagram for phasor $\mathbf{V_S}$, $\mathbf{V_1}$ and $\mathbf{V_2}$.
(d) For the phasors $\mathbf{V_S}$ and $\mathbf{V_2}$ which is leading and by how much angle?



20%

(a) At $\hat{V}_1$

$$\frac{\hat{V}_1 - 100}{10} + \frac{V_1}{-j10} + \frac{V_1 - V_2}{j5} = 0$$

$$0.1\hat{V}_1 - 10 + j0.1V_1 - j0.2V_1 + j0.2V_2 = 0$$

$$\boxed{(0.1 - j0.1)V_1 + (0 + j0.2)V_2 = 10}$$

At $\hat{V}_2$

$$\frac{V_2}{20} + \frac{V_2 - V_1}{j5} = 10$$

$$0.05V_2 - j0.2V_2 + j0.2V_1 = 10$$

$$\boxed{(0 + j0.2)V_1 + (0.05 - j0.2)V_2 = 10}$$

$$\begin{bmatrix} (0.1 - j0.1), & (0 + j0.2) \\ j0.2 & 0.05 - j0.2 \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

$\hat{V}_1 = 114 \angle -37.8$ $\qquad$ $V_2 = 89 \angle -26.6^\circ$ V

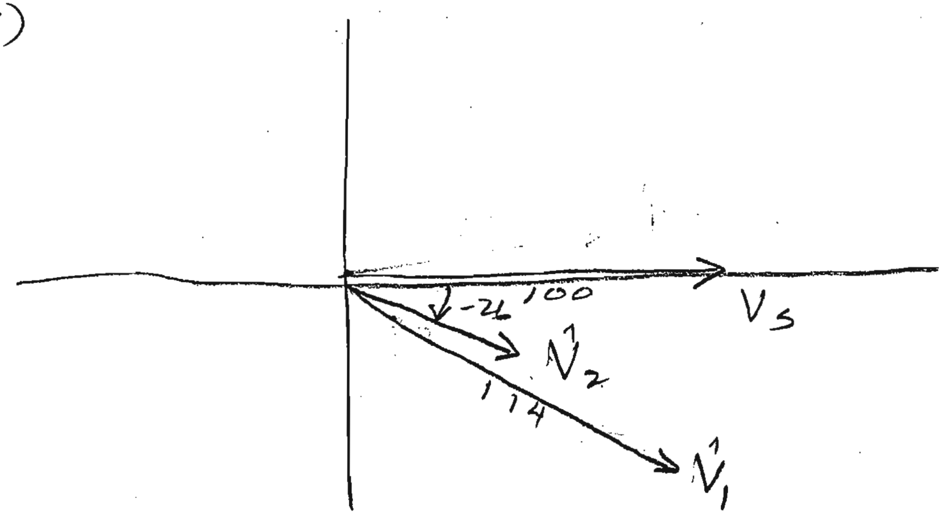
3A-5-2

(5)

(b)

$v_1(t) = 64.7\cos(\omega t + 6.3^\circ)$ V

(c)



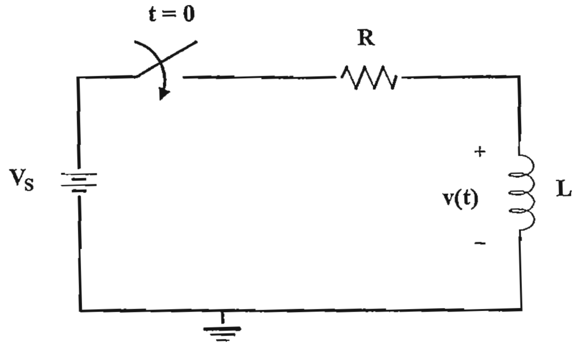
(d) $\vec{V}_s$ <u>leading</u> $\vec{V}_2$ by 26.6°

3A

(6) You are given the circuit of Figure 6A. The switch has been open for a very long time. The switch is closed at t = 0 and the resulting voltage, v(t), is shown in Figure 6B.

(a) Determine the time constant for the circuit.

(b) If R = 10 ohms, what is the value of L?



10%

Figure 6A: Circuit for problem 6.

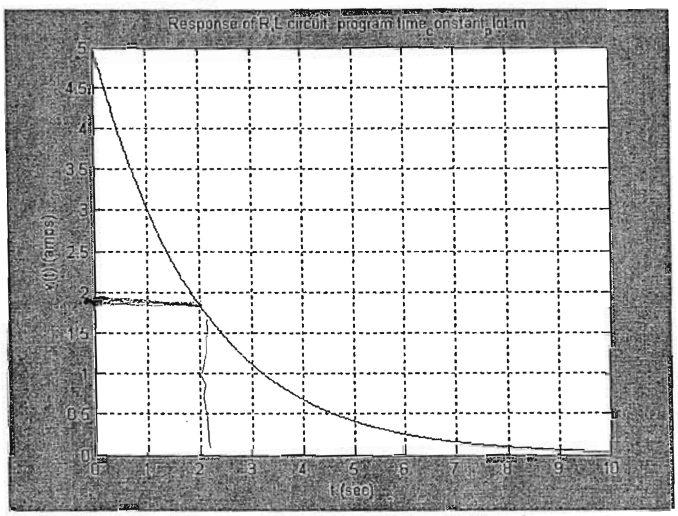


Figure 6B: Response from circuit of Figure 6A.

(a) $i = 5e^{-\frac{t}{\tau}}$ A $\qquad \tau = \frac{L}{R}$

At $t = \tau$,

$i(t) = 5e^{-1} = 1.84$ A

At 1.84, $t = \boxed{\tau = 2}$

(b) $\frac{L}{R} = 2 \quad \therefore \boxed{L = 20\ H}$