ECE 300

Test 3B,

Fall Term '07

wlg

(1) Consider the circuit shown in Figure 1. The switch has been in position “a” for a very long time and is switched to position “b” at t = 0.

(a) Develop and solve the differential equation for i(t). Do not use the step-by-step method.
(b) Sketch the current i(t) vs t approximately to scale.

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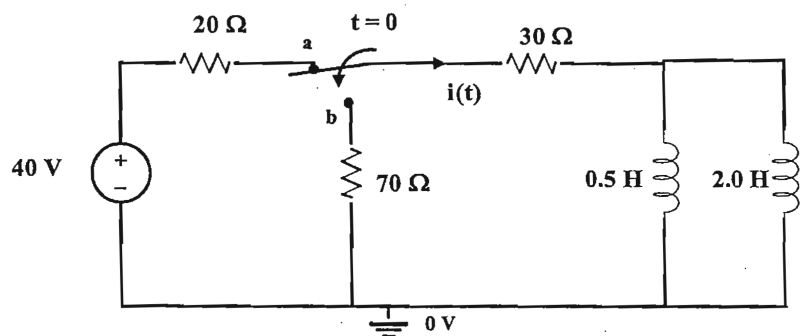


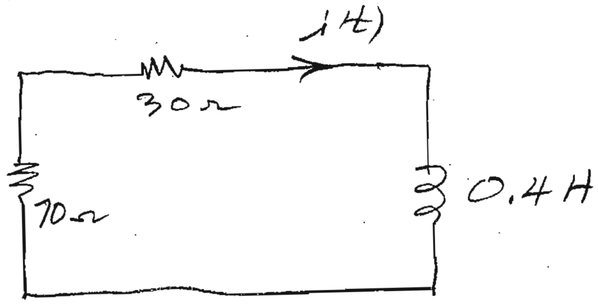
FOR $t < 0$

the coils look like a short.

$i(0^-) = \frac{40}{50} = 0.8\,A$

FOR $t > 0$

$i(0^+) = i(0^-) = 0.8\,A$



$100\,i + 0.4\frac{di}{dt} = 0$

$\frac{di}{dt} + 250\,i(t) = 0$ basic diff. eq.

The solution of the diff. eq. is of the form

$$i(t) = i_p(t) + i_c(t)$$

$i_p(t) = 0$ because the forcing function $= 0$

The characteristic equation is

$$s + 250 = 0$$

$$i(t) = i_c(t) = K e^{-250t} u(t) \text{ A}$$

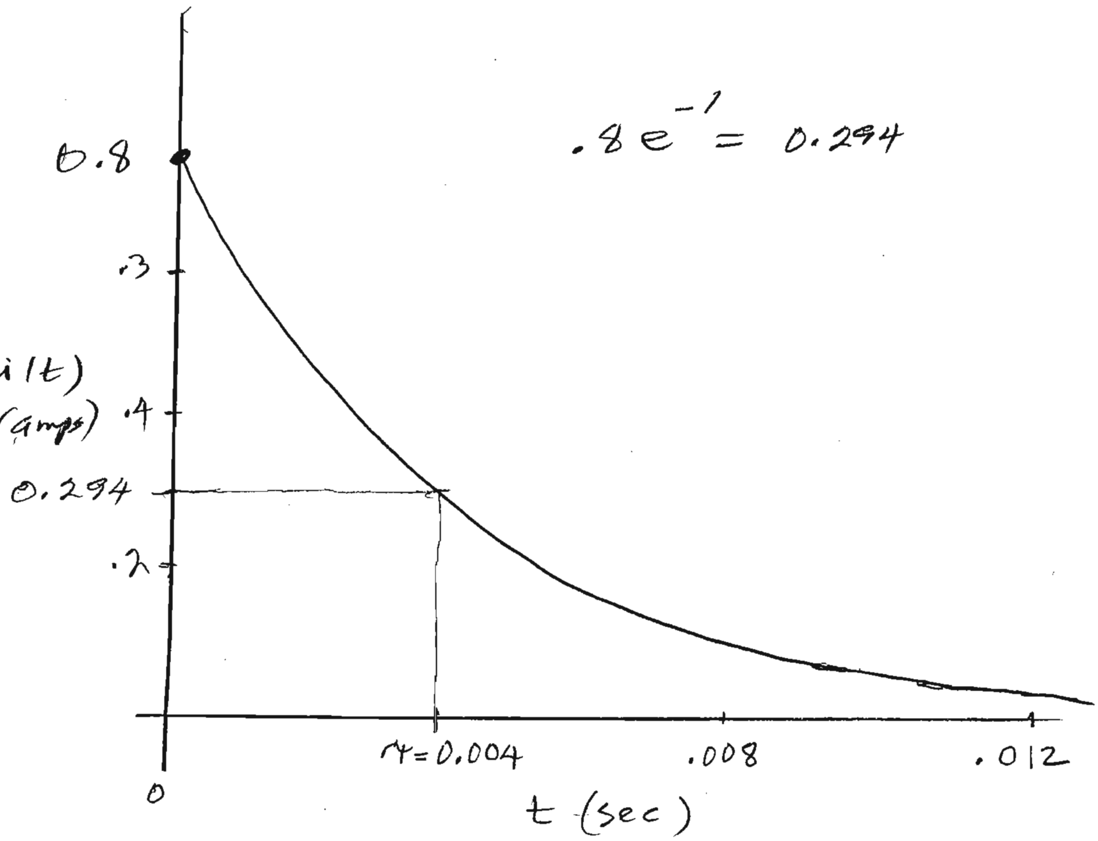
We know

$$i(0^+) = 0.8 = K e^{0} = K$$

$$\therefore \boxed{i(t) = 0.8 e^{-250t} u(t) \text{ A}}$$

1b)

$$.8e^{-1} = 0.294$$



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(2) You are given the AC circuit of Figure 2. Use a cosine reference. Use mesh analysis to solve for the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$. Give their values in polar form.

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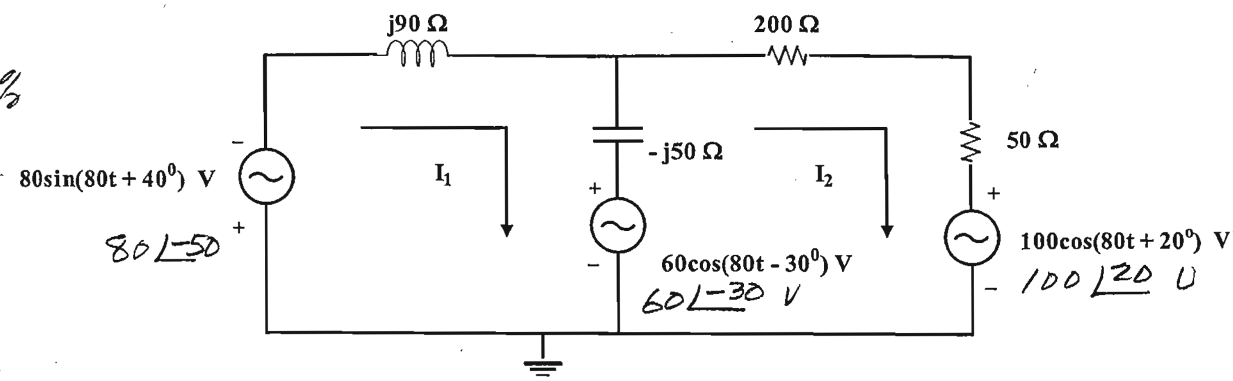

$80\sin(80t + 40) = 80\cos(80t - 50)$

As a phasor $80\angle{-50}$

For mesh #1

$$80\angle{-50} + j90\hat{I}_1 - j50\hat{I}_1 + j50\hat{I}_2 + 60\angle{-30} = 0$$

$$(j40)\hat{I}_1 + j50\hat{I}_2 = -80\angle{-50} - 60\angle{-30}$$
$$= 80\angle{130} + 60\angle{150}$$

mesh #2

$$-60\angle{-30} - j50\hat{I}_2 + j50\hat{I}_1 + 250 I_2 + 100\angle{20} = 0$$

$$j50 I_1 + (250 - j50) I_2 = 60\angle{-30} - 100\angle{20}$$

$$\begin{bmatrix} (0+j40) & (0+j50) \\ (0+j50) & (250-j50) \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 80\angle{130} + 60\angle{150} \\ 60\angle{-30} + 100\angle{-160} \end{bmatrix}$$

$\hat{I}_1 = 3.54\angle{63.37^\circ}$ A $\qquad \hat{I}_2 = 6.72\angle{-39.7^\circ}$ A

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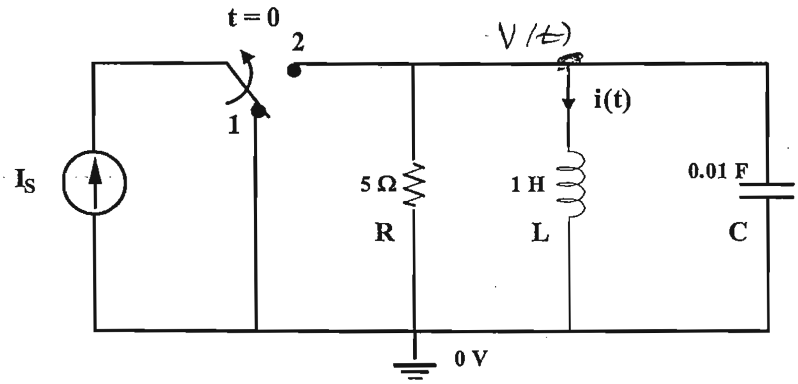
(3) You are given the parallel RLC circuit of Figure 3. The switch has been in position 1 for a very long time. All initial conditions on the circuit elements are zero for $t = 0^-$. At $t = 0$ the switch is moved from position 1 to position 2.



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(a) Develop the differential equation that can be used to solve for i(t).

(b) Give (determine) $i(0^+)$, and $\dfrac{di(0^+)}{dt}$.

(c) Which of the following forms would be used to find i(t) for this circuit? Explain your answer. You are not required to solve for A_1 and A_2.

- $i(t) = 10 + (A_1 + A_2 t)e^{-10t} \;\; u(t), \;\; A$
- $i(t) = 10 + A_1 e^{-2t} + A_2 e^{-5t} \;\; u(t), \;\; A$
- $i(t) = 10 + e^{-10t}\left[A_1 \cos(100t) + A_2 \sin(100t)\right] \;\; u(t), \;\; A$

For $t < 0$

$i(0^-) = 0 \quad \therefore \; i(0^+) = 0$ inductor

$V(0^-) = 0 \quad \therefore \; V(0^+) = 0$ capacitor

For $t > 0$

Nodal analysis

$$\frac{V}{R} + C\frac{dV}{dt} + i(t) = I_s$$

but $V(t) = L\dfrac{di}{dt}$

$$\frac{L}{R}\frac{di}{dt} + LC\frac{d^2 i}{dt^2} + i(t) = I_s$$

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i(t)}{LC} = \frac{I_s}{LC}$$

With numbers

$$\boxed{\frac{d^2 i}{dt^2} + 20\frac{di}{dt} + 100 i(t) = 100 I_s}$$

Characteristic Equation:

$$s^2 + 20s + 100 = 0$$

$$(s+10)(s+10) = 0$$

Form:

$$\boxed{i(t) = 10 + [A_1 + A_2 t]e^{-10t}\, u(t)\ A}$$

$$\boxed{i(0^+) = 0}$$

$$V(0^+) = 0 = L\frac{di(0^+)}{dt}$$

$$\boxed{\frac{di(0^+)}{dt} = 0}$$

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(4) You are given the circuit configuration of Figure 4. Assume **V** is the phasor voltage for $200\cos(100t + 20^\circ)$ V and **I** is the phasor current for $4\cos(100t - 10^\circ)$ A.
(a) Prepare the phasor diagram showing phasors **V** and **I**.
(b) Use inspection to determine if the load is inductive or capacitive. Explain your answer.

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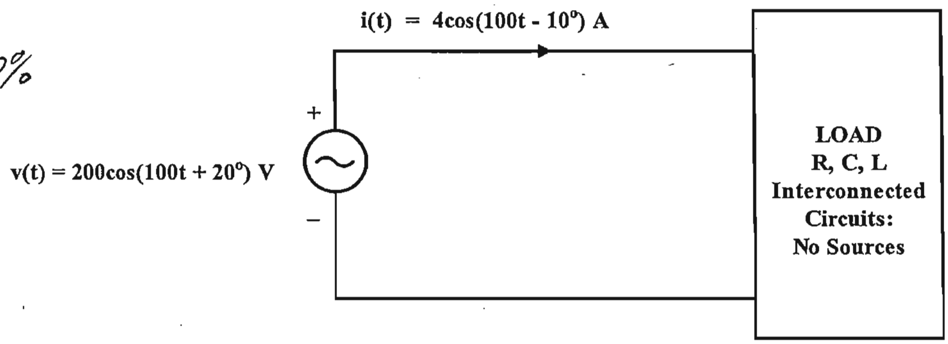
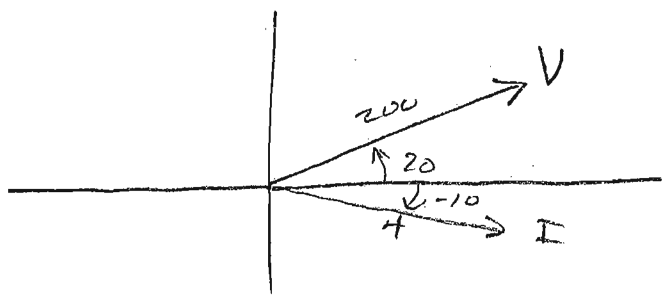


(a) $\vec{V} = 200\angle 20^\circ \qquad \vec{I} = 4\angle -10^\circ$



(b) Since $\vec{V}$ leads $\vec{I}$ the load is inductive.

Another View:

$$\vec{Z} = \frac{\vec{V}}{\vec{I}} = \frac{200\angle 20}{4\angle -10} = 50\angle 30\ \Omega$$

Since Z has a positive j part the load is inductive.

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(5) You are given the circuit shown in Figure 5. Assume a cosine reference.

(a) Use nodal analysis to find the phasor voltages $\mathbf{V_1}$ and $\mathbf{V_2}$. Express these voltages in polar form.
(b) Give the expression for $v_2(t)$.
(c) Prepare a phasor diagram for phasor $\mathbf{V_S}$, $\mathbf{V_1}$ and $\mathbf{V_2}$.
(d) For the phasors $\mathbf{V_S}$ and $\mathbf{V_1}$ which is leading and by how much angle?

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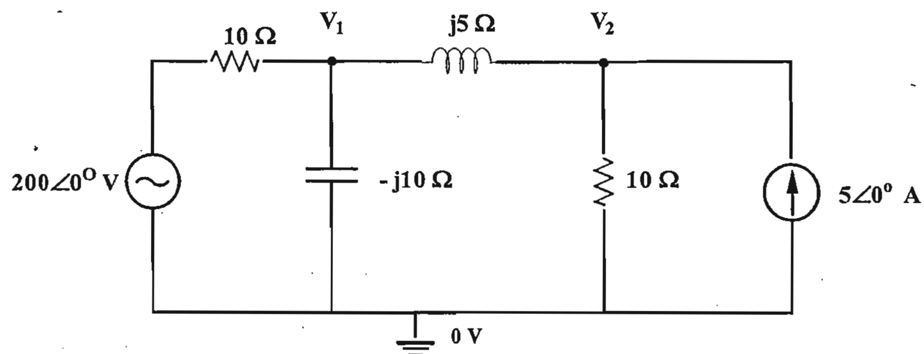


At V_1

$$\frac{V_1-200}{10}+\frac{V_1}{-j10}+\frac{V_1-V_2}{j5}=0$$

$$0.1V_1-20+j0.1V_1-j0.2V_1+j0.2V_2=0$$

$$\boxed{(0.1-j0.1)V_1+(0+j0.2V_2)=20}$$

At V_2

$$\frac{V_2}{10}+\frac{V_2-V_1}{j5}=5\angle 0$$

$$0.1V_2-j0.2V_2+j0.2V_1=5\angle 0$$

$$\boxed{0.2V_1+(0.1-j0.2)V_2=5\angle 0}$$

$$\begin{bmatrix}0.1-j0.1, & j0.2\\ j0.2 & 0.1-j0.2\end{bmatrix}\begin{bmatrix}\vec{V_1}\\ \vec{V_2}\end{bmatrix}=\begin{bmatrix}20\\ 5\end{bmatrix}$$

$V_1 = 126.9\angle{-23.1^\circ}\ V$ $\qquad$ $V_2 = 106.7\angle{-38.6^\circ}\ V$

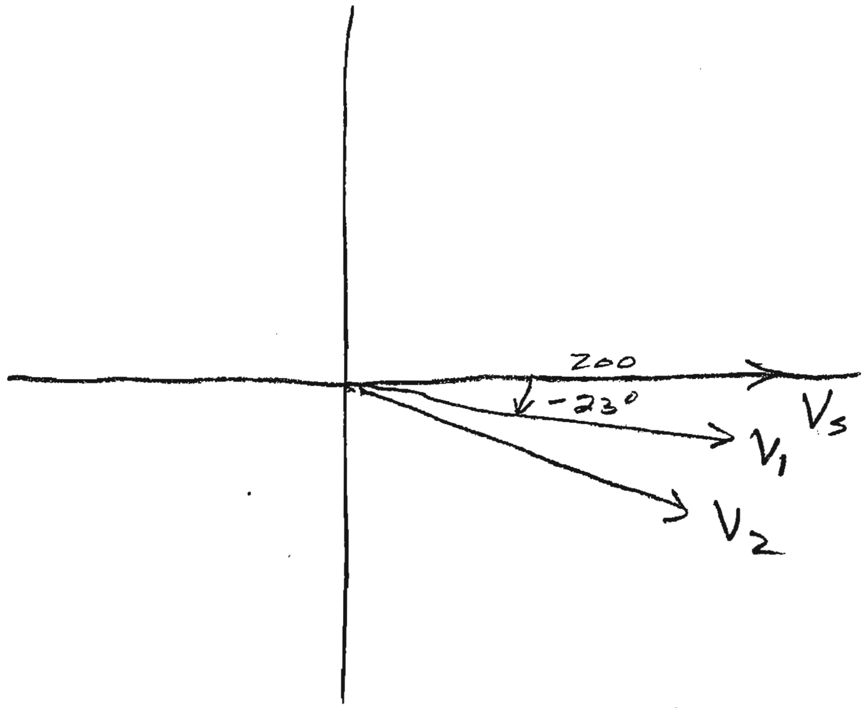
3B_5-2

(5)

(b)

$v_2(t) = 106.7\cos(\omega t - 38.6^\circ)\ V$

(c)



(d) V_s leads V_1 by 23°

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(6) Consider the circuit shown in Figure 6A. The switch has been open for a very long time and is closed at t = 0. Prior to t = 0, the capacitor is initially charged to 20 volts. The capacitor voltage for $t \geq 0$ is shown in Figure 6B.

(a) Determine the time constant for the circuit.

(b) If R = 20 Ω, what is the value of C?

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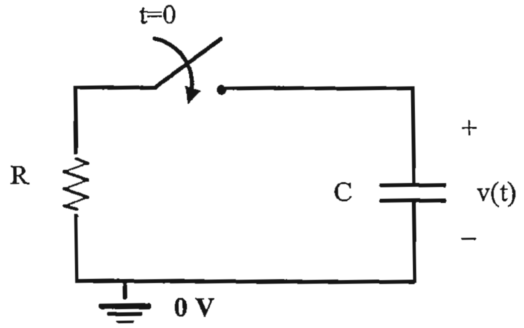

Figure 6B: Circuit for problem 6.

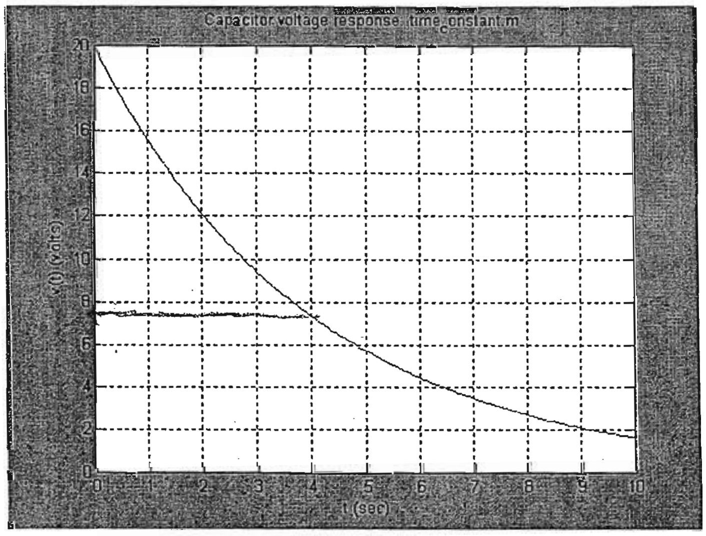

Figure 6B: Voltage response for the circuit of Figure 6B.

(a) $v(t) = 20e^{-\frac{t}{\tau}} \Big|_{t=\tau} = 20e^{-1} = 7.36$

At $v = 7.36$, $t = \boxed{\tau = 4}$

(b) $\tau = 4 = RC = 20\,C$

$C = \frac{4}{20} = \frac{1}{5} = 0.2\ F$