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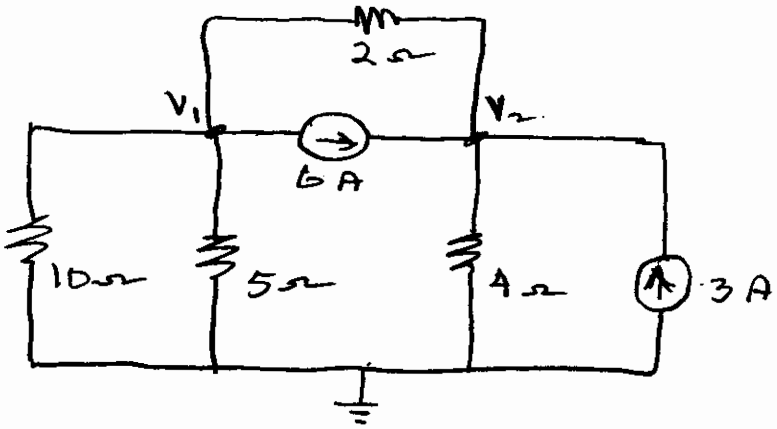
ECE 300
HW #3

3.2 Using nodal analysis, obtain V_1 & V_2 for the following circuit.



At V_1

(10)
$$\frac{V_1}{10} + \frac{V_1}{5} + 6 + \frac{V_1 - V_2}{2} = 0$$

gives

$$V_1 + 2V_1 + 60 + 5V_1 - 5V_2 = 0$$

clearing

$$\boxed{8V_1 - 5V_2 = -60}$$

At V_2

(4)
$$\frac{V_2}{4} + \frac{V_2 - V_1}{2} - 6 - 3 = 0$$

gives

$$V_2 + 2V_2 - 2V_1 = 36$$

clearing

$$\boxed{-2V_1 + 3V_2 = 36}$$

$$\begin{bmatrix} 8 & -5 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -60 \\ 36 \end{bmatrix}$$

$\boxed{V_1 = 0}$, $\boxed{V_2 = 12V}$

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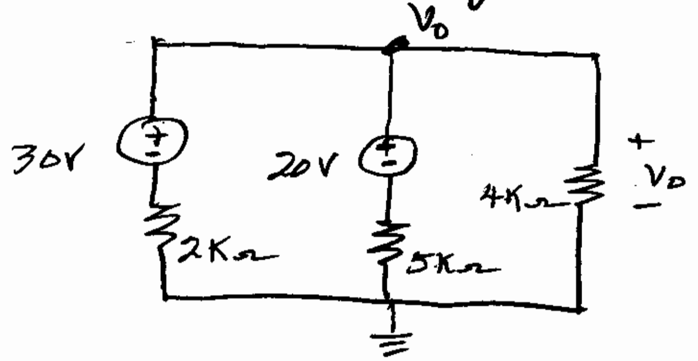
3.5 Use Nodal analysis to find V_o in the following circuit.



Just make V_o the node voltage.

$$\frac{V_o - 30}{2K} + \frac{V_o - 20}{5K} + \frac{V_o}{4K} = 0$$

20 com Dem

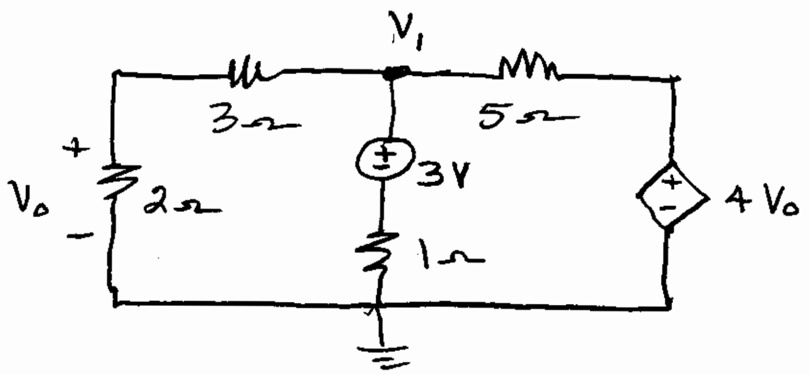
$$10V_o - 300 + 4V_o - 80 + 5V_o = 0$$

clearing

$$19V_o = 380$$

$$\boxed{V_o = 20\,V}$$

3.8 Use nodal analysis to find V_o.



$$\frac{V_1}{5} + \frac{V_1 - 3}{1} + \frac{V_1 - 4V_o}{5} = 0 \qquad (1)$$

but $V_o = 2\left(\frac{V_1}{5}\right)$ (2)

substitute (2) into (1)

$$\frac{V_1}{5} + \frac{V_1 - 3}{1} + \frac{V_1 - 8\frac{V_1}{5}}{5} = 0$$

OR

$$\frac{V_1}{5} + \frac{V_1 - 3}{1} + \frac{-3V_1}{25} = 0$$

com. dem 25

$$5V_1 + 25V_1 - 75 - 3V_1 = 0$$

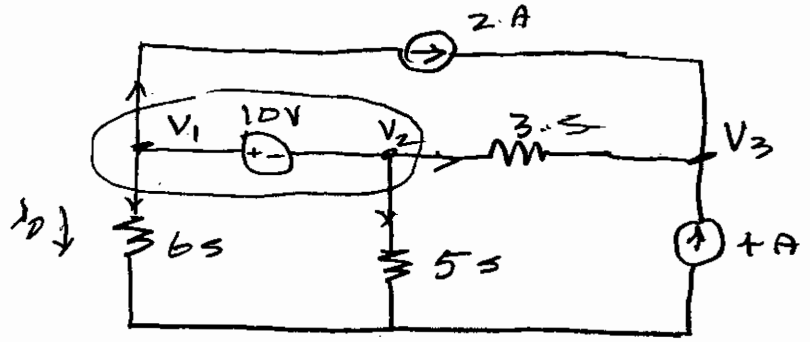
$$27V_1 = 75$$

$$V_1 = \frac{75}{27}$$

$$V_o = \frac{2}{5}\left(\frac{75}{27}\right) = 1.111 \text{ V}$$

$$\boxed{V_o = 1.111 \text{ V}}$$

3.15 Using nodal analysis, find i_0 and the power dissipated in each resistor in the following circuit.



At the super node

at super node

$$6V_1 + 2 + 5V_2 + (V_2 - V_3)3 = 0$$

$$\boxed{6V_1 + 8V_2 - 3V_3 = -2}$$

at V_3

$$3(V_3 - V_2) - 2 - 4 = 0$$

$$\boxed{0V_1 - 3V_2 + 3V_3 = 6}$$

Constraint Eq:

$$V_1 - 10 - V_2 = 0$$

OR

$$\boxed{V_1 - V_2 + 0V_3 = 10}$$

$$\begin{bmatrix} 6 & 8 & -3 \\ 0 & -3 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 10 \end{bmatrix}$$

$V_1 = 4.91\,V$ $V_2 = -5.09\,V$ $V_3 = -3.09\,V$

$$i_0 = V_1 \times 6 = 4.91 \times 6 = 29.46\,A$$

$$\boxed{i_0 = 29.46\,A}$$

3.15 cont.

$$P_{6\Omega} = V_1^2 \times 6 = (4.91)^2 \times 6 = 144.6\,W$$

$$\boxed{P_{6\Omega} = 144.6\,W}$$

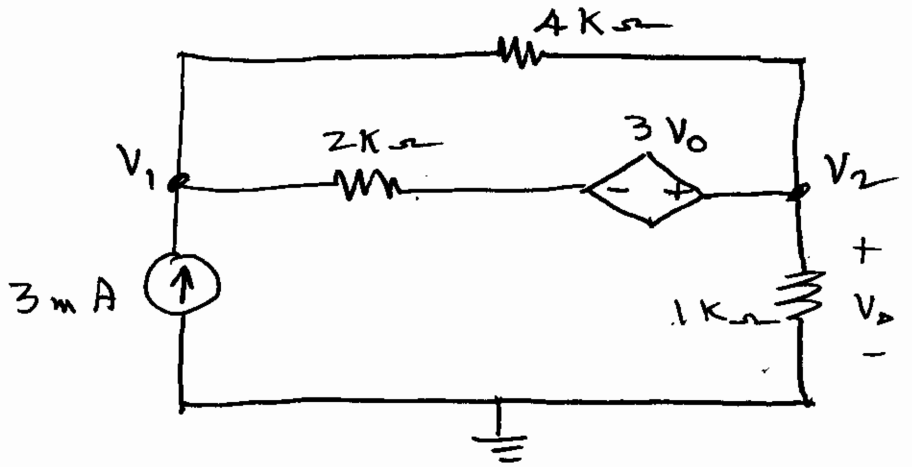
$$P_{5\Omega} = (V_2^2) \times 5 = (-5.09)^2 \times 5 = 129.5\,W$$

$$\boxed{P_{5\Omega} = 129.5\,W}$$

$$P_{3\Omega} = (V_3 - V_2)^2\, 3 = (-3.09 + 5.09)^2\, 3$$

$$\boxed{P_{3\Omega} = 12\,W}$$

3.21 Use nodal analysis to find V_1 and V_2 in the following circuit.



At V_1

$$\frac{V_1 + 3V_0 - V_2}{2K} + \frac{V_1 - V_2}{4K} - 3K^{-1} = 0$$

multiply thru by 4K

$$2V_1 + 6V_0 - 2V_2 + V_1 - V_2 = 12$$

$$V_0 = V_2$$

$$\boxed{3V_1 + 3V_2 = 12}$$

At V_2

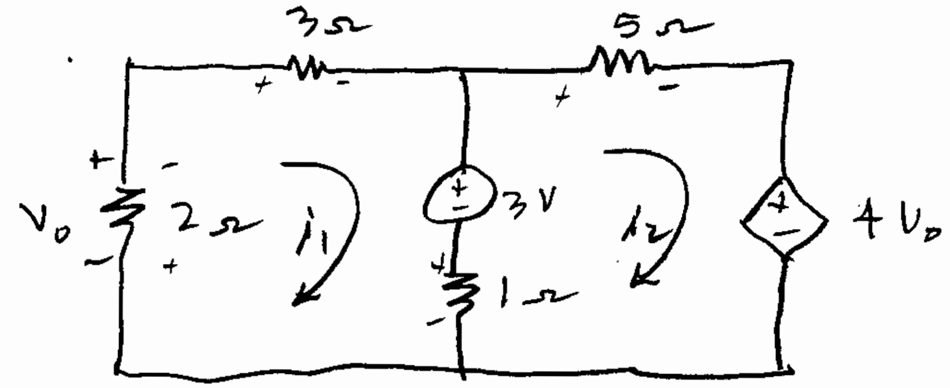
$$\frac{V_2}{1K} + \frac{V_2 - V_1}{4K} + \frac{V_2 - 3V_o - V_1}{2K}$$

$$4V_2 + V_2 - V_1 + 2V_2 - 6V_0 - 2V_1$$

$$\boxed{-3V_1 + V_2 = 0}$$

$V_1 = 1\,V$, $V_2 = 3\,V$

3.37 Find V_o in the following circuit by using mesh analysis.



$$2i_1 + 3i_1 + (i_1 - i_2) + 3 = 0$$

$$\boxed{6i_1 - i_2 = -3}$$

$$-(i_1 - i_2) + 5i_2 - 3 + 4V_o = 0$$

$$V_o = -2i_1$$

$$-i_1 + i_2 + 5i_2 - 8i_1 = 3$$

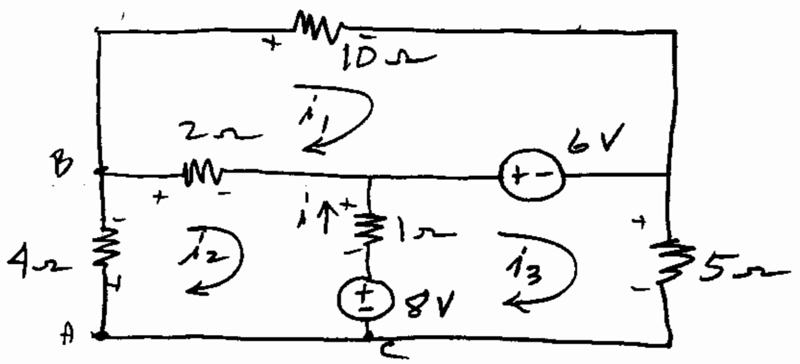
$$\boxed{-9i_1 + 6i_2 = 3}$$

$$i_1 = -0.555$$

$$V_o = -2i_1 = -1.111\text{ V}$$

$$\boxed{V_o = -1.111\text{ V}}$$

3.41 Use mesh analysis to find i in the following circuit.



At A, cw, Σ Drops = 0

$$4i_2 + 2(i_2 - i_1) + 1(i_2 - i_3) + 8 = 0$$

$$\boxed{-2i_1 + 7i_2 - i_3 = -8}$$

At B, cw, Σ drops = 0

$$10i_1 - 6 - 2(i_2 - i_1) = 0$$

$$\boxed{12i_1 - 2i_2 + 0i_3 = 6}$$

At C, cw, Σ Drops = 0

$$-8 - 1(i_2 - i_3) + 6 + 5i_3 = 0$$

$$\boxed{0i_1 - i_2 + 6i_3 = 2}$$

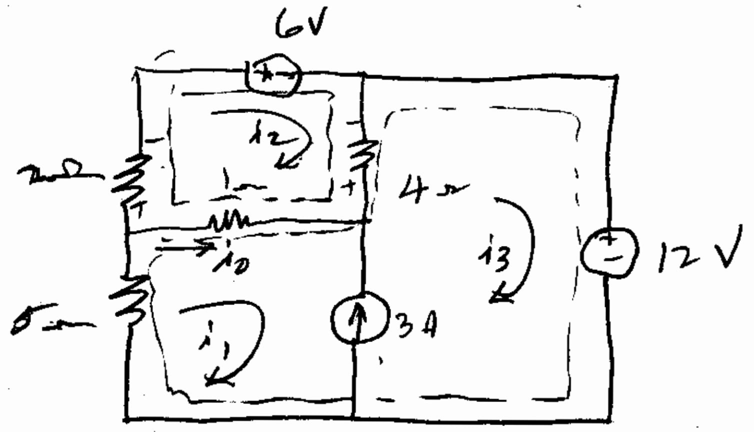
$$\begin{bmatrix} 12 & -2 & 0 \\ -2 & 7 & -1 \\ 0 & -1 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 2 \end{bmatrix}$$

$i_1 = 0.329$ $\quad i_2 = -1.026$ $\quad i_3 = 0.162$

$$i = i_3 - i_2 = 0.162 - (-1.026) =$$

$$\boxed{i = 1.88\ A}$$

3.44 Use mesh analysis to find i_o in the following circuit.



$$5i_1 + 1(i_1 - i_2) + 4(i_3 - i_2) + 12 = 0$$

$$6i_1 - 5i_2 + 4i_3 = -12$$

$$2i_2 + 6 + 4(i_2 - i_3) + 1(i_2 - i_1) = 0$$

$$-i_1 + 7i_2 - 4i_3 = -6$$

$$-i_1 + 0i_2 + i_3 = 3$$

$$\begin{bmatrix} 6 & -5 & 4 \\ -1 & 7 & -4 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -12 \\ -6 \\ 3 \end{bmatrix}$$

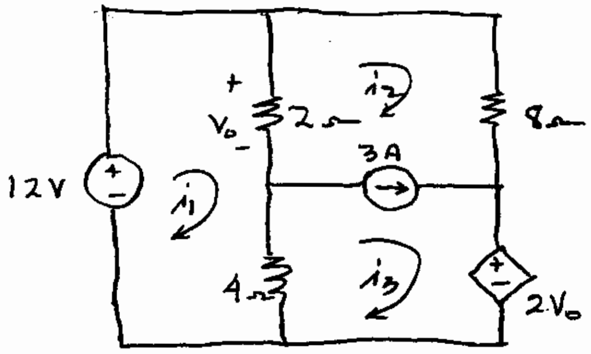
-1.733

$i_1 = -3.067A$, $i_2 = -1.333$, $i_3 = -0.066$

$$i_o = i_1 - i_2 = -3.067 + 1.333 = -1.734A$$

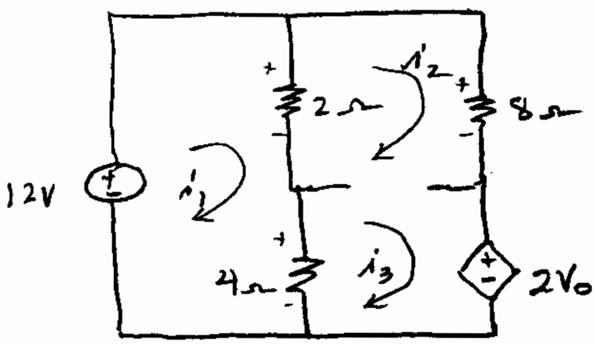
$$i_o = -1.734 \text{ A}$$

3.52 Use mesh analysis to find i_1, i_2, i_3.



Original circuit

Fig 1:



With current source removed

Fig 2:

From Figure 2 we write;

$$-12 + 2(i_1 - i_2) + 4(i_1 - i_3) = 0$$

or

$$\boxed{6i_1 - 2i_2 - 4i_3 = 12} \qquad (1)$$

$$-4(i_1 - i_3) - 2(i_1 - i_2) + 8i_2 + 2V_o = 0 \qquad (2)$$

with $V_o = 2(i_1 - i_2)$ into (2)

$$-4(i_1 - i_3) - 2(i_1 - i_2) + 8i_2 + 2[2(i_1 - i_2)] = 0 \quad (3)$$

clearing

$$\boxed{-2i_1 + 6i_2 + 4i_3 = 0} \qquad (4)$$

constraint equation is

$$i_3 - i_2 = 3$$

or

$$\boxed{0i_1 - i_2 + i_3 = 3} \qquad (5)$$

$$\begin{bmatrix} 6 & -2 & -4 \\ -2 & 6 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 3 \end{bmatrix}$$

$\boxed{i_1 = 3.5\text{ A}}$, $\boxed{i_2 = -0.5\text{ A}}$, $\boxed{i_3 = 2.5\text{ A}}$

3.61

Calculate the current gain i_o / i_s in the circuit below:

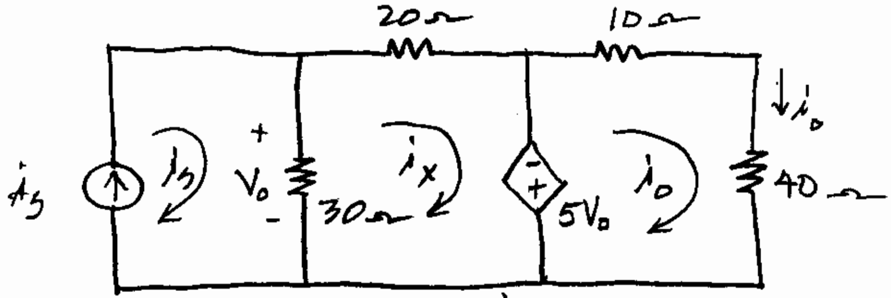


Figure 1

Nodal analysis would probably be easier but I will work this using mesh

Redrawing Figure 1 with the current source removed gives;

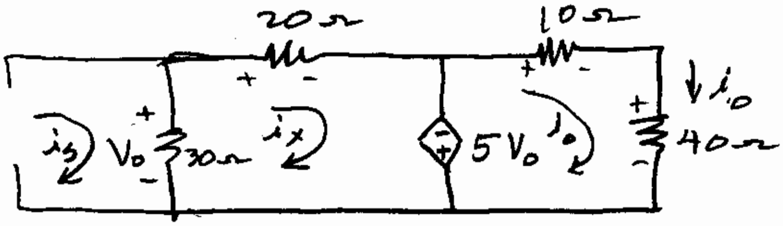


Figure 2

From Figure 2 we write

$$-30(i_s - i_x) + 20 i_x - 5V_o = 0 \qquad (1)$$

but

$$V_o = 30(i_s - i_x) \qquad (2)$$

substitute (2) into (1)

$$-30(i_s - i_x) + 20 i_x - 5[30(i_s - i_x)] = 0 \qquad (3)$$

clearing gives,

$$-30 i_s + 30 i_x + 20 i_x - 150 i_s + 150 i_x = 0$$

or

$$\boxed{-180 i_s + 200 i_x + 0 i_s = 0} \qquad (*)$$

From Figure 2, second mesh,

$$5V_o + 10i_o + 40i_o = 0 \quad (5)$$

substituting Eq (2) into (5) gives

$$5[30(i_s - i_x)] + 50i_o = 0$$

$$\boxed{150i_s - 150i_x + 50i_o = 0} \quad (6)$$

so, we have from (4) and (6)

$$-180i_s + 200i_x + 0i = 0 \quad (7)$$

$$150i_s - 150i_x + 50i = 0 \quad (8)$$

Strategy: Eliminate i_x from (7) & (8)

multiply (7) by 1.5; multiply (8) by 2

$$-270i_s + 300i_x + 0i_o = 0 \quad (9)$$

$$300i_s - 300i_x + 100i_o = 0 \quad (10)$$

Add (9) and (10)

$$30i_s + 100i_o = 0$$

or

$$100i_o = -30i_s$$

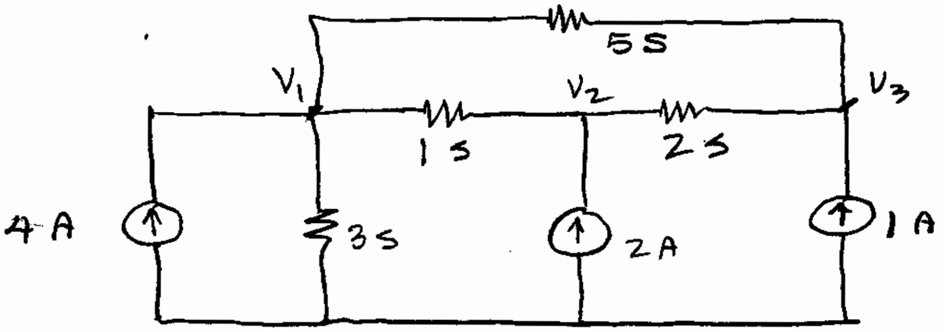
$$i_o = -0.3i_s$$

or

$$\frac{i_o}{i_s} = -0.3$$

3.68

By inspection, write the node-voltage equations for the following.



The equations will be of the form;

$$\begin{bmatrix} G_{11} & -G_{12} & -G_{13} \\ -G_{21} & G_{22} & -G_{23} \\ -G_{31} & -G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \Sigma i_1 \\ \Sigma i_2 \\ \Sigma i_3 \end{bmatrix}$$

G_{11} = sum of admittance tied to node V_1

G_{22} = sum of admittance tied to node V_2

G_{33} = sum of admittance tied to node V_3

$G_{12} = G_{21}$ = sum of admittance connected between nodes V_1 and V_2

$G_{13} = G_{31}$ = sum of admittance connected between nodes V_1 and V_3

$G_{23} = G_{32}$ = sum of admittance connected between nodes V_2 and V_3

We have

$G_{11} = 3 + 1 + 5 = 9S$

$G_{22} = 2 + 1 = 3S$

$G_{33} = 2 + 5 = 7S$

3.68 continued

$G_{12} = G_{21} = -1\ S$

$G_{13} = G_{31} = -5\ S$

$G_{23} = G_{32} = -2\ S$

We have

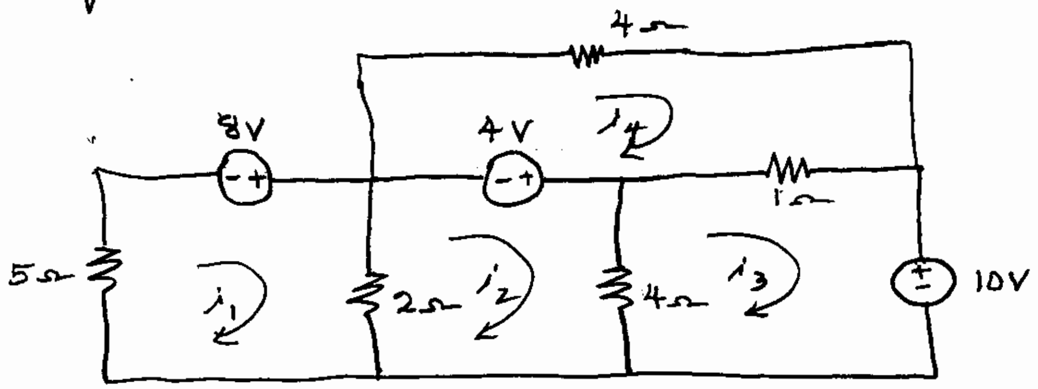
$$\begin{bmatrix} 9 & -1 & -5 \\ -1 & 3 & -2 \\ -5 & -2 & 7.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

solving for V_1, V_2, V_3

$V_1 = 2.33\,V$, $V_2 = 3.27\,V$, $V_3 = 2.75\,V$

3.72

By inspection, write the mesh equations for the following circuit



From mesh 1

$$7i_1 - 2i_2 = 8$$

From mesh 2

$$6i_2 - 4i_3 = 4$$

From mesh 3

$$-4i_2 + 5i_3 - i_4 = -10$$

From mesh 4

$$-i_3 + 5i_4 = -4$$

OR

$$\begin{bmatrix} 7 & -2 & 0 & 0 \\ 0 & 6 & -4 & 0 \\ 0 & -4 & 5 & -1 \\ 0 & 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -10 \\ -4 \end{bmatrix}$$