Desk Copy

ECE 300
Spring Semester, 2005
HW Set #7

Due: March 15, 2005 Name Green
wlg AM PM Print(last, first)

Use Engineering Paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. <u>Be sure to show how you got your answers.</u> Each problem counts 5 points.**

7.5 $v(t) = 4e^{12t}$ **V u(t)**

7.7 $i(t) = 20e^{-(13/8)t}$ **A u(t)**

7.12 $i(t) = 4e^{-2t}$ **A u(t)**

7.15 (a) 0.25 sec (b) 0.5 msec

7.24 (a), (b), (c)
(c) $x(t) = r(t-1) - r(t-2) - r(t-3) + r(t-4)$
(a) and (b) on your own

7.26 (a), (b)
(b) $v_2(t) = [2u(t-2) - r(t-2) + r(t-4)]$ **V**

7.42 (a) $v_0(t) = 8(1 - e^{-0.25t})$ **V**

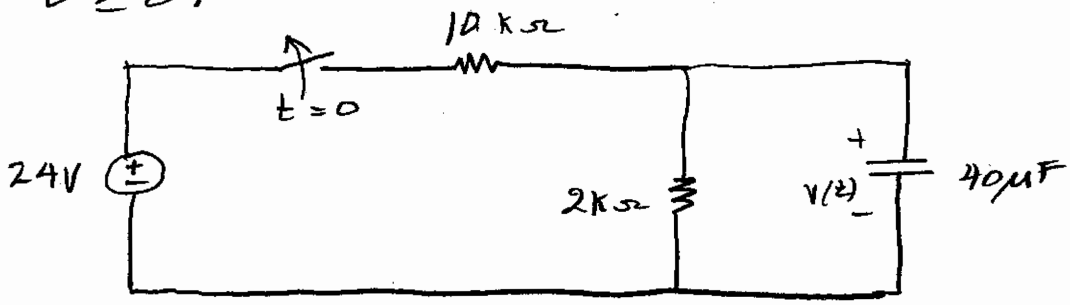
(b) $v_0(t) = 8e^{-(1/12)t}$ **V u(t)**

7.45 $v(t) = 1.25(1 - e^{-0.2t})$ **V u(t) ;** $i(t) = 0.125e^{-0.2t}$ **A u(t)**

7.53 (a) $i(0) = 5$ **A;** $i(t) = 5e^{-0.5t}$ **A u(t) ; (b)** $i(0) = 6$ **A,** $i(t) = 6e^{-(2/3)t}$ **A u(t)**

7.56 $v(t) = -4e^{-20t}$ **V u(t)**

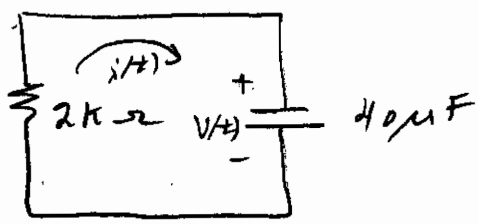
7.6 Switch has been closed for a long time. Opened at $t=0$: Find $v(t)$; $t \geq 0$.



<u>Before $t=0$:</u>

$$V(0^-) = \frac{24 \times 2K}{2K + 10K} = 4\ V$$

<u>$t \geq 0$</u>



$$R\,i(t) + v(t) = 0$$

$$RC\frac{dv}{dt} + v(t) = 0$$

$$\frac{dv}{dt} + \frac{v(t)}{RC} = 0$$

$$V(t) = V(0)\,e^{-\frac{t}{\tau}}$$

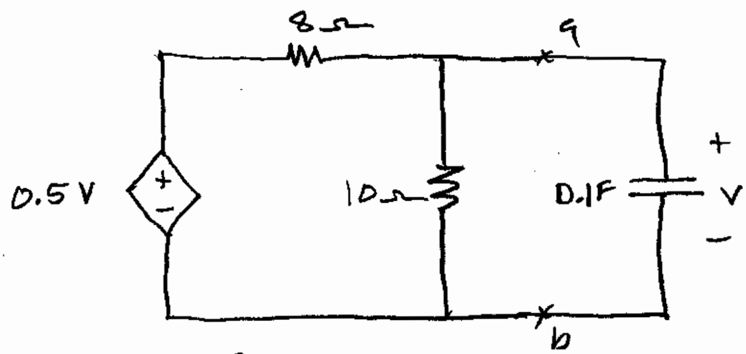
$\tau = RC$

$\tau = 2 \times 10^3 \times 40 \times 10^{-6}$

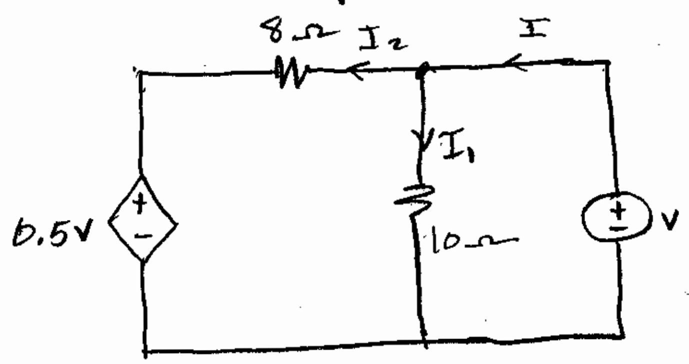
$\tau = .08$

$$\boxed{v(t) = 4\,e^{-12.5t}\,u(t)}$$

7.7 $V(0) = 20\text{ V}$ in the circuit below.
Find $V(t)$, $t \geq 0$.



Must find R_{TH} looking into a-b with the capacitor removed.



$$I = I_1 + I_2$$

$$I = \frac{V}{10} + \frac{V - 0.5V}{8} = \frac{V}{10} + \frac{0.5V}{8}$$

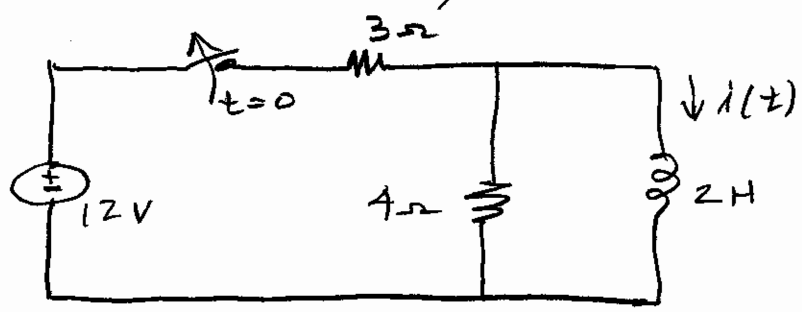
$$I = V\left(\frac{1}{10} + \frac{0.5}{8}\right] = \left(\frac{13}{80}\right) V$$

$$\frac{V}{I} = R_{eq} = \frac{80}{13}\ \Omega$$

$$\tau = \frac{80}{13} \times 0.1 = \frac{8}{13}$$

$$\boxed{V(t) = 20e^{-\frac{13t}{8}} \qquad t \geq 0}$$

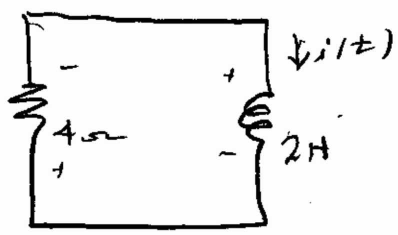
7.12 switch has been closed for a long time. Find $i(t)$, $t \geq 0$



$t<0$; circuit is in steady state.

$i(0) = \frac{12}{3} = 4\ A$

$t \geq 0$



$$L\frac{di}{dt} + Ri(t) = 0$$

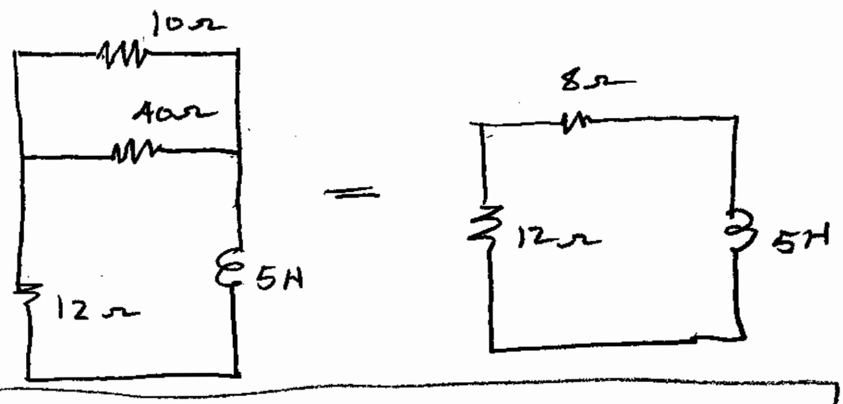
$$\frac{di}{dt} + i(t)\frac{R}{L} = 0 \qquad \tau = \frac{L}{R} = 0.5$$

$$i(t) = i(0)e^{-\frac{t}{\tau}}$$

$$\boxed{i(t) = 4e^{-2t} \qquad t \geq 0}$$

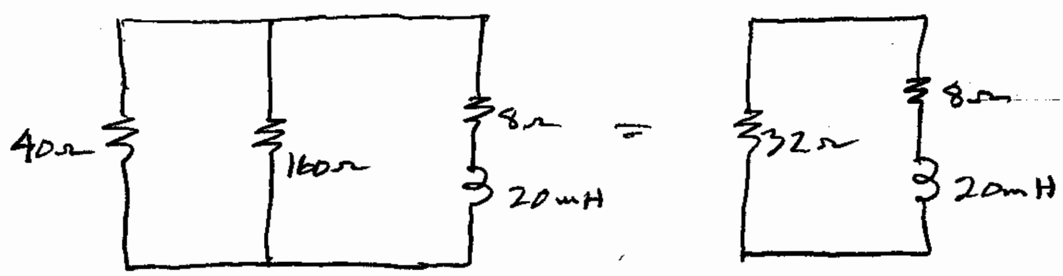
7.15 Find the time constant for each of the following circuits.

(a)



$$\boxed{\tau = \frac{L}{R_{eq}} = \frac{5}{20} = 0.25 \text{ sec}}$$

(b)



$$\tau = \frac{L}{R_{eq}} = \frac{20\text{mH}}{40}$$

$$\boxed{\tau = 0.5 \times 10^{-3} \text{ sec} = 0.5 \text{ msec}}$$

7.24

Express the following in terms of singularity functions.

(a) $\boxed{v(t) = -5\,u(t)}$

(b) $$i(t) = \begin{cases} 0 & t < 1 \\ -10 & 1 < t < 3 \\ 10 & 3 < t < 5 \\ 0 & t > 5 \end{cases}$$

$$i(t) = -10[u(t-1) - u(t-3)] + 10[u(t-3) - u(t-5)]$$

$$\boxed{i(t) = -10u(t-1) + 20u(t-3) - 10u(t-5)}$$

(c) $$x(t) = \begin{cases} t-1 & 1 < t < 2 \\ 1 & 2 < t < 3 \\ 4-t & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = (t-1)[u(t-1) - u(t-2)] + [u(t-2) - u(t-3)] + (4-t)[u(t-3) - u(t-4)]$$

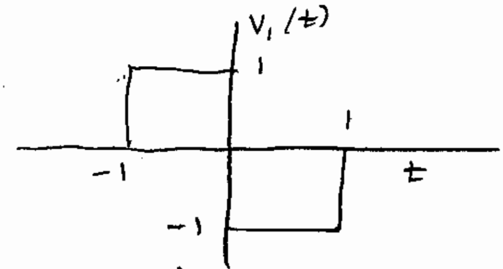
$$(t-1)u(t-1) - (t-2)u(t-2) - (t-3)u(t-3) + (t-4)u(t-4)$$

$$\boxed{x(t) = r(t-1) - r(t-2) - r(t-3) + r(t-4)}$$

7.26

Express the following wave forms in terms of singular functions.

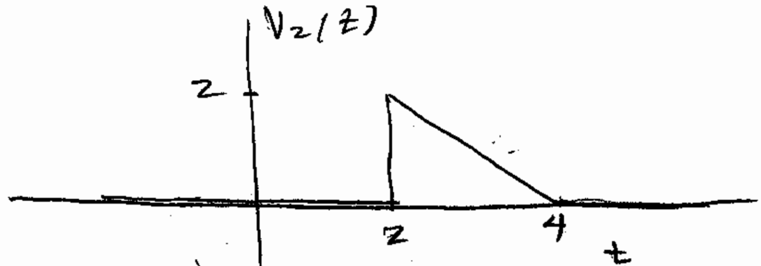
(a)

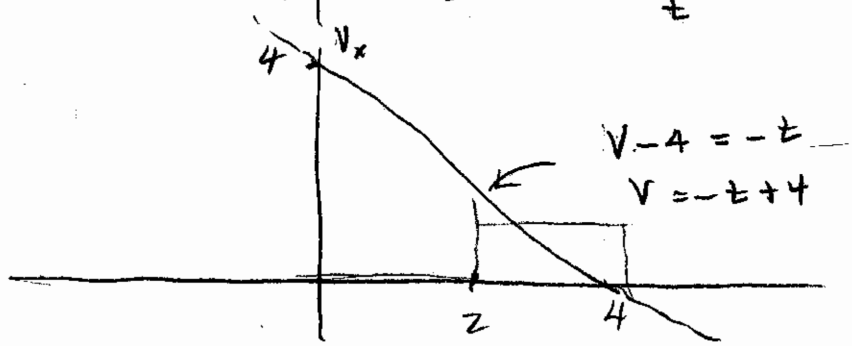


by inspection

$$v_1(t) = u(t+1) - 2u(t) + u(t-1)$$

(b)





$$-(t-4)\left[u(t-2) - u(t-4)\right]$$

$$-(t-4)u(t-2) + (t-4)u(t-4)$$

$$-(t-2)u(t-2) + 2u(t-2) + (t-4)u(t-4)$$

so

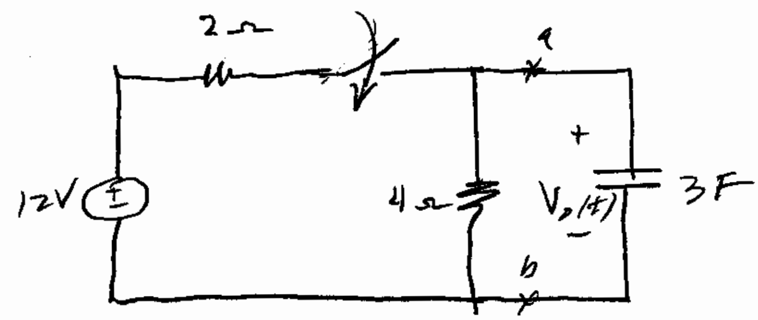
$$2u(t-2) - (t-2)u(t-2) + (t-4)u(t-4)$$

$$\boxed{2u(t-2) - r(t-2) + r(t-4)}$$

7.42 Consider the circuit below.

(a) The switch has been open for a long time and is closed at $t=0$. Find $V_o(t)$.

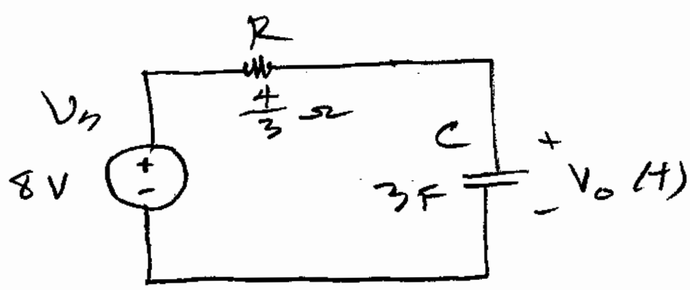
(b) Suppose that the switch has been closed for a long time and is opened at $t=0$. Find $V_o(t)$



(a) Method 1: Direct differential equation approach.

We note $V_o(0) = 0$.

After the switch is closed we can look into terminals a-b and form a Thevenin equivalent. This is shown below.



The d.e. for the circuit is

$RC = \tau = 4$

7.42 cont.

$$\frac{dV_o(t)}{dt} + \frac{v(t)}{RC} = \frac{V_s}{RC}$$

or

$$\frac{dV_o}{dt} + 0.25V(t) = 2$$

$$V_h = Ke^{-.25t}$$

$$V_F = K_F$$

$$\frac{K_F}{4} = 2$$

$$K_F = 8$$

$$V_o(t) = Ke^{-.25t} + 8$$

$$V_o(0) = 0 = K + 8$$

$$K = -8$$

$$V_o(t) = 8 - 8e^{-.25t}$$

<u>method II</u>

using

$$V_o(t) = V(\infty) + (V(0) - V(\infty))e^{-\frac{t}{\tau}}$$

$$\tau = RC = \frac{4}{3} \cdot 3 = 4 \text{ sec}$$

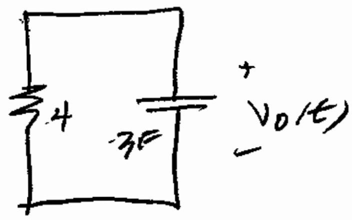
$$V(0) = 0, \quad V(\infty) = 8$$

so

$$\boxed{V_o(t) = 8 - 8e^{-0.25t} \qquad t \geq 0}$$

7.42

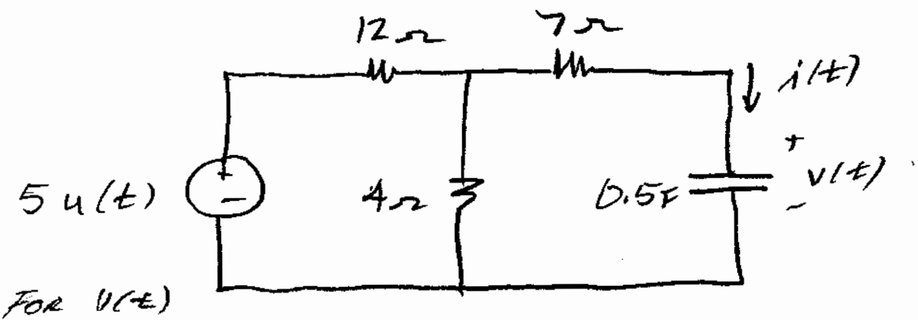
(b) $v_o(0) = 8$



$v(\infty) = 0, \quad \tau = 4 \times 3 = 12$ sec

$v(t) = 8 e^{-\frac{t}{12}} \quad t \geq 0$

7.45 Find the step responses for $v(t)$ and $i(t)$ when $v_s = 5u(t)$ V in the following circuit.



For $v(t)$

$$v(0) = 0$$

$$v(\infty) = \frac{5 \times 4}{4+12} = \frac{5}{4} = 1.25$$

$$R_{eq} = 7 + \frac{4 \times 12}{4+12} = 7 + 3 = 10\,\Omega$$

$$\tau = R_{eq} C = 10 \times 0.5 = 5$$

$$\therefore \boxed{v(t) = 1.25 - 1.25 e^{-0.2t} \quad t \geq 0}$$

For $i(t)$

$$\boxed{i(0) = 0 \quad t < 0}$$

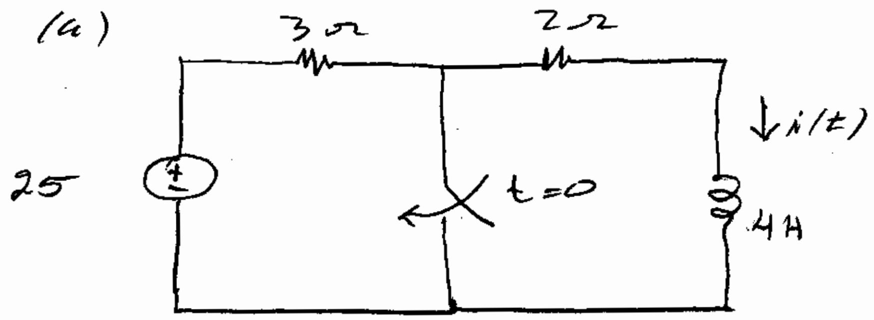
$$i(t) = C\frac{dv_c}{dt} = 0.5 \times 0.2 \times 1.25 e^{-0.2t} \quad t > 0$$

$$\boxed{i(t) = 0.125 e^{-0.2t} \quad t > 0}$$

7.53

Determine the inductor curr $i(t)$ for both $t < 0$ and $t > 0$ for each of the following circuits

(a)



$$i(0^-) = \frac{25}{5} = 5 \text{ Amps} = i(0^+)$$

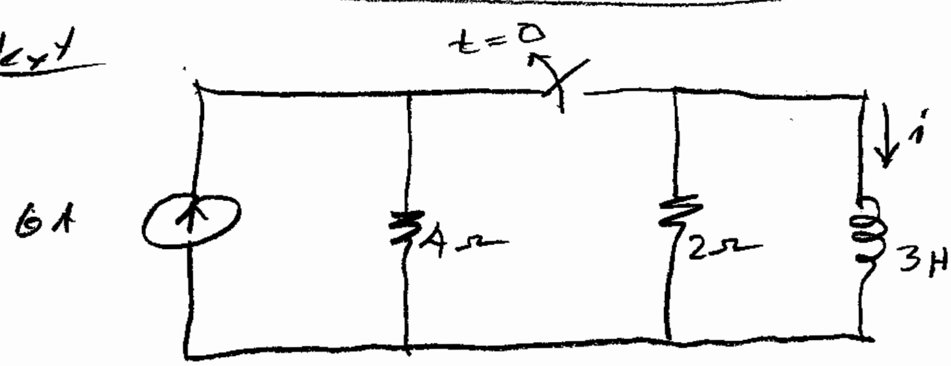
After the switch is closed;

$$i(t) = i(\infty) + (i(0^+) - i(\infty))e^{-\frac{t}{\tau}}$$

$$i(\infty) = 0; \quad \tau = \frac{L}{R} = \frac{4}{2} = 2 \text{ sec}$$

$$\boxed{i(t) = 5e^{-2t} \text{ A} \quad t \geq 0}$$

Next



7.53

By inspecting the circuit we see

$$\boxed{\begin{aligned} i(0^-) &= 6A \\ i(0^+) &= 6A \end{aligned}}$$

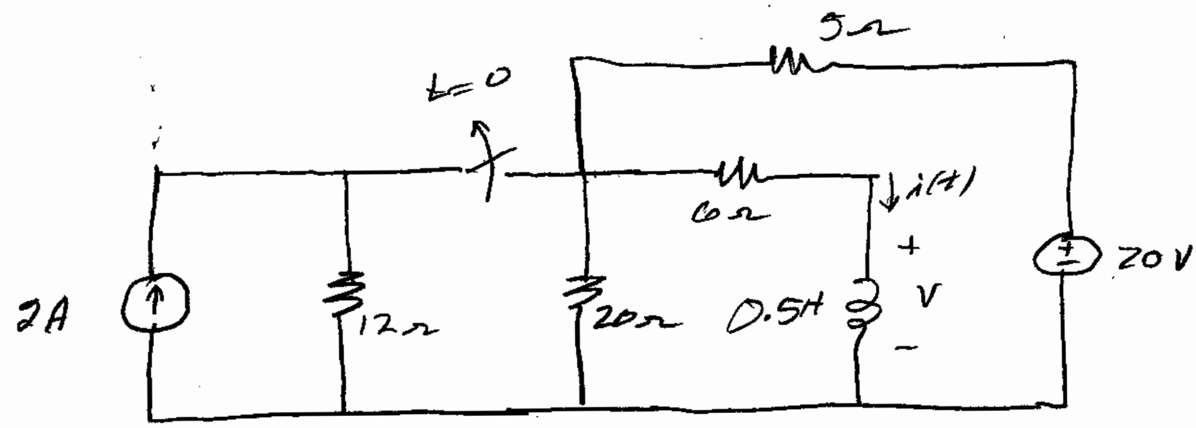
$$i(\infty) = 0$$

$$\frac{L}{R} = \frac{3}{2}$$

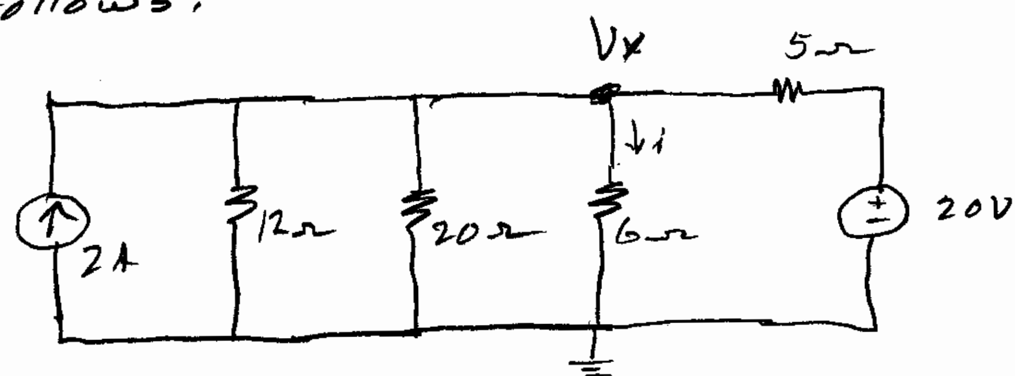
$$i(t) = i(\infty) + (i(0) - i(\infty))e^{-\frac{t}{\tau}}$$

$$\boxed{i(t) = 6e^{-\frac{2}{3}t} \qquad t \geq 0}$$

7.56 For the following circuit find $v(t)$ for $t>0$



Before $t=0$, the circuit is as follows:



Writing a node equation:

$$\frac{V_x}{12} + \frac{V_x}{20} + \frac{V_x}{6} + \frac{V_x - 20}{5} = 2$$

60

$$5V_x + 3V_x + 10V_x + 12V_x - 240 = 120$$

$$30V_x = 360$$

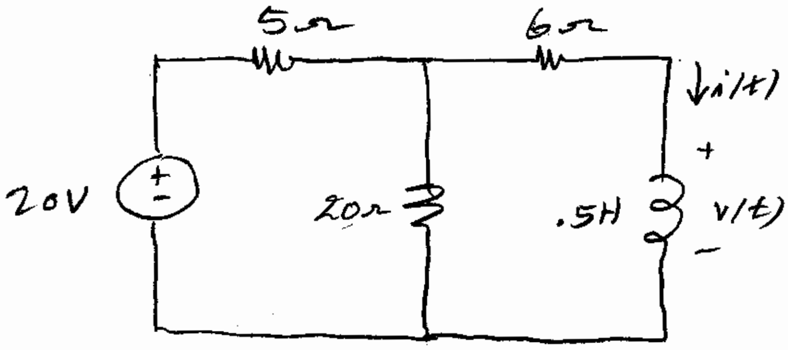
$$V_x = 12$$

$$i(0^-) = \frac{12}{6} = 2\,A$$

$$i(0^+) = 2\,A$$

7.56 Cont.

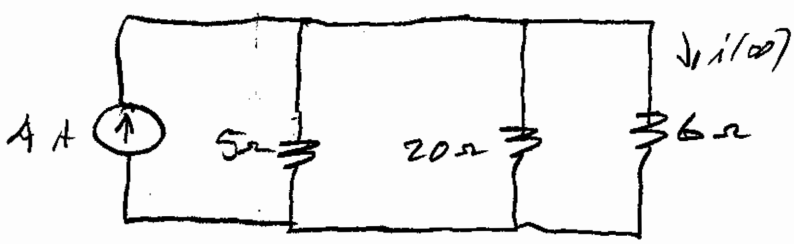
for $t > 0$



$$R_{eq} = 6 + \frac{20 \times 5}{20+5} = 6 + 4 = 10\ \Omega$$

$$\tau = \frac{L}{R} = \frac{0.5}{10} = 0.05$$

For $i(\infty)$



$$i(\infty) = \frac{4 \times 5 \| 20}{5 \| 20 + 6} = \frac{4 \times 4}{4+6} = 1.6\ A$$

$$i(t) = i(\infty) + (i(0) - i(\infty))e^{-\frac{t}{\tau}}$$

$$= 1.6 + (2 - 1.6)e^{-20t}$$

$$i(t) = 1.6 + 0.4e^{-20t}$$

$$V_L = L\frac{di}{dt} = 0.5(-.8)e^{-20t}$$

$$\boxed{V_L(t) = -4e^{-2t}\ V \qquad t > 0}$$