Desk Copy

ECE 300
Spring Semester, 2005
HW Set #10

Due: April 14, 2005 Name Green
wlg AM PM Print(last, first)

Version 2

Use Engineering Paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers. Each problem counts 5 points.**

11.4 $P_{supplied} = 42.7$ W, $P_{20} = 30.5$ W, $P_{10} = 12.2$ W; $P_c = P_L = 0$

11.13 $Z_L = 12.8 + j49.6$ ohms, $P_{max} = 90.1$ W

11.15 $Z_L = 5 - j0.5$ ohms, $V_{th} = 78.97\angle 71.57$ V, $P_o = 89.97$ W

11.22 $I_{RMS} = 2.449$ A

11.33 $I_{RMS} = 6.667$ A

11.49 (a) $S = 4 + j2.37$ kVA; (b) $S = 1.6 - j1.2$ kVA; (c) $S = 0.46 + j1.27$ kVA
(d) $S = 110.8 + j166.2$ VA

11.51 (a) pf = 0.9956 lagging; (b) P = 15.56 W; (c) Q = 1.466 VAR
(d) S = 15.63 VA; (e) $S = 15.56 + j1.466$ VA

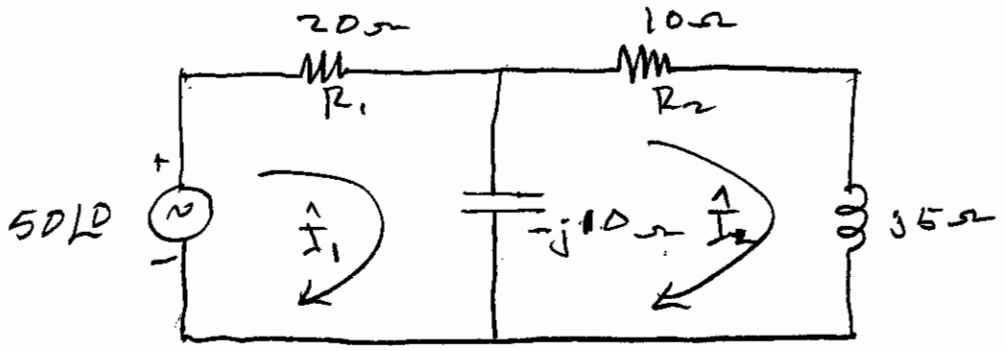
15.56 $S = 4.54 + j1.4$ VA

11.60 $V_o = 7.1\angle 32.3^\circ$ kV pf = 0.8454 lagging

11.74 (a) pf = 0.8992 lagging; (b) C = 5.74 mF

H.W. # 10

11.4 Find the average power absorbed by each element (device) in the following circuit.



Use mesh analysis to find $\hat{I}_1$ and $\hat{I}_2$.

$$\begin{bmatrix} 20-j10 & j10 \\ j10 & 10-j5 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0 \\ 0 \end{bmatrix}$$

$\hat{I}_1 = 1.74\angle 12.2^\circ$ $\qquad$ $\hat{I}_2 = 1.56\angle -51.3^\circ$ A

$P_{20} = \frac{1}{2}|\hat{I}_1|^2 R_1 = \underline{30.3}$ W

$P_{10} = \frac{1}{2}(1.56)^2 \times 10 = \underline{12.17}$ W

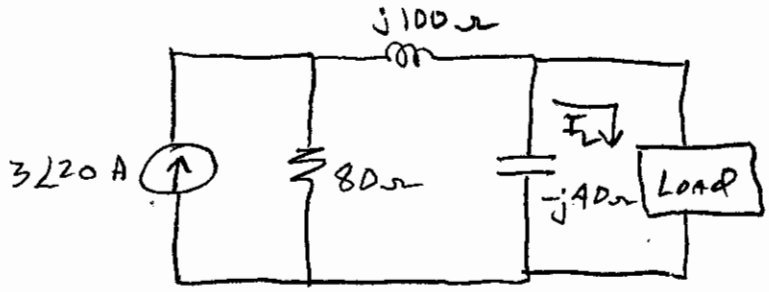
$P_{supplied} = \frac{50 \times 1.74}{2} \cos(12.2)$

$P_{supplied} = 42.6$ W

11.13 For the following circuit find;

(a) LOAD impedance for MAXIMUM pwr transfer.

(b) the value of power delivered to the load of part (a)



(a) FOR maximum pwr transfer make

$$Z_L = Z_{TH}^*$$

$$\hat{Z}_{TH} = \frac{(80+j100)(40\angle -90)}{80+j100-j40}$$

$$\hat{Z}_{TH} = 12.8 - j49.6\ \Omega$$

$$\therefore Z_L = 12.8 + j49.6\ \Omega$$

(b)

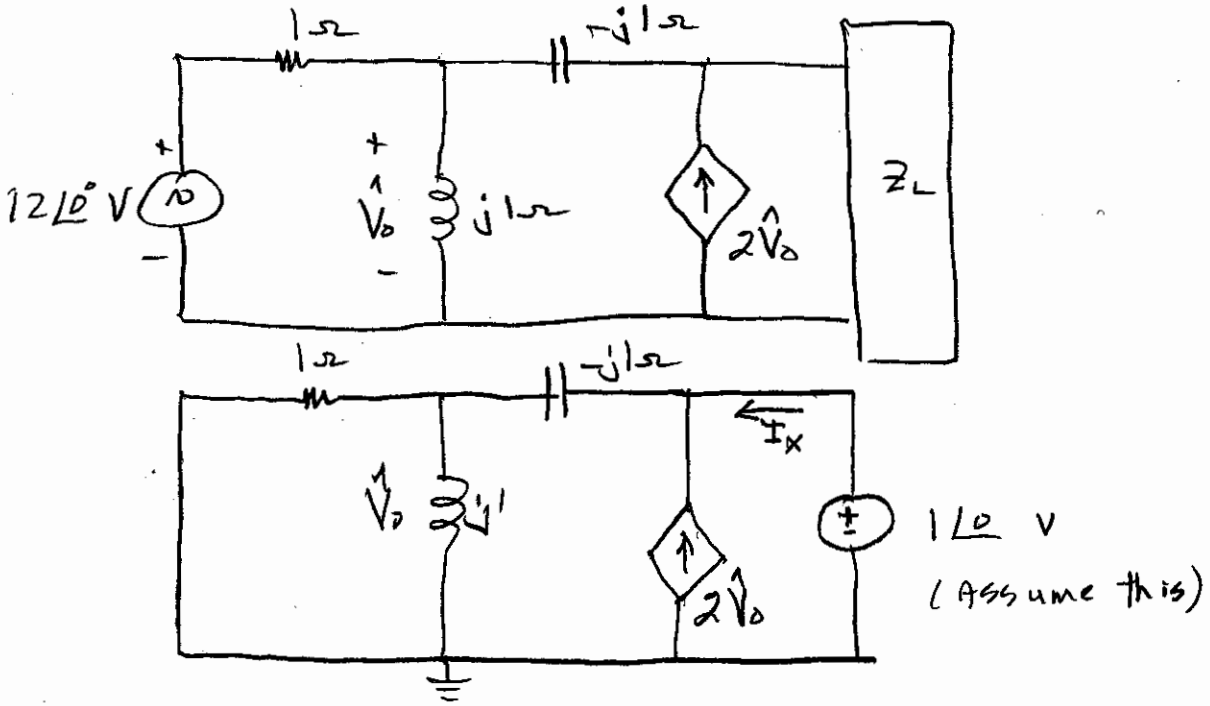
$$\hat{V}_{TH} = \left(\frac{(3\angle 20^\circ)(80)}{(80+j60)}\right)(40\angle -90) = 96\angle -46.9\ V$$

$$\hat{I}_L = \frac{V_{TH}}{2R}\ ;\ P_L = \frac{\hat{I}_L}{2} R_L = \frac{|V_{TH}|^2}{8R_L}$$

$$P_L = \frac{(96)^2}{8 \times 12.8} = 90\ W$$

$$P_L = 90\ W$$

11.15 Given the circuit below. Find Z_L for maximum power transfer and find the power delivered to the resulting load resistance.



$$\frac{\hat{V}_o}{1} - j\frac{V_o}{1} + j\frac{(\hat{V}_o - 1)}{1} = 0$$

$$\hat{V}_o - jV_o + jV_o - j = 0$$

$$I_x + 2V_o = \frac{1 - V_o}{-j} = j - jV_o$$

$$I_x = j - jV_o + 2V_o = j + V_o(-2 - j)$$

$$I_x = j + j(-2 - j) = j - 2j + 1 = 1 - j$$

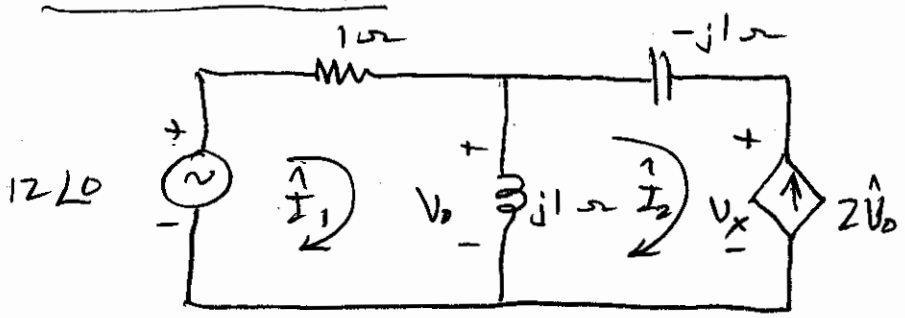
$$Z_{TH} = \frac{1}{I_x} = \frac{1}{1-j} = 0.5 + j0.5 \ \Omega$$

11.15 continued

For maximum power transfer make

$$\boxed{Z_L = Z_{TH}^* = 0.5 - j0.5}$$

Now find V_{TH}



$$(1+j)I_1 - jI_2 = 12 \qquad (1)$$

$$-jI_1 + V_x = 0 \qquad (2)$$

$$I_2 = -2V_o = -2j(I_1 - I_2)$$

$$(1-2j)I_2 = -2jI_1$$

$$I_2 = \frac{-j2I_1}{1-j2} \qquad (3)$$

Put (3) into (1)

$$(1+j)I_1 - j\left(\frac{-j2I_1}{1-j2}\right) = 12$$

Gives

$$I_1 = \frac{12(1-j2)}{1-j}$$

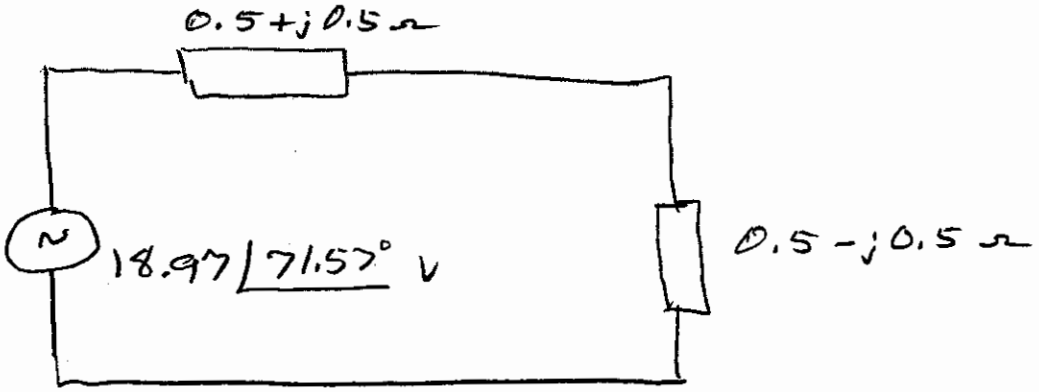
From (2),

$$V_x = V_{TH} = jI_1$$

11.15 CONTINUED

$$V_{TH} = \frac{12j\,(1-j2)}{(1-j)} = \frac{12\,(2+j)}{(1-j)}$$

$$\hat{V}_{TH} = 18.97 \angle 71.57^\circ \text{ V}$$

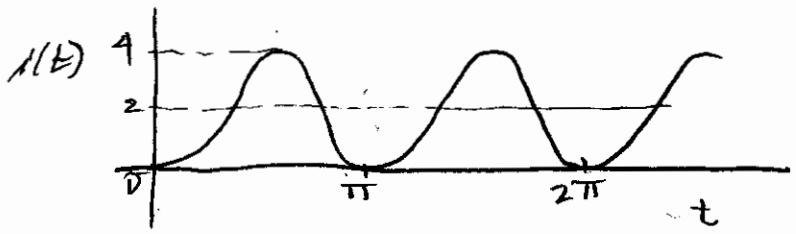


$$P_o = \frac{|\hat{V}_{TH}|^2}{8 \times 0.5} = \frac{(18.97)^2}{4}$$

$$P_o = 89.97 \text{ W}$$

11.22

Find the rms value of the signal below.



$i(t) = 2 - 2\cos 2t$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi}(2-2\cos 2t)^2 dt}$$

$$(2-2\cos 2t)^2 = 4 - 8\cos 2t + 4\cos^2 2t$$

$$4 - 8\cos 2t + 4\cos^2 2t = 4 - 8\cos 2t + \frac{4}{2}(1+\cos 4t)$$

So,

$$(2-2\cos 2t)^2 = 4 - 8\cos 2t + 2 + 2\cos 4t$$

$$= 6 - 8\cos + 2\cos 4t$$

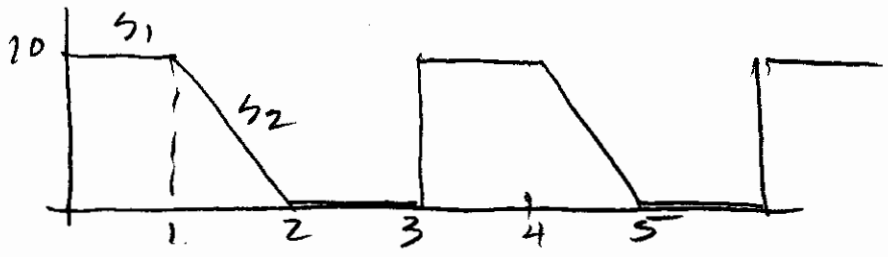
$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi}(6 - 8\cos 2t + 2\cos 4t)dt}$$

Reduces to (because of integrating the cosine)

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} 6\,dt} = \sqrt{\frac{1}{\pi}6\pi} = \sqrt{6}$$

$I_{rms} = 2.449$ A

11.33 Find the rms of the following signal.



$s_1 = 10$; $s_1^2 = 100$

$s_2 = -10(t-2)$; $s_2^2 = 100t^2 - 400t + 400$

$$I_{rms} = \sqrt{\frac{1}{3}\left[\int_0^1 100\,dt + \int_1^2 (100t^2 - 400t + 400)\,dt\right.}$$

$$I_{rms} = \sqrt{\frac{1}{3}\left[100 + \frac{100t^3}{3}\Big|_1^2 - 200t^2\Big|_1^2 + 400t\Big|_1^2\right.}$$

$$= \sqrt{\frac{1}{3}\left[100 + \frac{800}{3} - \frac{100}{3} - 800 + 200 + 400\right.}$$

$$= \sqrt{\frac{1}{3}\left[\frac{700}{3} - 200\right]}$$

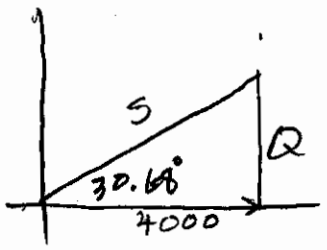
$$I_{rms} = 6.67 \text{ A}$$

21.49 Find the complex power of the following cases.

(a) $P = 4000\ W$, P.f. = .86 lagging

$\cos\theta = .86$

$\theta = 30.68^\circ$

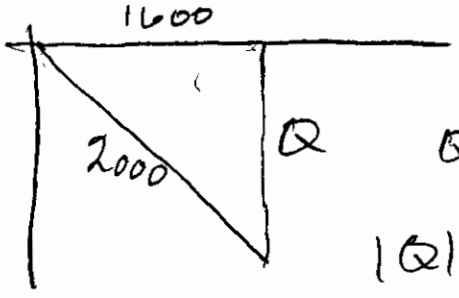


$\tan 30.68 = \frac{Q}{4000}$

$Q = 2373$ VARs

$\therefore \tilde{S} = (4000 + j2373)$ KVA

(b) $S = 2$ KVA $P = 1.6$ KW (capacitive)



$Q^2 = 2000^2 - 1600^2$

$|Q| = \sqrt{2000^2 - 1600^2}$

$= 1200$

$\tilde{S} = 1600 - j1200$ KVA

11.49

(c) $\hat{V}_{rms} = 208 \angle 20^\circ \text{ V}$

$\hat{I}_{rms} = 6.5 \angle -50^\circ \text{ A}$

$\hat{S} = \hat{V}_{rms} I_{rms}^*$

$\hat{S} = 208 \times 6.5 \angle 70$

$\hat{S} = 1352 \angle 70$

$\hat{S} = 462.4 + j\,1270$

$\hat{S} = (0.464 + j1.27)\ \text{KVA}$

(d) $\hat{V}_{rms} = 120 \angle 30^\circ$; $\hat{Z} = 40 + j60$

$\hat{S} = \frac{|V_{rms}|^2}{Z^*} = \frac{120^2}{40 - j60}$

$\hat{S} = (0.1107 + j0.1662)\ \text{KVA}$

OR

$\hat{S} = (110.7 + j166.2)\ \text{VA}$

11.51 continued

(b) $P_{source} = \frac{16 \times 1.95}{2} \cos(45 - 39.62)$

$$P_{source} = 15.53 \text{ W}$$

(c) Q

$$Q = \frac{16 \times 1.95}{2} \sin(45 - 39.62)$$

$$Q = 1.46 \text{ VARs}$$

(d) $$S = \frac{16 \times 1.95}{2}$$

$$S = 15.6 \text{ VA}$$

(e) $$\vec{S} = P + jQ$$

$$\vec{S} = (15.53 + j\,1.46) \text{ VA}$$

11.51 For the entire circuit calculate

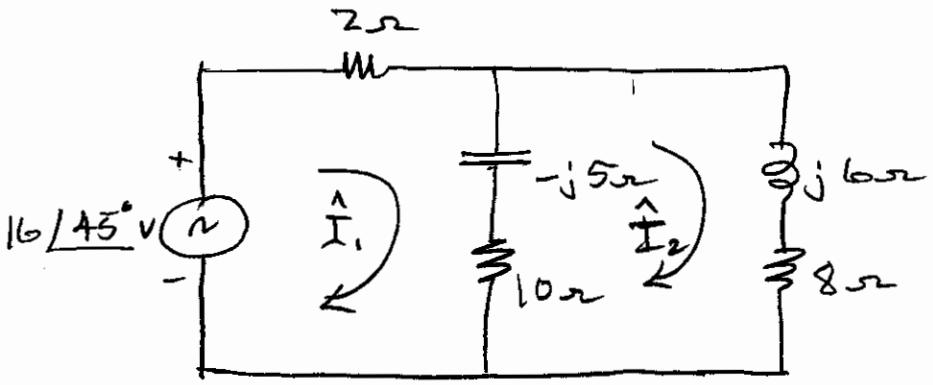
(a) power factor

(b) average power delivered by the source

(c) the reactive power

(d) the apparent power

(e) the complex power



$$\begin{bmatrix} 12-j5 & -10+j5 \\ -10+j5 & 18+j \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 16\angle 45 \\ 0 \end{bmatrix}$$

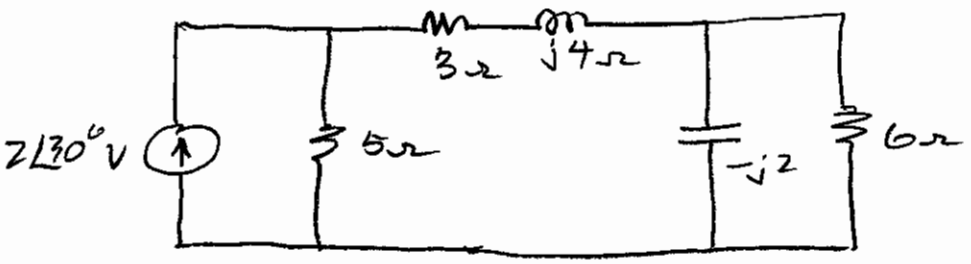
$\hat{I}_1 = 1.505 + j1.246 = 1.95\angle 39.62°$

$\hat{I}_2 = 1.21\angle 9.89°$ A

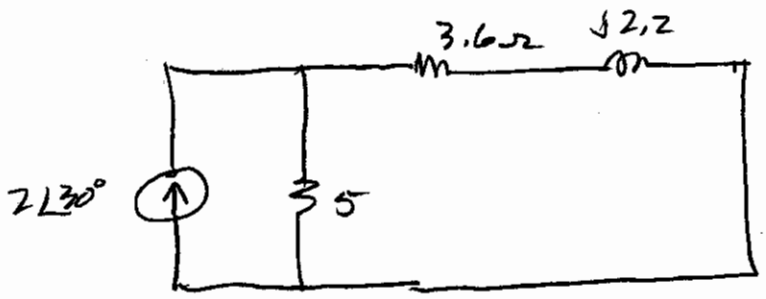
(a) $\hat{V} = 16\angle 45$; $\hat{I}_1 = 1.95\angle 39.62$

$pf = \cos(45 - 39.62) = 0.9956$ lagging

15,56 FIND the complex power delivered by the current source in the following circuit.



Reduces to



$$Z = 5 \parallel (3.6 + j2.2)$$

$$Z = 2.376 \angle 17.08$$

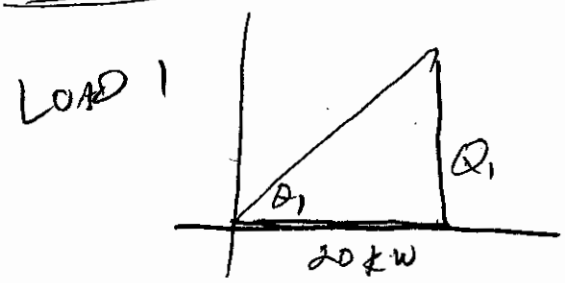
$$\hat{S} = \frac{1}{2} \hat{V} \hat{I}^*$$

$$\hat{V} = Z \hat{I}$$

$$\hat{S} = \frac{1}{2} |\hat{I}|^2 Z = \frac{1}{2}(2)^2 \times 2.376 \angle 17.08 \text{ VA}$$

$$\hat{S} = 4.54 + j1.356 \text{ VA}$$

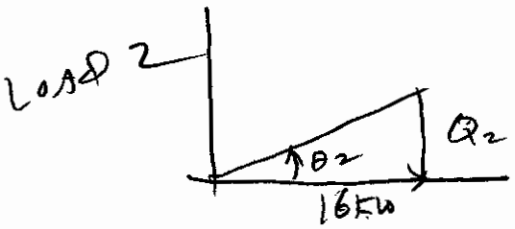
11.60 (See text for problem) Find $\tilde{V}_o$ and P.f.



$\cos\theta = .8$

$\theta_1 = 36.87°$

$Q_1 = 20k \tan 36.87 = 15$ KVARs



$\cos\theta_2 = .9$

$\theta_2 = 25.84°$

$Q_2 = 16k \tan 25.84 = 7.75$ KVARs

$\tilde{S}_{TOTAL} = (20+16)\text{KW} + j(15+7.75)\text{KVARs}$

$\tilde{S}_{TOTAL} = (36 + j\,22.75)$ VA

$\tilde{S} = I_{rms}^2 \tilde{Z}$

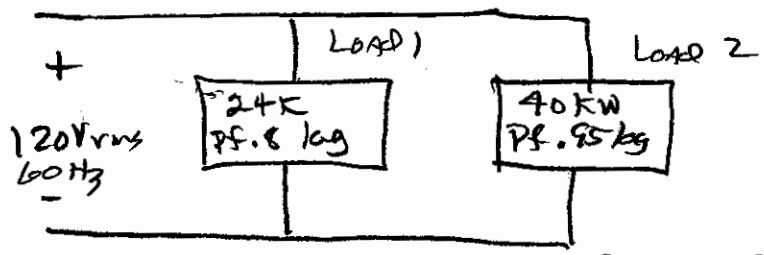
$\tilde{Z} = \dfrac{(36+j22.75)K}{6^2} = (1 + j0.632)K\Omega$

$Z = 1000 + j632\ \Omega$

$\tilde{V}_o = \tilde{I}\tilde{Z} = 6(1000 + j632) = 7.098\angle 32.3°$ V

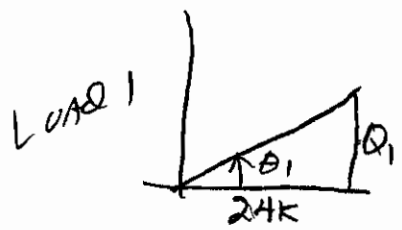
$pf = \cos(\theta_v - \theta_I) = \cos(32.3) = 0.845$ lagging

11.74 Given



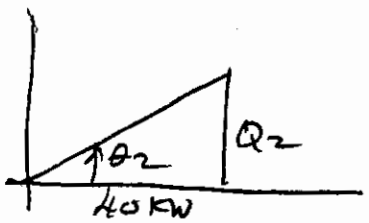
(a) Find the power factor of the combined load
(b) What C placed in parallel with the load to change the pf to 1?

(a)



$\cos\theta_1 = 0.8$

$Q_1 = 24K \tan(\cos^{-1}.8) = 18\ KVA$



$\cos\theta_2 = .95$

$Q_2 = 40K \tan(\cos^{-1}.95) = 13.15\ KVA$

$\hat{S}_{TOTAL} = (40+24)K + j(18+13.15)K$

$\hat{S}_{TOTAL} = (64 + j\,31.15)\ KVA = 71.18\angle 25.95$

$pf = \cos(25.95) = 0.899$ lagging

(b)

$\hat{S} = \frac{V_{rms}^2}{Z^*} = -j31.15 = \frac{120^2}{j/\omega C} = -j120^2\omega C$

$C = \frac{31.15 \times 10^3}{120^2 \times 377} = 5.74\ mF$