ECE 300

Spring 2000

Lecture Notes #4

- Voltage Division
- Current Division
- Nodal Analysis

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lecture #4
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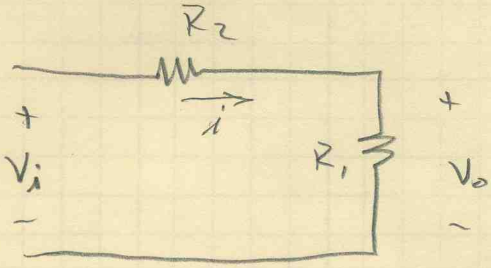
ECE 300

Voltage Division

Consider



$$V_o = R_1 \times i$$

$$i = \frac{V_i}{R_1 + R_2}$$

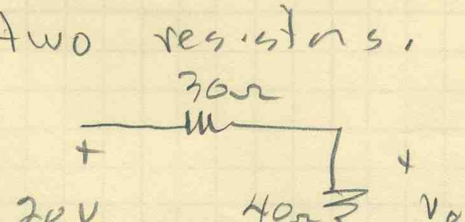
so

$$V_o = \frac{R_1 V_i}{R_1 + R_2}$$

Rule for Resistors (2) in series

The output across one of the resistors is equal to the input voltage x that resistor, divided by the sum of the resistors.

This can be extended to more than two resistors.



$$V_o = \frac{20 \times 40}{30 + 40}$$

$$V_o = 11.43\ V$$

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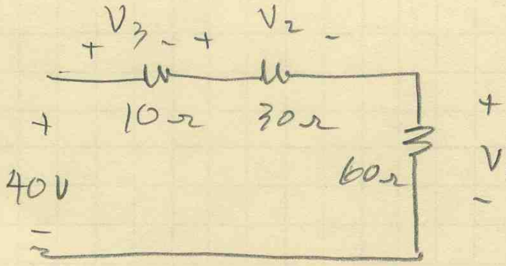
Ex



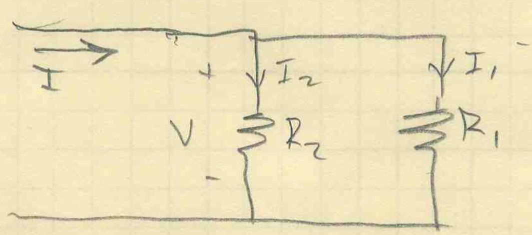
$$V_1 = \frac{40 \times 60}{10+30+60} = 24V$$

$$V_2 = \frac{40 \times 30}{10+30+60} = 12V$$

$$V_3 = \frac{40 \times 10}{10+30+60} = 4V$$

$$V_1 + V_2 + V_3 = 40V = \text{input voltage}$$

Current Division



$$I_1 = \frac{V}{R_1} \quad ; \quad V = I \times R_{eq}$$

$$I_1 = \frac{I \times R_{eq}}{R_1} = \frac{I \times \cancel{R_1} R_2}{\cancel{R_1}(R_1+R_2)} = \frac{I \times R_2}{R_1+R_2}$$

$$\boxed{I_1 = \frac{I \times R_2}{R_1+R_2}}$$

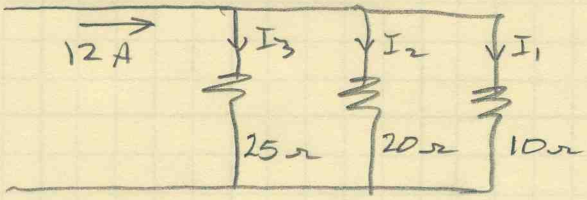
$$I_2 = \frac{I \times R_1}{R_1+R_2}$$

If we have n resistors in parallel numbered as $R_1, R_2, R_3, R_4 \cdots R_{n-1}, R_n$ then

$$I_i = \frac{I \times R_{eq}}{R_i}$$

where $R_1 \le R_i \le R_n$

Example



$$G_{eq} = G_1 + G_2 + G_3 = \frac{1}{10} + \frac{1}{20} + \frac{1}{25}$$

$$G_{eq} = 0.1 + 0.05 + 0.04 = 0.19$$

$$R_{eq} = \frac{1}{G_{eq}} = 5.263\,\Omega$$

$$I_1 = \frac{12 \times R_{eq}}{10} = \frac{12}{10 \times 0.19} = 6.3158\ A$$

$$I_2 = \frac{12 \times}{0.19 \times 20} = 3.1579\ A$$

$$I_3 = \frac{12}{.19 \times 25} = 2.5263$$

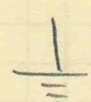
$$I \stackrel{?}{=} 2.3158 + 3.1579 + 2.5263 = 12\ A$$

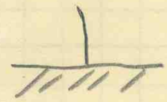
Greatest use of voltage division and current splitting is with 2 resistors.

Nodal Analysis

As we have noted, every linear, planar circuit has n nodes. In nodal analysis we assign one node as the reference node, assign zero potential to this node. This is also, sometimes called the ground node.

Earth Ground

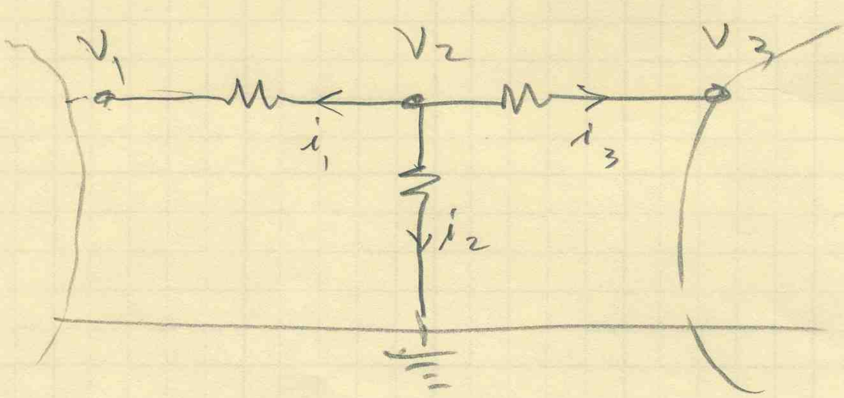




chassis ground.

We assign voltages to the remaining n-1 nodes. We apply KCL at each node to express the currents.

Suppose you have the following situation;

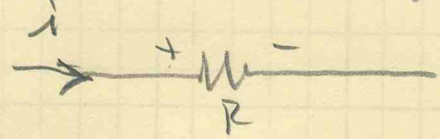


It is clear that at node V_2 we have, using Σi's leaving $= 0$,

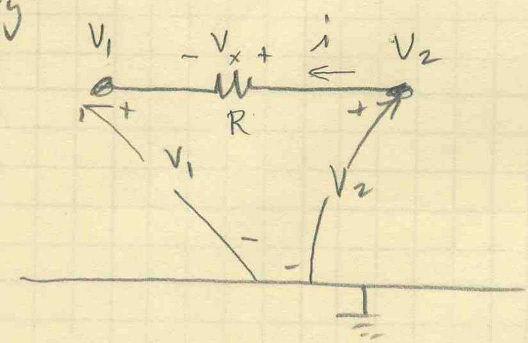
$$i_1 + i_2 + i_3 = 0$$

We want to express each of the currents in terms of the assigned voltages and resistors. We do this using Ohm's Law.

Recall from the default sign convention,



Current goes from the high potential to the low potential. Consider the following



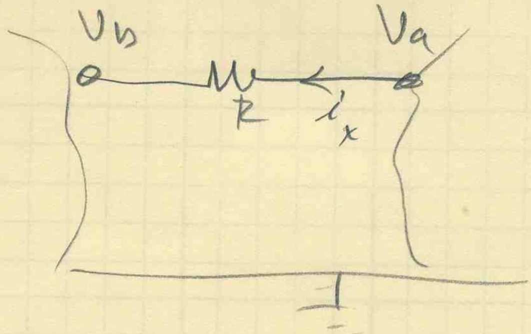
$$-V_2 + V_x + V_1 = 0$$

$$V_x = \frac{V_2 - V_1}{R}$$

$$i = \frac{V_x}{R} = \frac{V_2 - V_1}{R}$$

Writing the current this way is a fundamental key to nodal analysis.

We do not draw the arrows for V_1 and V_2 and we do not show V_x nor signs for V_x. Rather, we look at

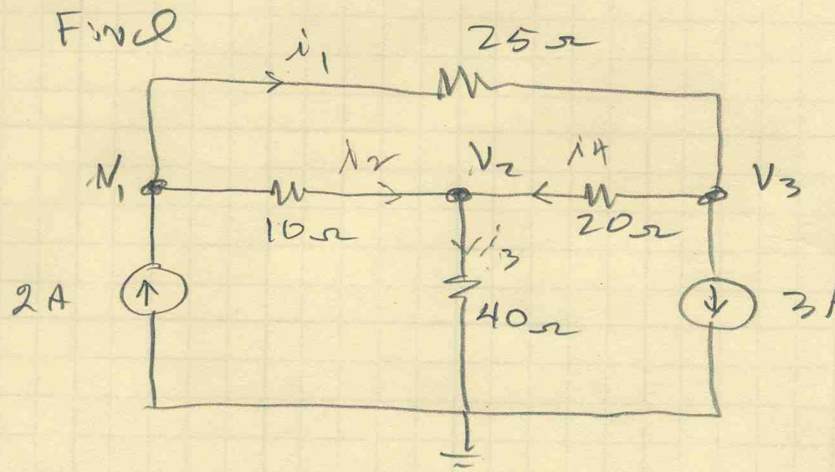


$$i_x = \frac{V_a - V_b}{R}$$

To go along with the text, we will use only current sources in our circuits.

Example 1:

Find



Find V_1, V_2, V_3. Then find i_1 & i_3

& use Σi's leaving a node equal zero.

At Node 1

$$\frac{V_1 - V_3}{25} + \frac{V_1 - V_2}{10} - 2 = 0$$

x25

$$V_1 - V_3 + 2.5V_1 - 2.5V_2 = 50$$

$$\boxed{3.5V_1 - 2.5V_2 - V_3 = 50}$$

At Node 2

$$\frac{V_2 - V_1}{10} + \frac{V_2}{40} + \frac{V_2 - V_3}{20} = 0$$

x40

$$4V_2 - 4V_1 + V_2 + 2V_2 - 2V_3 = 0$$

$$\boxed{-4V_1 + 7V_2 - 2V_3 = 0}$$

At Node 3

$$\frac{V_3 - V_2}{20} + \frac{V_3 - V_1}{25} + 3 = 0$$

x 25

$$1.25V_3 - 1.25V_2 + V_3 - V_1 = -75$$

$$\boxed{-V_1 - 1.25V_2 + 2.25V_3 = -75}$$

$$\begin{bmatrix} 3.5 & -2.5 & -1 \\ -4 & 7 & -2 \\ -1 & -1.25 & 2.25 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ -75 \end{bmatrix}$$

$V_1 = -34.55$ V

$V_2 = -40$ V

$V_3 = -70.91$ V

$$i_1 = \frac{V_1 - V_3}{25} = \frac{-34.55 + 70.91}{25} = 1.45\ A$$

$$i_2 = \frac{V_1 - V_2}{10} = \frac{-34.55 + 40}{10} = 0.545\ A$$

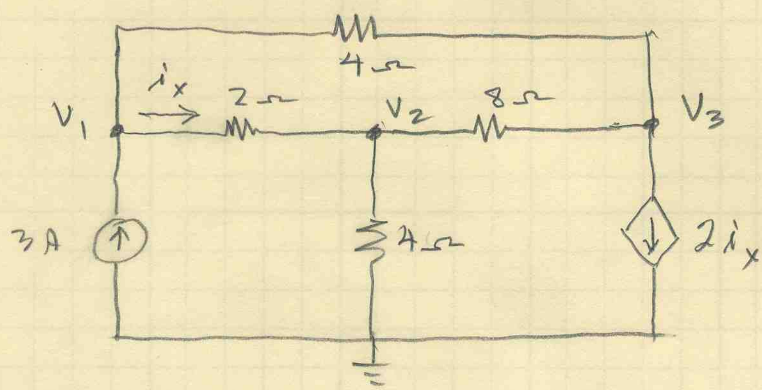
$$i_3 = \frac{V_2}{40} = \frac{-40}{40} = -1\ A$$

$$i_4 = \frac{V_3 - V_2}{20} = \frac{-70.91 + 40}{20} = -1.546\ A$$

Check

$$i_3 = i_2 + i_4 = 0.545 - 1.545 = -1\ A$$

Example 3.2 3rd Ed



At V_1

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} - 3 = 0$$

×4

$$2V_1 - 2V_2 + V_1 - V_3 = 12$$

$$\boxed{3V_1 - 2V_2 - V_3 = 12}$$

At Node V_2 $\quad \Sigma i$ leaving $= 0$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{4} + \frac{V_2 - V_3}{8} = 0$$

$\times 8$

$$4V_2 - 4V_1 + 2V_2 + V_2 - V_3 = 0$$

$$\boxed{-4V_1 + 7V_2 - V_3 = 0}$$

At Node V_3

$$\frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} + 2i_x = 0$$

$$i_x = \frac{V_1 - V_2}{2}$$

$$\frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} + \frac{\cancel{2}(V_1 - V_2)}{\cancel{2}} = 0$$

$\times 8$

$$V_3 - V_2 + 2V_3 - 2V_1 + 8V_1 - 8V_2 = 0$$

$$\boxed{6V_1 - 9V_2 + 3V_3 = 0}$$

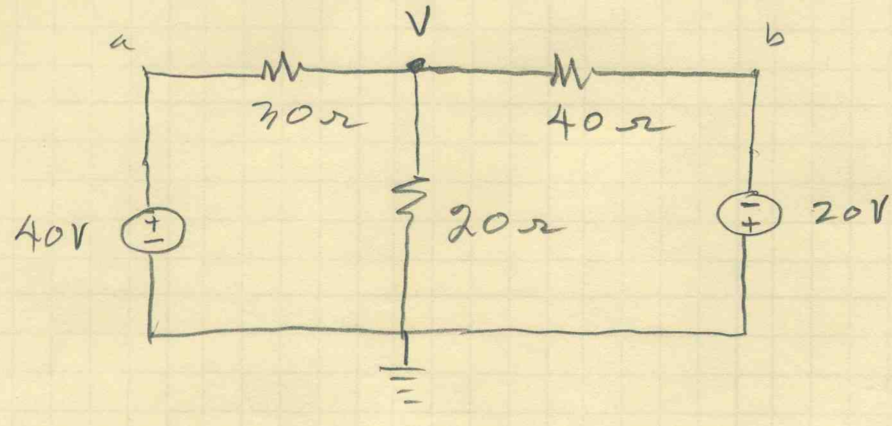
$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ -6 & -9 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$V_1 = 4.8\,V$ $\quad V_2 = 2.4\,V$, $V_3 = -2.4\,V$

Nodal Analysis With Voltage Sources

Example

Find the node voltage indicated in the following circuit.



a & b are nodes but we don't need to solve for voltages at these points. If we know V we can find the currents through the 30Ω & 40Ω resistors. This is shown later.

At Node V ; Σi leaving = 0

$$\frac{V-40}{3\not{0}} + \frac{V+20}{4\not{0}} + \frac{V}{2\not{0}} = 0$$

$\times 12$

$$4V - 160 + 3V + 60 + 6V = 0$$

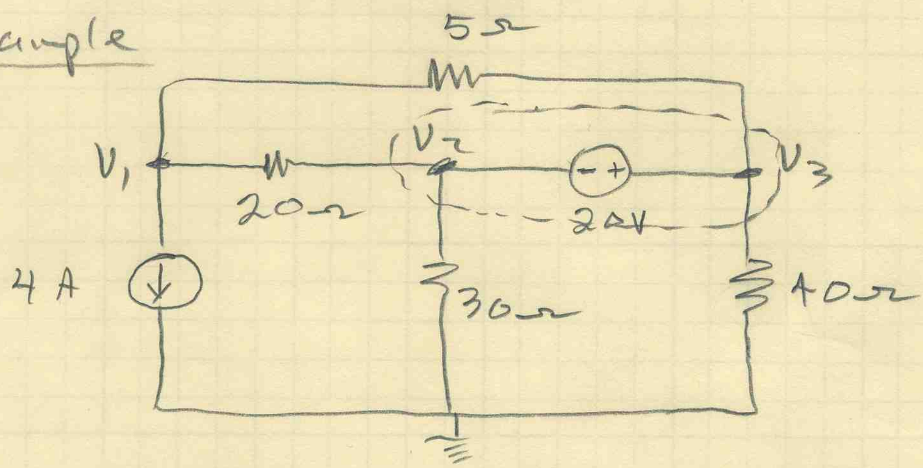
$$13V = 100$$

$$\boxed{V = 7.69\ V}$$

Knowing V, we can solve for everything else in the circuit.

Supernodes

Example



We assign node voltages V_1, V_2, V_3

We place a surface around V_2, V_3 and the source (20V) as shown.

We treat the surface as a node insofar as writing KCL.

At Node V_1

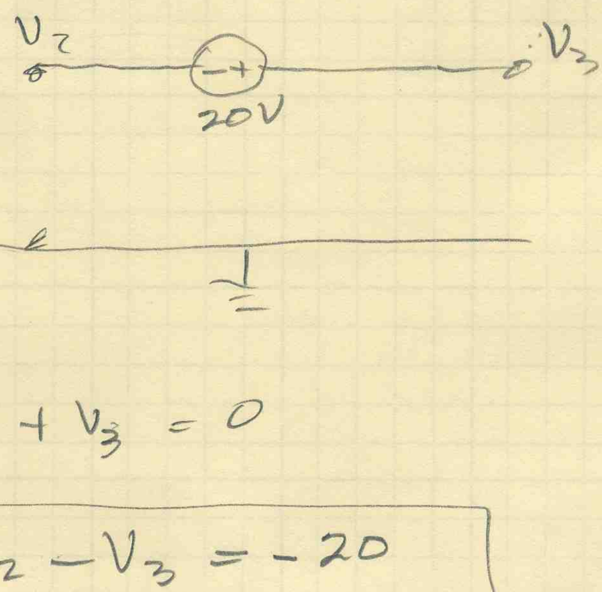
$$\boxed{\frac{V_1 - V_2}{20} + \frac{V_1 - V_3}{5} + 4 = 0}$$

At the super Node

$$\boxed{\frac{V_2 - V_1}{20} + \frac{V_2}{30} + \frac{V_3}{40} + \frac{V_3 - V_1}{5} = 0}$$

3 unknowns, 2 equations.

Third equation comes from the constraint



$$-V_2 - 20 + V_3 = 0$$

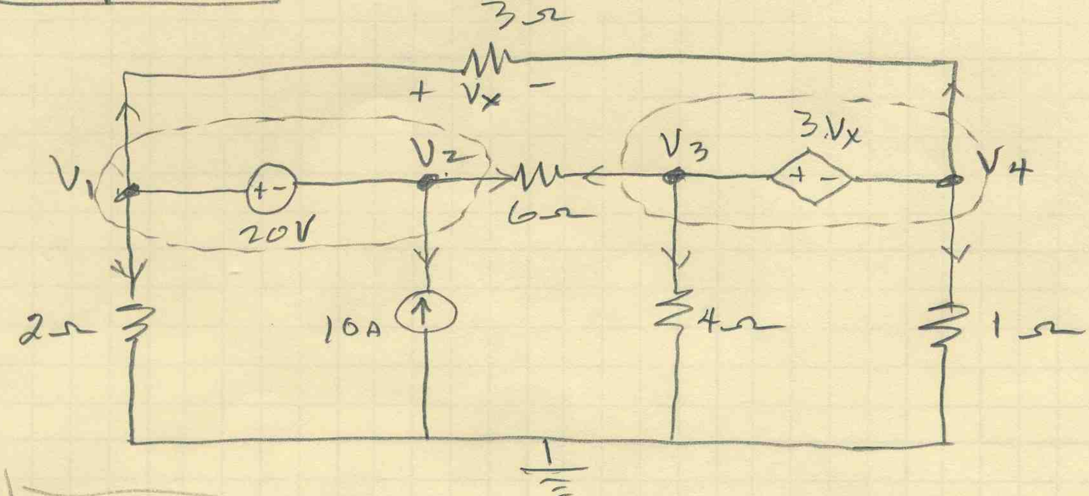
OR

$$\boxed{V_2 - V_3 = -20}$$

3rd equation

solve these, for V_1, V_2, V_3

Example 3.4 3rd Ed.



$$\boxed{\frac{V_1 - V_4}{3} + \frac{V_1}{2} + \frac{V_2 - V_3}{6} - 10 = 0} \quad (1)$$

$$\boxed{\frac{V_3 - V_2}{6} + \frac{V_3}{4} + \frac{V_4 - V_1}{3} + \frac{V_4}{1} = 0} \quad (2)$$

Constraints

$$\boxed{-V_1 + 20 - V_2 = 0} \quad (3)$$

$$-V_3 + 3V_x - V_4 = 0$$

$$-V_1 + V_x + V_4 = 0$$

$$V_x = V_1 - V_4$$

$$V_x = V_1 - V_4$$

so

$$\boxed{-V_3 + 3(V_1 - V_4) - V_4 = 0} \quad (4)$$

Careful algebra leads to the correct solution