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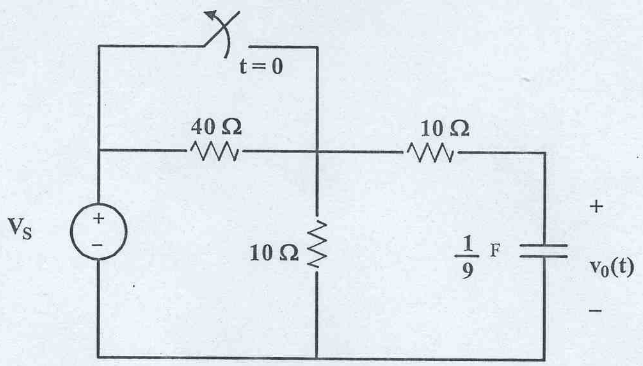
ECE 300
Test 3B
Spring 2006

(1) The circuit shown in Figure 1 is in steady state before the switch opens at t = 0. The input is 10 V, constant. The output is the voltage across the capacitor, $v_o(t)$.

**Required:** Determine the expression for the output voltage $v_o(t)$ for $t \geq 0$.




FOR $t < 0$

$$V_o(0^-) = V_s = 10\,V$$

FOR $t > 0$

Voltage across the capacitor cannot change instantaneously so: $V_o(0^+) = V_o(0^-) = 10\,V$

$$V_o(\infty) = \frac{V_s \times 10}{40+10} = \frac{10 \times 10}{50} = 2\,V$$

$$R_{eq} = 10 + \frac{40 \times 10}{40+10} = 18\,\Omega$$

$$\tau = R_{eq}C = \frac{18}{9} = 2$$

$$V_o(t) = V_o(\infty) + [V_o(0^+) - V_o(\infty)]e^{-\frac{t}{\tau}}$$

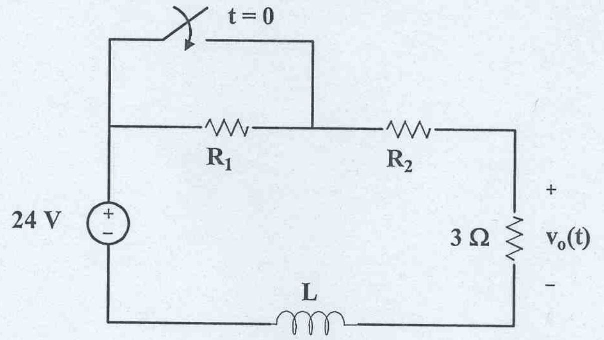
$$V_o(t) = 2 + 8e^{-0.5t}, \; V$$

Test B

(2) The circuit shown in Figure 2 is in steady state before the switch closes at t = 0. The input to the circuit is a constant dc voltage of 24 V. The output, $v_o(t)$, is the voltage across the 3 Ω resistor and is given by

$$v_o(t) = 6 - 3e^{-0.35t}\ V; \quad t \geq 0$$

**Determine** the (a) value of the inductor L, (b) the values of the resistor $R_1$ and $R_2$.




Look at $V_o(\infty)$

$$V_o(\infty) = 6 - 3e^{-.35t}\Big|_{t=\infty} = 6V$$

From the ckt;

$$V_o(\infty) = \frac{24 \times 3}{R_2 + 3} = 6$$

$$72 = 6R_2 + 18$$

$$\boxed{R_2 = 9\,\Omega}$$

Look at $V_o(0^-) = V_o(0^+)$

From the equation; $V_o(0^-) = 6 - 3e^{-.35t}\Big|_{t=0}$

$$V_o(0^-) = V_o(0^+) = 3\ V$$

but $V_o(0^-) = \left(\frac{24}{R_1 + R_2 + 3}\right) \times 3 = 3$

Test B 

(2)

40

$$\frac{24}{R_1 + 9 + 3} = 1$$

$$24 = R_1 + 12$$

$$R_1 = 12\,\Omega$$

$$\tau = \frac{1}{.35} = \frac{L}{R_{eq}}$$

$$R_{eq} = R_2 + 3 = 9 + 3 = 12\,\Omega$$

$$\tau = 2.86 = \frac{L}{12}$$

$$L = 12 \times 2.86 = 34.32\ H$$

i.

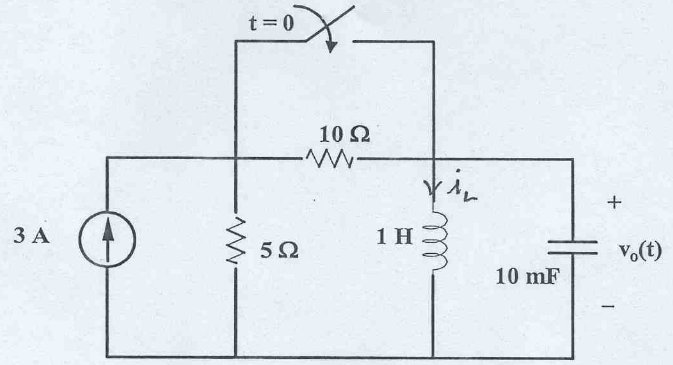
$R_1 = 12\,\Omega$

$R_2 = 9\,\Omega$

$L = 34.32\ H$

(3) T3A & T3B

(3) You are given the circuit of Figure 3. **Determine** the expression for the voltage $v_o(t)$.

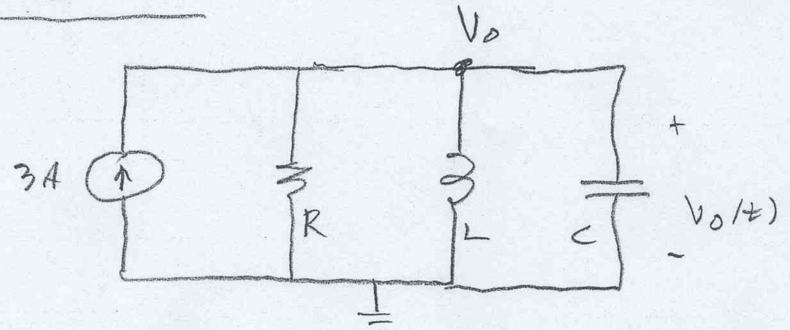



FOR $t < 0$

$$i_L(0^+) = i_L(0^-) = \frac{3 \times 5}{10+5} = 1A$$

$$V_o(0^+) = V_o(0^-) = 0$$

FOR $t > 0$




PARALLEL RLC CKT

Using nodal:

$$\frac{V_o}{R} + C\frac{dV_o}{dt} + \frac{1}{L}\int_0^t V_o\,dt + 1 = 3 \qquad (1)$$

(I.C. $= i_L(0^+)$)

Will need $\frac{dV_o(0^+)}{dt}$ in solving the D.E.

Go ahead and get it now from (1)

(3)

$$C\frac{dV(0^+)}{dt} = 3 - 1 - \frac{V_o(0^+)}{R} \quad (V_o(0^+) \to 0)$$

$$\frac{dV(0^+)}{dt} = \frac{2}{C} = \frac{2}{10\times10^{-3}} = 200 \text{ V/s}$$

so;

$$V_c(0^+) = 0 \quad , \quad \frac{dV(0^+)}{dt} = 200 \text{ V/s} \qquad (2)$$

Go to (1) and take $\frac{d(\cdot)}{dt}$, this gives

$$\frac{1}{R}\frac{dV_o}{dt} + C\frac{d^2V_o}{dt^2} + \frac{V_o}{L} = 0$$

$$\frac{d^2V_o}{dt^2} + \frac{1}{RC}\frac{dV_o}{dt} + \frac{V_o}{LC} = 0$$

Put in values;

$$\frac{d^2V_o}{dt^2} + 20\frac{dV_o}{dt} + 100V_o = 0$$

$$s^2 + 20s + 100 = 0$$

$$(s+10)(s+10) = 0$$

$$V_o = (A + Bt)e^{-10t} \qquad (3)$$

$$V_o(0) = A = 0$$

$$V_o(t) = Bte^{-10t}$$

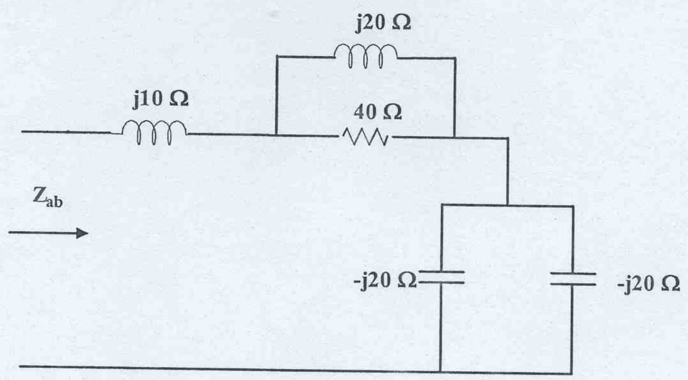
$$\left.\frac{dV_o}{dt}\right|_{t=0} = 200 = \left[-10Bte^{-10t} + Be^{-10t}\right]\Big|_{t=0}$$

$$B = 200$$

$$\boxed{V_o(t) = 200te^{-10t} \text{ V}} \quad \text{Ans}$$

Test 3B

(4) You are given the circuit of Figure 4. **Determine** the impedance, $Z_{ab}$.




$$(-j20) \parallel (-j20) = \frac{(-j20)(-j20)}{-j20 - j20} = -j10$$

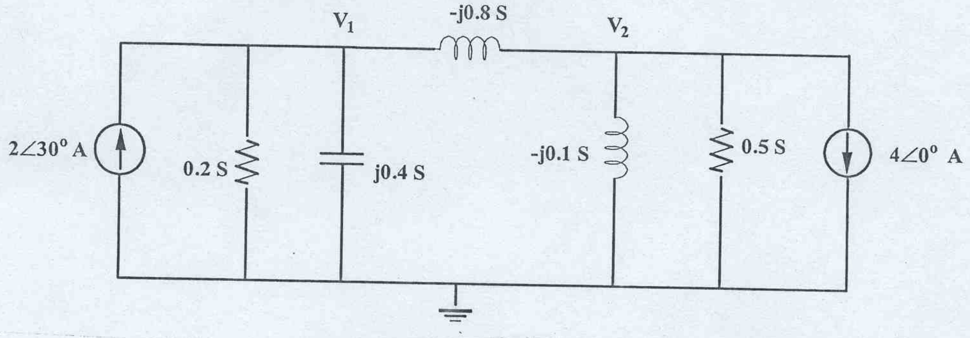
$-j10$ in series with $+j10 = 0$

so

$$Z_{ab} = 40 \parallel j20 = \frac{40(j20)}{40 + j20}$$

$$Z_{ab} = 8 + j16 = 17.89 \angle 63.43^\circ \ \Omega$$

(5) You are given the circuit of Figure 5. **Solve** for the phasor voltages $V_1$ and $V_2$. Express your answers in polar form. **Note:** The unit for the circuit parameters is given in Seiman (S).



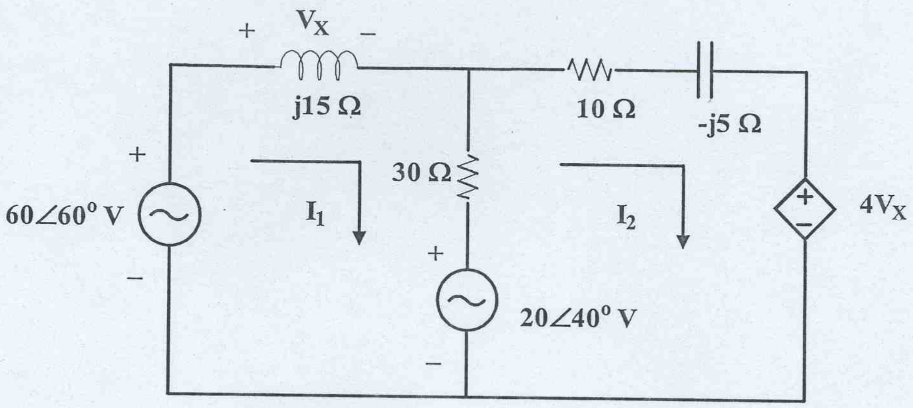

By inspection

$$\begin{bmatrix} (0.2 - j0.4), & j0.8 \\ j0.8, & (0.5 - j0.9) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2\angle 30 \\ -4\angle 0 \end{bmatrix}$$

$V_1 = 5.16 \angle 95.45^\circ$ V

$V_2 = 0.4 \angle 135^\circ$ V

(6) **Solve** for the phasor currents $I_1$ and $I_1$ indicated in the circuit of Figure 6. Express your answers in polar form.




$$(30+j15)\hat{I}_1 - 30\hat{I}_2 = 60\angle 60 - 20\angle 40$$

$$-30\hat{I}_1 + (40-j5)\hat{I}_2 = 20\angle 40 - 4V_x$$

$$= 20\angle 40 - 4(j15)I_1$$

$$(-30+j60)I_1 + (40-j5)I_2 = 20\angle 40$$

$$\begin{bmatrix} (30+j15, & -30 \\ (-30+j60), & (40-j5) \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 60\angle 60 - 20\angle 40 \\ 20\angle 40 \end{bmatrix}$$

$$I_1 = 0.99\angle -24.04^\circ \text{ A}$$

$$I_2 = 1.40\angle -63.98^\circ \text{ A}$$