Desk Copy

ECE 300
Spring Semester, 2007
HW Set #8

Due: March 27, 2007 : **Name**______________________
wlg Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers.** Each problem counts 25 points.

The following **problems are not from the textbook.**

(8.1) You are given the parallel RLC source free circuit of Figure 8.1. V_0 (capacitor initial voltage is 10 V and I_0 (the inductor initial current) is 0.

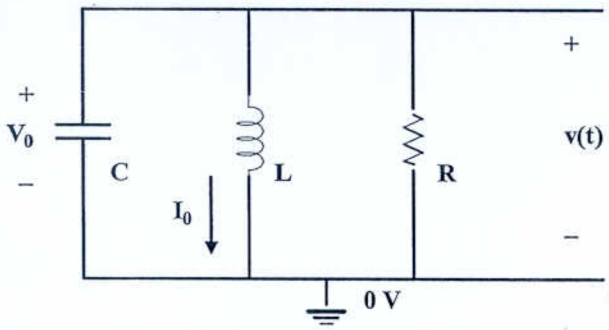

Figure 8.1: Circuit for problems 8.1 and 8.2.

(a) Develop the 2nd order differential equation, that can be used to solve for v(t), in terms of the circuit parameters, R, L and C .

(b) If R = 500 Ω, C = 2 μF, and L = 3.125 H;

- i) Give the characteristic equation in numerical form.
- ii) Give the numerical values of ξ, and ω_n.
- iii) Which category of damping does the circuit fall into?

(c) The solution for v(t) will be of the form

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Find s_1, s_2, A_1, A_2 and write out the solution for v(t)

Answer I got: $v(t) = [-3.33e^{-200t} + 13.33e^{-800t}]\ u(t)$ V

(d) Use MATLAB to check the solution of your differential equation. See the example at the end of the homework set that shows how to do this.

(e) Hand sketch the response you expect for v(t). This does not need to be accurate. However to give you a little insight, find the max/min by taking the derivative of v(t) and solving for zero slope. You should be able to give a reasonable estimate of how long it takes for v(t) to approach steady state.

(f) Write a MATLAB program that can be used to plot v(t). Compare the output with your estimate with your computer output. Include your program and plot with your homework.

(8.2) For the circuit of Figure 8.1, the parameters are now as follows.

$C = 1\ \mu F$, $s_1 = -150+j989$ rad/s, $s_2 = -150-j989$ rad/s

This cond true bc

The initial capacitor voltage $V_0 = 2$ V. All other initial conditions are zero.

$\frac{dv}{dt} = - \frac{V(0^+)/R}{C}$

$= -$

(a) Find R and L and confirm that the solution for v(t) is of the form

$$v(t) = e^{-\xi\omega_n t}[B_1 \cos \omega_d t + B_2 \sin \omega_d t]$$

It is not necessary to derive this: State why it is of this form.
Give the values of ξ, ω_n and ω_d.

(b) Use appropriate initial conditions to find the numerical form of v(t). My answer is,

$$v(t) = e^{-150t}[2 * \cos 989t + 0.3033 * \sin 989t]\ u(t)\ V$$

(c) Use MATLAB to check the solution of your differential equation. See the example at the end of the homework set that shows how to do this.

(d) Estimate how long it will take for v(t) to reach steady state. Call this T_{settle}.
Use $T_{settle} = 5\tau$ when $\tau = \xi\omega_n$. Estimate how many periods of ω_d are between 0 to T_{settle}.

(e) Sketch your expected response. You should show w_d and T_{settle}.

(f) Write a MATLAB program for plotting v(t). Compare your plot with your sketch. Include your MATLAB program and plot with your homework.

There is a lot to be done here but there is a lot you can learn. Learning is what I hope you accomplish.

See the following page for an example on how to use MATLAB to solve a differential equation.

Using MATLAB to solve a linear differential equation:

Example:

$$\frac{d^2v(t)}{dt^2}+8\frac{dv(t)}{dt}+12v(t)=5$$

Given:

$$with\ \ v(0^+)=2,\quad \frac{dv(0^+)}{dt}=0$$

Use the following code in MATLAB

```
>>
>>
>> dsolve('D2v + 8*Dv + 12*v = 5', 'v(0) = 2', 'Dv(0) = 0')

ans =

5/12+19/8*exp(-2*t)-19/24*exp(-6*t)

>>

-
```

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8.1 (a)
For the given circuit we can use nodal analysis to write

$$\frac{v}{R} + C\frac{dv}{dt} + \frac{1}{L}\int_0^t v(t)\,dt + I_0 = 0 \quad (A)$$

Taking the derivative gives

$$\frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} + \frac{v(t)}{L} = 0$$

OR

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v(t)}{LC} = 0$$

(b) Putting in the given values of $R, L, \& C$

$$\frac{d^2v}{dt^2} + \frac{dv}{(.5\times10^3\times2\times10^{-6})\,dt} + \frac{v(t)}{3.125\times2\times10^{-6}} = 0$$

OR

$$\frac{d^2v}{dt^2} + 1000\frac{dv}{dt} + 16\times10^4 = 0$$

i) Characteristic equation is

$$s^2 + 1000s + 16\times10^4 = 0$$

(8.1) continued

ii) characteristic equation in parameter form is

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Comparing coefficients:

$$\omega_n^2 = 16 \times 10^4$$

$$\underline{\omega_n = 400}$$

$$2\zeta\omega_n = 1000$$

$$\zeta = \frac{1000}{2\omega_n}$$

$$\underline{\zeta = 1.25}$$

iii) since $\zeta > 1$ the response is overdamped

(iii) The roots (values of s_1 & s_2) of the C.E. come from

$$s^2 + 1000s + 160{,}000 = 0$$

$$(s + 200)(s + 800) = 0$$

so

$$v(t) = [A_1 e^{-200t} + A_2 e^{-800t}]\ V$$

(8.1) continued

We use initial conditions to find A_1 & A_2. We need

$$v(0^+), \quad \frac{dv(0^+)}{dt}$$

We know $v(0^+) = 10\,V$, $I_0 = 0$

To find $\frac{dv(0^+)}{dt}$ go to Eq(A)

$$\frac{v(0^+)}{R} + C\frac{dv(0^+)}{dt} + \cancel{\frac{1}{L}\int_0^{t=0^+} v(t)\,dt} + I_0 = 0$$

$\xrightarrow{} 0$

Since $I_0 = 0$

$$\frac{dv(0^+)}{dt} = -\frac{v(0^+)}{RC} = \frac{-10}{0.5\times10^3\times2\times10^{-6}} = -10{,}000$$

Thus

$$v(0^+) = 10 = \left[A_1 e^{-200t} + A_2 e^{-800t}\right]_{t=0^+} = A_1 + A_2$$

$$\boxed{A_1 + A_2 = 10}$$

$$\frac{dv}{dt} = -200A_1 e^{-200t} - 800A_2 e^{-800t}$$

$$\frac{dv(0^+)}{dt} = -10{,}000 = -200A_1 - 800A_2$$

$$\boxed{200A_1 + 800A_2 = 10{,}000}$$

So we have

$$\begin{bmatrix} 1 & 1 \\ 200 & 800 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10{,}000 \end{bmatrix}$$

$$A_1 = -3.333, \quad A_2 = 13.333$$

So

$$v(t) = -3.33e^{-200t} + 13.33e^{-800t}$$

(d)

Check with MATLAB

```
>>
>>
>> dsolve('D2v + 1000*Dv + 160000*v = 0',  'v(0) = 10', 'Dv(0) = -10000')

ans =

40/3*exp(-800*t)-10/3*exp(-200*t)

>>
```

Checks

(e) We have e^{-200t}, e^{-800t}

e^{-200t} will take the longest to decay. τ for this term is $\tau_1 = \frac{1}{200} = 0.005$. Allow about $5\tau_1$ or .025 plus ≈ .006 for $5\tau_2$

$5\times\tau_{eff} \doteq .03$

We find $\frac{dv}{dt} = 0$ gives $t_{max/min} = 0.0046\,s$

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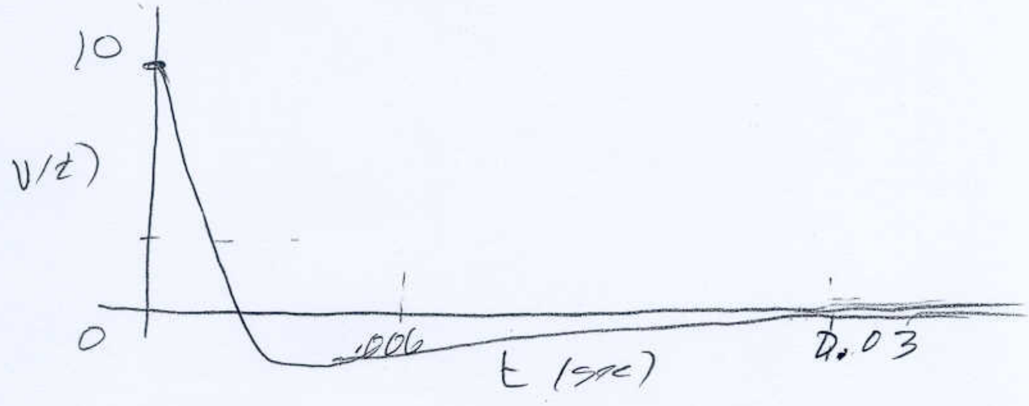
(8.1) continued



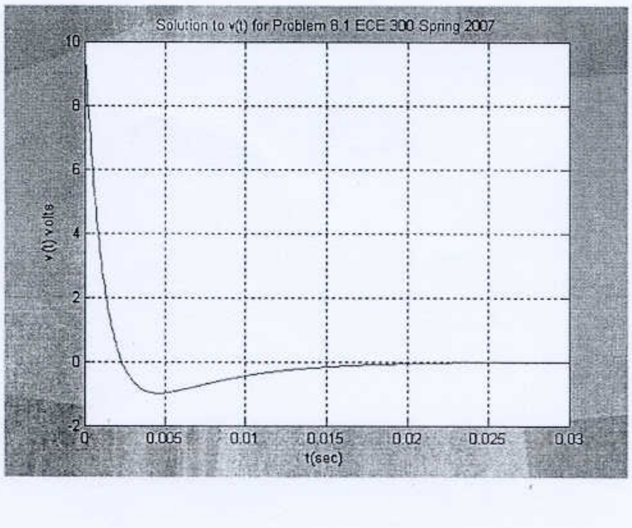
(f)

```
%  Program to check HW8.1 Spring 2007
%  History:  Written on Office Computer; wlg;
%            Program Name:  8_1_S07.m


t = 0:0.00005:0.03;

v = -3.33*exp(-200*t) + 13.33*exp(-800*t);

plot(t,v)
grid
ylabel('v(t) volts')
xlabel('t(sec)')
title('Solution to v(t) for Problem 8.1 ECE 300 Spring 2007')
```



(8.2)

(a)

With reference to the circuit of Figure 8.1 we have

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v(t)}{LC} = 0$$

Now the characteristic equation comes from

$$(s+150+j989)(s+150-j989)=0$$

which gives

$$s^2 + 300s + 1000621 = 0$$

compare to the C.E. of

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

Equate coefficients:

$$\frac{1}{LC} = 1000621$$

$$L = \frac{1}{1\times10^{-6} \times 1.000621\times10^{+6}}$$

$$\boxed{L = 0.9994\ H}$$

(8.2) continued

Also $\frac{1}{RC} = 300$

$$R = \frac{1}{.3\times10^3 \times 1\times10^{-6}} = 3.333\,K\Omega$$

$\boxed{R = 3.333K\Omega}$, $\boxed{L = 0.9994\ H}$

We know the general standard 2nd order form is

$$s^2 + 2\zeta w_n s + w_n^2 = 0$$

compare with

$$s^2 + 300s + 1000621 = 0$$

$$w_n^2 = 1000621$$

$$\boxed{w_n = 1000.31\ \text{rad/s}}$$

$$2\zeta w_n = 300$$

$$\zeta = \frac{150}{1000.31} = 0.14995$$

$$\boxed{\zeta = 0.14995}$$

System is underdamped so we know that $v(t)$ has the form

$$v(t) = e^{-\zeta w_n t}\left[B_1 \cos w_d t + B_2 \sin w_d t\right]$$

(8.2) continued

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\omega_d = 1000.31\sqrt{1-(0.14995)^2}$$

$$\boxed{\omega_d = 989 \text{ rad/s}}$$ (as expected)

$$\boxed{\omega_n = 1000.31 \text{ rad/s}}$$

$$\boxed{\zeta = 0.14995}$$

(b) We know

$$v(t) = e^{-150t}\left[B_1 \cos 989t + B_2 \sin 989t\right]$$

$$v(0^+) = 2 = e^{-150t}\left[B_1 \cos 989t + B_2 \sin 989t\right]\Big|_{t=0} = B_1$$

$$\therefore \underline{\underline{B_1 = 2}}$$

$$\frac{dv(0^+)}{dt} = 0 = e^{-150t}\left[-\omega_d B_1 \sin\omega_d t + \omega_d B_2 \cos\omega_d t\right] - 150e^{-150t}\left[B_1 \cos\omega_d t + B_2 \sin\omega_d t\right]\Big|_{t=0}$$

$$0 = \omega_d B_2 - 150 B_1 = 989 B_2 - 300$$

$$B_2 = \frac{300}{989} = 0.3033$$

(8.2)

b.

$$v(t) = e^{-150t}\left[2\cos 989t + 0.3033\sin 989t\right]u(t)\ V$$

(c) We use the following

$$\frac{d^2v}{dt} + 300\frac{dv}{dt} + 1000621\, v(t) = 0$$

with $v(0^+) = 2$, $\frac{dv(0^+)}{dt} = 0$

```
>>
>>
>>   dsolve('D2v + 300*Dv + 1000621*v = 0', 'v(0) = 2', 'Dv(0) = 0')

ans =

2*exp(-150*t)*cos(989*t)+300/989*exp(-150*t)*sin(989*t)

>>
```

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The answer agrees with the above.

(d) We know $\tau = \frac{1}{\zeta\omega_n} = \frac{1}{150}$

We know the response will be settled in 5τ or

$$T_{settle} = 5\tau = \frac{5}{150} = 0.0333\ \text{sec}$$

We know

$$\omega_d = 989 = 2\pi f_d = \frac{2\pi}{T_d}$$

(8.2)

T_d is the period of ω_d

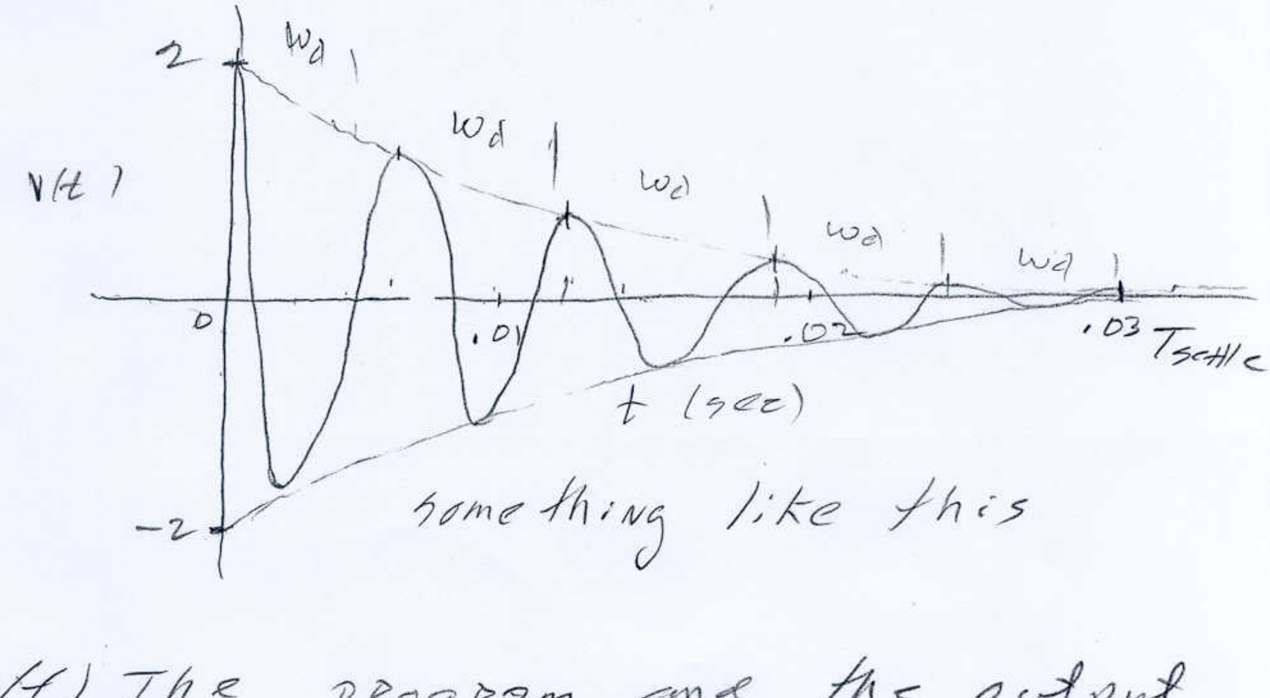
$$T_d = \frac{2\pi}{989} = 0.0063$$

During the time to settle, T_{settle}, we will have the following number of cycles for ω_d.

$$\frac{T_{settle}}{T_d} = \frac{.0333}{.0063} \doteq 5 \text{ cycles}$$

(e)

We would think the respons would have the following form:



something like this

(f) The program and the output are on the following page.

(8.2) cont.

```
% Program to plot solution to DE for problem 8.2 HW ECE 300 S07
% History: On office computer. March 21, 2007: wlg
%         Program Name: S07_8_2.m

t = 0:0.00001:.03;

v = [2*exp(-150*t).*cos(989*t)+300/989*exp(-150*t).*sin(989*t)];
x1=2*exp(-150*t);
x2=-2*exp(-150*t);

plot(t,v,t,x1,'-.',t,x2,'-.')

grid
ylabel('v (volts)')
xlabel('t(sec)')
title('Solution to Parallel RLC Circuit: Underdamped')
```