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**ECE 300**
**Spring Semester, 2007**
**HW Set #9**

**Due: March 29, 2007 : version 2**

**Name** wlg
Print (last, first)

wlg

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers.** Each problem counts 20 points.

8.35 Ans: $v(t) = [12 - e^{-t}(4\cos 2t + 2\sin 2t)]\, u(t)$ V

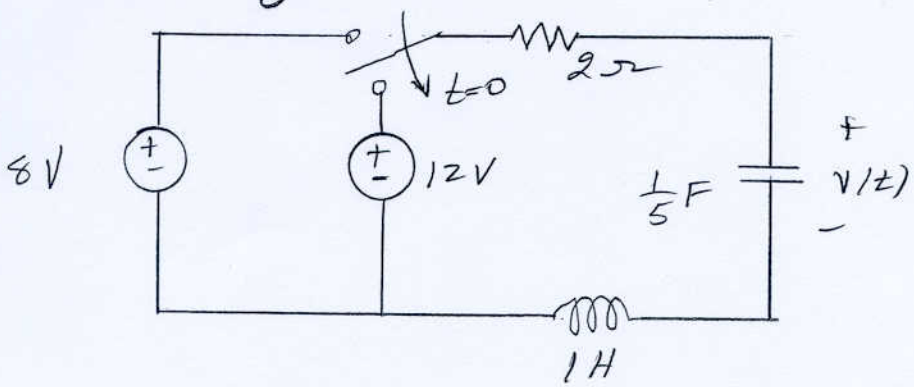
8.47 Ans: $v_o(t) = 200te^{-10t}\, u(t)$ V

8.57 Ans: (a) $s^2 + 20s + 36 = 0$
(b) $i_x(t) = [-0.75e^{-2t} - 1.25e^{-18t}]\, u(t)$ amps : $v_r(t) = [6e^{-2t} + 10e^{-18t}]\, u(t)$ volts

Wlg

H.W. #9
ECE 300
Sp. 2007

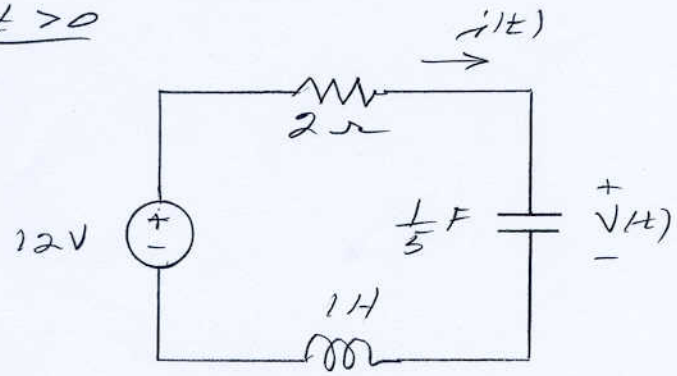
8.35 Determine $v(t)$ for $t > 0$ for the following circuit.




$t < 0$

$$v(0^-) = v(0^+) = 8\ V$$

$t > 0$




$$Ri + L\frac{di}{dt} + v(t) = 12$$

$$RC\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + v(t) = 12$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v(t)}{LC} = \frac{12}{LC}$$

6.35 cont

putting in numbers

$$\frac{d^2v}{dt^2} + 2\frac{dv}{dt} + 5v(t) = 60$$

$$(s+1+j2)(s+1-j2) = 0$$

$$\zeta\omega_n = 1, \quad \omega_d = 2$$

$$v(t) = v_{ss} + v_t$$

$$v_{ss} = k_{ss}$$

$$5k_{ss} = 60 \longrightarrow k_{ss} = 12 = v_{ss}$$

$$v(t) = 12 + e^{-t}[B_1 \cos\omega_d t + B_2 \sin\omega_d t]$$

$$v(0^+) = 8 = 12 + e^{-t}[B_1 \cos\omega_d t + B_2 \sin\omega_d t]\Big|_{t=0}$$

$$8 = 12 + B_1$$

$$B_1 = -4$$

$$i(0^-) = i(0^+) = C\frac{dv}{dt}$$

$$i(0^+) = 0 \therefore \frac{dv(0^+)}{dt} = 0$$

$$\left.\frac{dv}{dt}\right|_{t=0} = \left.\begin{aligned} & e^{-t}[-\omega_d B_1 \sin\omega_d t + \omega_d B_2 \cos\omega_d t] \\ & - e^{-t}[B_1 \cos\omega_d t + B_2 \sin\omega_d t)\end{aligned}\right\}_{t=0}$$

$$0 = \omega_d B_2 - B_1$$

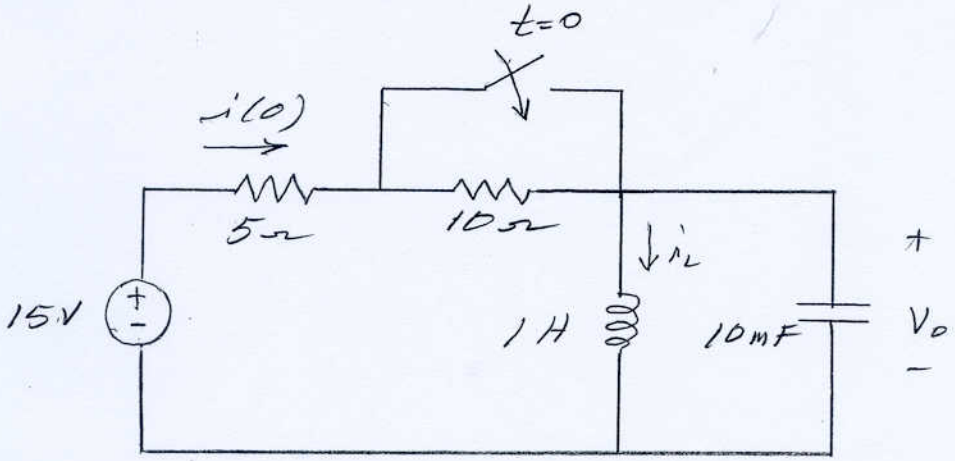
8.35 (cont)

$$B_2 = \frac{B_1}{\omega_d} = \frac{-4}{2} = -2$$

$\therefore$

$$v(t) = \left[12 - e^{-t}\left[4\cos 2t + 2\sin 2t\right]\right] u(t) \text{ V}$$

8.47 Find the output voltage $v_o(t)$ for the following circuit.

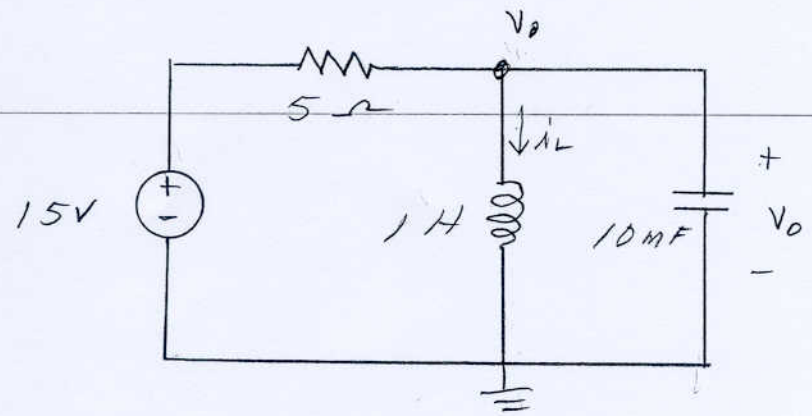



$t < 0$

$i(0^-) = 1A, \quad \therefore i_L(0^-) = 1\text{ A} \therefore v_o(0^-) = 0$

Now look at the circuit for $t > 0$:

8.47 (cont.)




Nodal Analysis

$$\frac{V_o - 15}{5} + \frac{1}{L}\int_0^t V_o(t) + i_L(0) + C\frac{dV_o}{dt} = 0 \quad (1)$$

At $t = 0^+$

$$\frac{V_o(0^+) - 15}{5} + 1 + 10\times10^{-3}\frac{dV_o(0)}{dt} = 0$$

$$V_o(0^+) = 0$$

$$10\times10^{-3}\frac{dV_o(0^+)}{dt} = 2$$

$$\frac{dV_o(0^+)}{dt} = \underline{\underline{200}} \; ; \qquad V_o(0^+) = 0$$

Take the derivative of (1)

$$.2\frac{dV_o}{dt} + V_o(t) + 10\times10^{-3}\frac{d^2V_o}{dt^2} = 0$$

$$\frac{d^2V_o}{dt^2} + 20\frac{dV_o}{dt} + 100\,V_o = 0$$

8.47 cont

Char. Eq.

$$s^2 + 20s + 100 = 0$$

$$(s+10)(s+10) = 0$$

repeated roots

$$v_o(t) = (A_1 + A_2 t)e^{-\zeta\omega_n t}$$

$$v_o(0) = 0 = (A_1 + A_2 t)e^{-10t}\Big|_{t=0} = A_1$$

$\therefore A_1 = 0$

$$v_o(t) = A_2 t e^{-10t}$$

$$\frac{dv_o}{dt} = -10A_2 t e^{-10t} + A_2 e^{-10t}$$

$$\frac{dv_o}{dt}\Big|_{t=0} = 200 = \left[-10A_2 t e^{-10t} + A_2 e^{-10t}\right]\Big|_{t=0}$$

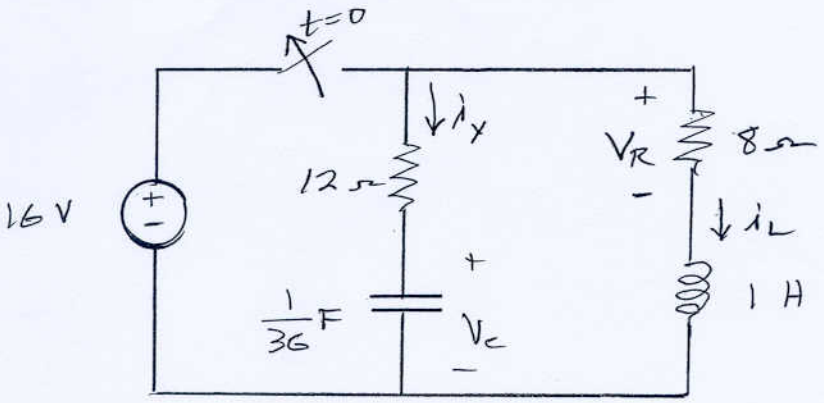
$$200 = A_2$$

$\therefore$ $\boxed{v_o(t) = 200 t e^{-10t} u(t) \quad V}$

8.57 Given the circuit below.

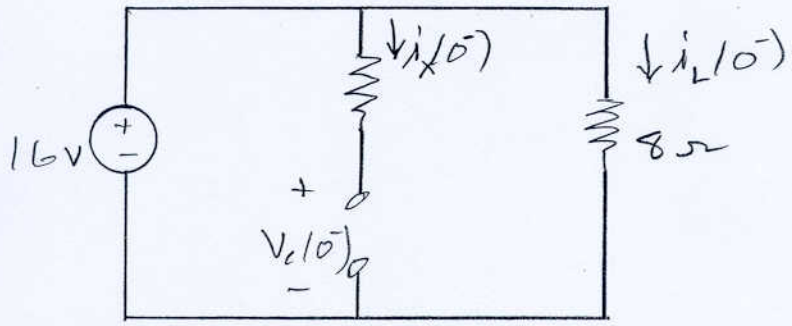
(a) Find the characteristic equation.

(b) Find $i_x$ and $V_R$




$t < 0$

Circuit becomes




$V_c(0^-) = V_c(0^+) = 16\ V$ ✱

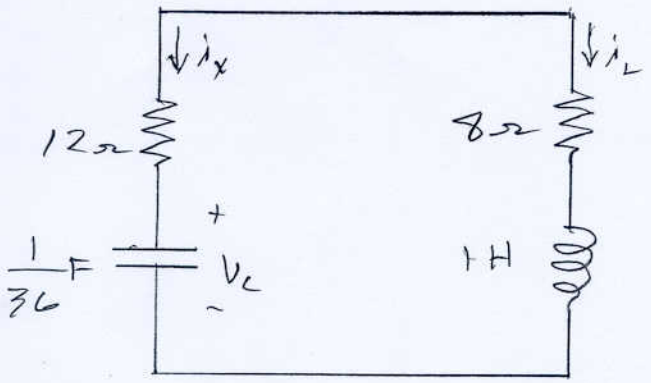
$i_L(0^-) = i_L(0^+) = 2\ A$ ✱

Now for $t > 0$

8.57 continued

8.57

$t > 0$




$$20 i_x + 36 \int_0^t i_x \, dt + v_c(0^+) + L \frac{di_x}{dt} = 0 \quad (1)$$

At $t = 0^+$

$$20 i_x(0^+) + 16 + \frac{d i_x(0^+)}{dt} = 0$$

$$i_x(0^+) = -i_2(0^+) = -2 \text{ A}$$

into (1)

$$20 \times (-2) + 16 + \frac{d i_x(0^+)}{dt} = 0$$

so

$$\boxed{\frac{d i_x(0^+)}{dt} = 24 \text{ V/s}}$$

initial conditions

$$\boxed{i_x(0^+) = -2}$$

Take the derivative of (1) with respect to t.

$$\frac{20di_x}{dt} + 36i_x + \frac{d^2i_x}{dt^2} = 0$$

or

$$\frac{d^2i_x}{dt^2} + 20\frac{di_x}{dt} + 36i_x(t) = 0$$

(a) The characteristic equation is

$$s^2 + 20s + 36 = 0$$

(b) Characteristic equation factors as

$$(s+2)(s+18) = 0$$

overdamped

$$i_x(t) = A_1e^{-2t} + A_2e^{-18t}$$

$$i_x(0) = -2 = A_1e^{-2t} + A_2e^{-18t}\Big|_{t=0}$$

$$A_1 + A_2 = -2$$

8.57 continued

$$\frac{di_x}{dt} = -2A_1 e^{-2t} - 18A_2 e^{-18t}$$

$$\frac{di_x(0)}{dt} = 24 = \left[-2A_1 e^{-2t} - 18A_2 e^{-18t}\right]\Big|_{t=0}$$

$$24 = -2A_1 - 18A_2$$

$$2A_1 + 18A_2 = -24$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 18 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} -2 \\ -24 \end{bmatrix}$$

$$A_1 = -0.75, \quad A_2 = -1.25$$

$$i_x(t) = \left[-0.75e^{-2t} - 1.25e^{-18t}\right] u(t) \text{ A} \quad *$$

$$V_R = -8 i_x(t)$$

$$V_R = \left[6e^{-2t} + 10e^{-18t}\right] u(t) \text{ V} \quad *$$