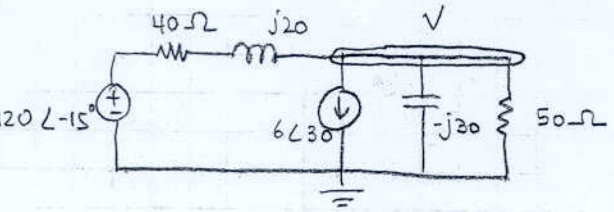
4/17/2007

HW #11 Solution.

#10.7



FIND V using Nodal analysis

$$\frac{V-(120\angle-15^\circ)}{40+j20} + 6\angle 30^\circ + \frac{V}{50} + \frac{V}{-j30} = 0$$

$$\frac{V}{40+j20} + \frac{V}{50} + \frac{V}{-j30} = -6\angle 30^\circ + \frac{120\angle-15^\circ}{40+j20}$$

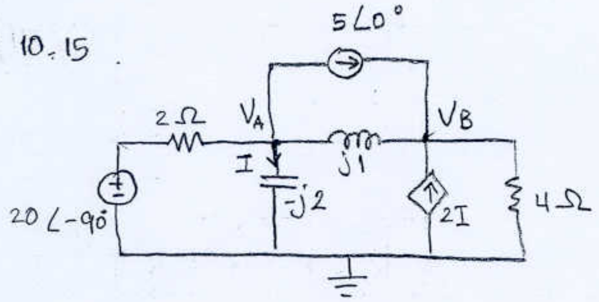
$$= -3.2 - j4.78$$

$$= 5.75\angle-123.80^\circ$$

$$\therefore\ V = -111.7 - j54.34$$

$$= 124.22\angle-154.06^\circ\ V$$

10.15



FIND I using nodal analysis

NODE A : $\dfrac{V_A - (20\angle -90^\circ)}{2} + 5\angle 0^\circ + \dfrac{V_A}{-j2} + \dfrac{V_A - V_B}{j1} = 0$

$$\frac{V_A}{2} + \frac{V_A}{-j2} + \frac{V_A}{j1} - \frac{V_B}{j1} = -5\angle 0^\circ + 10\angle -90^\circ$$

$$\left(\tfrac{1}{2} - j\tfrac{1}{2}\right)V_A + jV_B = -5 - j10 \quad \text{———} \; ①$$

NODE B : $\dfrac{V_B}{4} - 2I - 5\angle 0' + \dfrac{V_B - V_A}{j1} = 0$ but $I = \dfrac{V_A}{-j2}$

$$\frac{V_B}{4} - \cancel{2}\frac{V_A}{-j\cancel{2}} + \frac{V_B}{j1} - \frac{V_A}{j1} = 5\angle 0^\circ$$

$$\left(\tfrac{1}{4} - j\right)V_B = 5\angle 0 \quad \text{———} \; ②$$

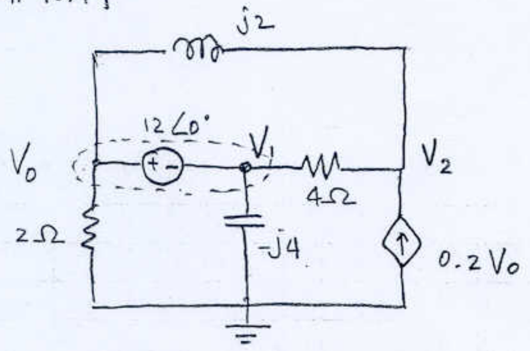
Solving FOR eqn ① & ② :

$$\begin{bmatrix} 1/2 - j\,1/2 & j \\ 0 & 1/4 - j \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -5 - j10 \\ 5 \end{bmatrix}$$

$$V_A = 10.88 - j11.47$$
$$V_B = 1.18 + j4.7$$

$$\therefore \quad I = \frac{V_A}{-j2} = 7.90 \angle 43.49^\circ \text{ A}$$

10.19



FIND V_0 using nodal analysis.

At supernode:

$$\frac{V_0}{2} + \frac{V_0 - V_2}{j2} + \frac{V_1 - V_2}{4} + \frac{V_1}{-j4} = 0$$

$$\frac{V_0}{2} + \frac{V_0}{j2} + \frac{V_1}{4} + \frac{V_1}{-j4} - \frac{V_2}{j2} - \frac{V_2}{4} = 0$$

$$\left(\frac{1}{2} - j\frac{1}{2}\right)V_0 + \left(\frac{1}{4} + \frac{j}{4}\right)V_1 + \left(-\frac{1}{4} + j\frac{1}{2}\right)V_2 = 0 \quad \text{—— (1)}$$

At node V_2:

$$-0.2\,V_0 + \frac{V_2 - V_1}{4} + \frac{V_2 - V_0}{j2} = 0$$

$$-\frac{1}{5}V_0 - \frac{V_0}{j2} - \frac{V_1}{4} + \frac{V_2}{4} + \frac{V_2}{j2} = 0$$

$$\left(-\frac{1}{5} + j\frac{1}{2}\right)V_0 - \frac{1}{4}V_1 + \left(\frac{1}{4} - j\frac{1}{2}\right)V_2 = 0 \quad \text{—— (2)}$$

Also,

$$V_0 - V_1 = 12 \quad \text{—— (3)}$$

Solving for (1), (2), & (3):

$$\begin{bmatrix} 1/2 - j\,1/2 & 1/4 + 1/4\,j & -1/4 + j\,1/2 \\ -1/5 + j\,1/2 & -1/4 & 1/4 - j\,1/2 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

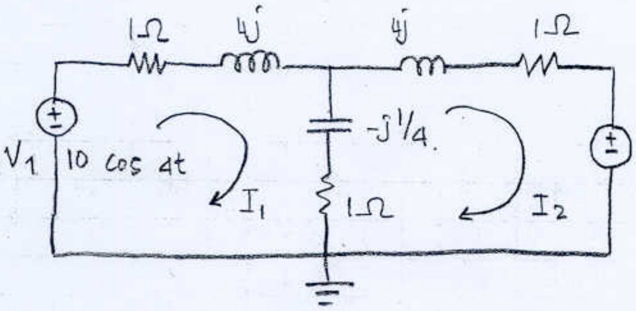
#10.19 cont'd

$$V_0 = 4.91 + j5.9 = \underline{\underline{7.682 \angle 50.19° \ V}}$$

$$V_1 = -7.08 + j5.9 = 9.22 \angle 140.2°$$

$$V_2 = 1.42 + j3.62 = 3.89 \angle 68.58°$$

#10.28



$V_2 = 20\cos(4t-30)$

Find $i_1(t)$ and $i_2(t)$

$\omega = 4.$

Mesh 1: $\quad -10\angle 0^\circ + (1+4j)I_1 + (1-j/4)(I_1 - I_2) = 0$

$$(1+4j)I_1 + (1-j/4)I_1 + (-1+j/4)I_2 = 10\angle 0^\circ$$

$$(2+j3.75)I_1 + (-1+j0.25)I_2 = 10\angle 0^\circ \quad \text{—— (1)}$$

Mesh 2: $\quad 20\angle -30^\circ + (1-j/4)(I_2 - I_1) + (1+4j)I_2 = 0$

$$(-1+j/4)I_1 + (2+3.75j)I_2 = -20\angle -30^\circ \quad \text{—— (2)}$$

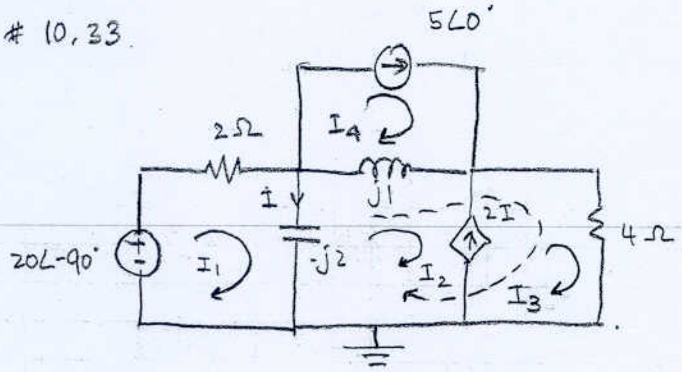
Solving for (1) & (2):

$$\begin{bmatrix} 2+j3.75 & -1+j0.25 \\ -1+j0.25 & 2+j3.75 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ -20\angle -30^\circ \end{bmatrix}$$

$I_1 = 2.74\angle -41.07^\circ \longrightarrow i_1(t) = 2.74\cos(4t - 41.07^\circ)\text{ A}$

$I_2 = 4.11\angle 92^\circ \longrightarrow i_2(t) = 4.11\cos(4t + 92^\circ)\text{ A}$

10.33



FIND I using Mesh analysis.

Mesh 1 : $-20\angle{-90°} + 2I_1 - j2(I_1 - I_2) = 0$

$$(2-j2)I_1 + j2(I_2) = 20\angle{-90°} \quad \text{——} ①$$

Supermesh :

$$j1(I_2 - I_4) + 4I_3 - j2(I_2 - I_1) = 0 \qquad \text{but } I_4 = 5.$$

$$j1(I_2 - 5) + 4I_3 - j2(I_2 - I_1) = 0$$

$$j2I_1 + (-j1)I_2 + 4I_3 = j5 \quad \text{——} ②$$

Constraint :

$$I_2 - I_3 = -2I \qquad \text{and} \quad I = I_1 - I_2$$
$$= -2(I_1 - I_2)$$

$$\Longrightarrow 2I_1 - I_2 - I_3 = 0 \quad \text{——} ③$$

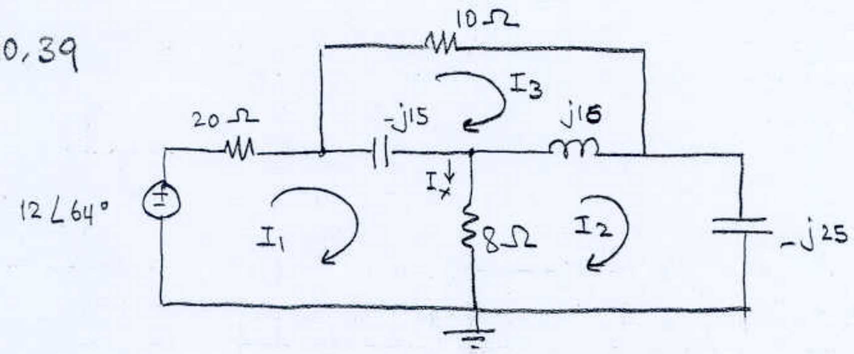
Solving for ①, ②, & ③ :

$$\begin{bmatrix} 2-j2 & j2 & 0 \\ j2 & -j1 & 4 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20\angle{-90} \\ j5 \\ 0 \end{bmatrix}$$

$I_1 = 6.91\angle{-141.9°}$
$I_2 = 14.8\angle{-139°}$
$I_3 = 1.21\angle{76°}$

$\therefore \underline{\underline{I = I_1 - I_2 = 7.9\angle{43.49°}\ A}}$

10.39



FIND all I's.

Mesh 1 : $-12\angle 64° + 20\,I_1 - j15\,(I_1 - I_3) + 8\,(I_1 - I_2) = 0$

$$(28 - j15)\,I_1 - 8\,I_2 + j15\,I_3 = 12\angle 64° \quad \text{——— } (1)$$

Mesh 2 : $8\,(I_2 - I_1) + j16\,(I_2 - I_3) - j25\,I_2 = 0$

$$-8I_1 + (8 - j9)\,I_2 - j16\,I_3 = 0 \quad \text{——— } (2)$$

Mesh 3 : $-j15\,(I_3 - I_1) + 10\,I_3 + j16\,(I_3 - I_2) = 0$

$$j15\,I_1 - j16\,I_2 + (10 + j)\,I_3 = 0 \quad \text{——— } (3)$$

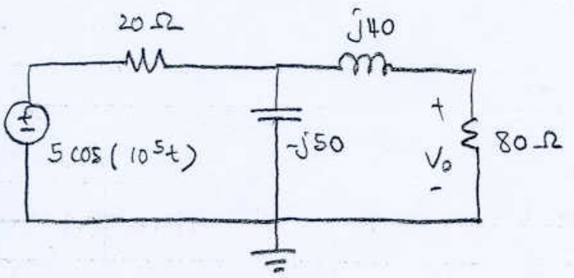
Solving eqn (1), (2) & (3) :

$$\begin{bmatrix} 28 - j15 & -8 & j15 \\ -8 & 8 - j9 & -j16 \\ j15 & -j16 & 10 + j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12\angle 64° \\ 0 \\ 0 \end{bmatrix}$$

$I_1 = 0.38 \angle 109.6°$
$I_2 = 0.344 \angle 124.4°$
$I_3 = 0.145 \angle -60.42$

$I_x = I_1 - I_2 = 0.1 \angle 48.5°$

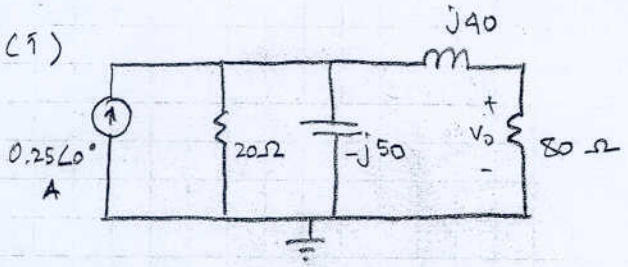
#10.50



$\omega = 10^5$

FIND V_0 using source transformation.

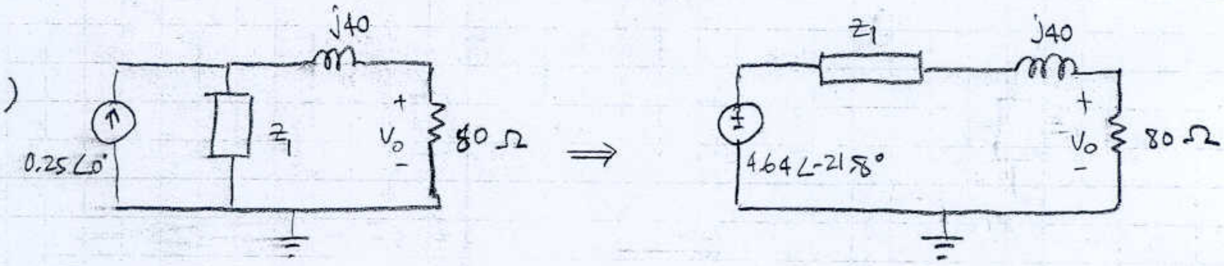
(i)



Let:

$$Z_1 = 20 \,//\, -j50 = \frac{(20)(-j50)}{20 - j50} = 17.24 - j6.9$$

(ii)



$$V_s = (0.25\angle 0^\circ)(17.24 - j6.9)$$
$$= 4.31 - j1.72$$
$$= 4.64\angle -21.8^\circ \text{ V}$$

(iii) Using voltage division,

$$V_0 = (4.64\angle -21.8^\circ)\,\frac{80}{80 + j40 + (17.24 - j6.9)}$$

$$\Rightarrow \underline{V_0 = 3.615\angle -40.6^\circ \text{ V}} \Rightarrow \underline{v_0(t) = 3.615\cos(10^5 t - 40.6^\circ) \text{ V}}$$

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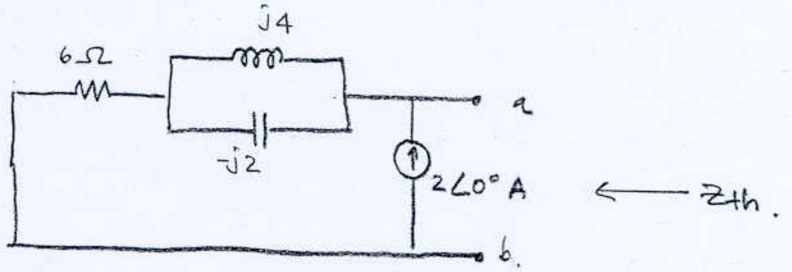
10.56 (a)



$$Z_{TH} = 6 + (j4 \parallel -j2)$$

$$= 6 + \frac{(j4)(-j2)}{j4 - j2}$$

$$= 6 + \frac{8}{j2} = 6 - j4 = \underline{\underline{7.211 \angle -33.68° \ \Omega}}$$

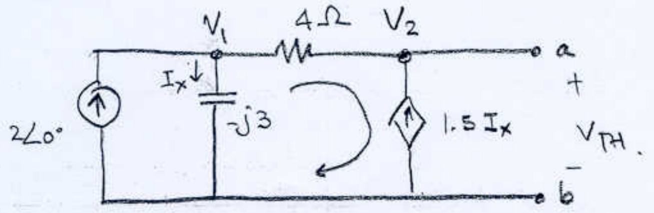
By placing a short circuit at terminal a-b,

$$I_N = 2\angle 0° \text{ A}$$

$$\therefore \quad V_{TH} = Z_{TH} \cdot I_N = (7.211 \angle -33.68°)(2\angle 0°)$$

$$\underline{\underline{V_{TH} = 14.42 \angle -33.69° \ \text{V}}}$$

10.61



FIND V_{TH} & Z_{TH}.

(i) to find V_{TH}.

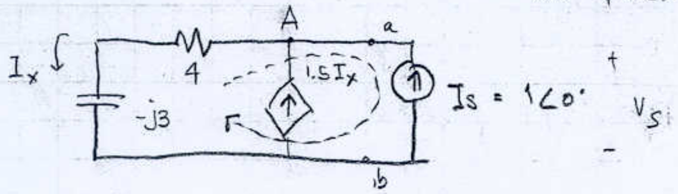
At node V_1, using KCL:

$$-2 + I_x - 1.5 I_x = 0$$
$$-0.5 I_x = 2$$
$$I_x = -4A$$

then we apply KVL:

$$-I_x(-j3) - 1.5 I_x (4) + V_{TH} = 0$$
$$V_{TH} = I_x(-j3) + 1.5 I_x (4)$$
$$= (-4)(-j3) + 1.5(-4)(4)$$
$$\therefore V_{TH} = -24 + j12 \text{ Volts}$$

(ii) to FIND Z_{TH}, WE deactivate the independent source. and place 1A at terminal ab:



At NODE A : $I_x = 1.5 I_x + I_s$

$$-0.5 I_x = 1 A$$
$$I_x = -2A.$$

Apply KVL on outer loop Clockwise DIRECTION :

$$(4 - j3)(-I_x) + V_s = 0$$
$$V_s = I_x (4 - j3)$$
$$= (-2)(4 - j3) = (-8 + j6) V.$$

$$\therefore Z_{TH} = \frac{V_s}{I_s} = (-8 + j6)\ \Omega$$