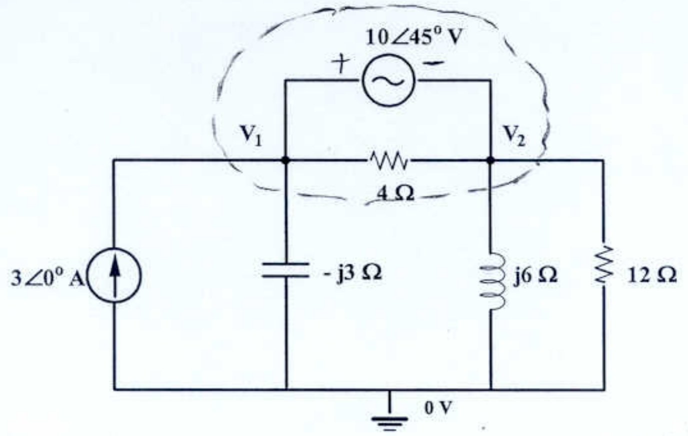
wlg

ECE 300
Test 3A
Spring 2007

(1) You are given the circuit of Figure 1. Use Nodal Analysis to find the phasor voltages V_1 and V_2. Express your answers in polar form.



Super node as shown:

$$\frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12} = 3$$

$$j0.333V_1 - j0.1667V_2 + 0.0833\,V_2 = 3$$

$$j0.333\,V_1 + (0.0833 - j0.1667)\,V_2 = 3$$

Constraint

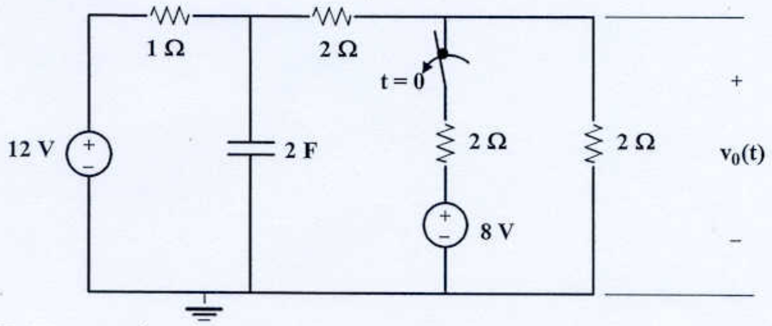
$$V_1 - 10\angle 45 - V_2 = 0$$

$$\begin{bmatrix} j0.333, & 0.0833 - j.1667 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 10\angle 45 \end{bmatrix}$$

$V_1 = 25.8\angle -70.43°\,V$ $\quad V_2 = 31.45\angle -87.13°\ V$

Test A

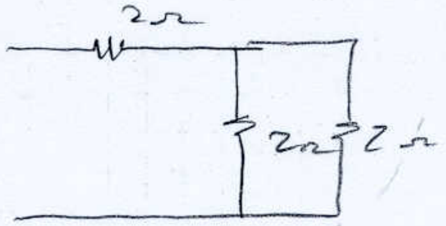
(2) You are given the circuit shown in Figure 2. The switch has been closed for a very long time and is opened at t=0. Use the step-by-step method to find $v_o(t)$.



$t<0$

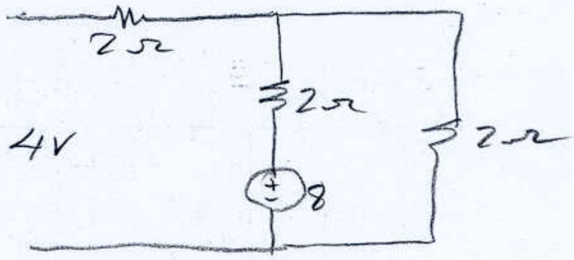
(make a Thevenin)

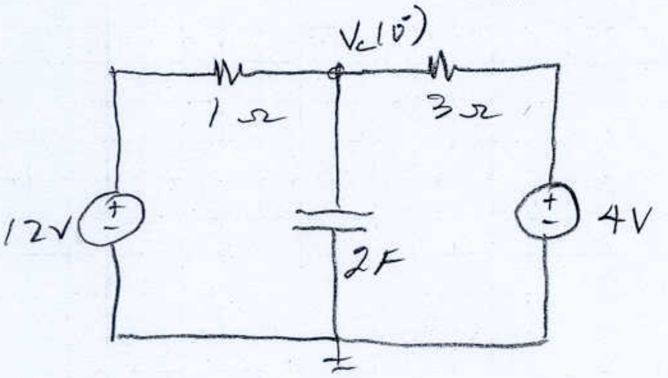
R_{TH}



$R_{TH} = 3\Omega$

$V_{TH} = 4V$





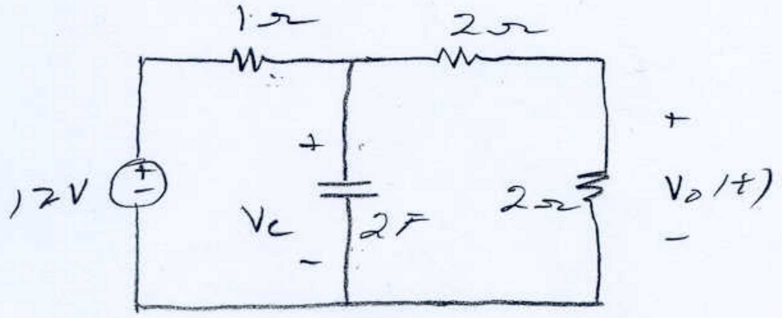
$$\frac{V_c - 12}{1} + \frac{V_c - 4}{3} = 0 \Rightarrow 3V_c - 36 + V_c - 4 = 0$$

$$V_c(0^-) = 10\ V$$

Test A

(2) 2.2

$\underline{t > 0}$



$V_c(0^-) = V_c(0^+)$

$V_o(0^+) = \dfrac{V_c(0^+) \times 2}{2+2} = 0.5\, V_c(0^+)$

$\underline{V_o(0^+) = 5\text{ V}}$

$V_o(\infty) = \dfrac{12 \times 2}{5} = \dfrac{24}{5}\text{ V} = 4.8\text{ V}$

$R_{eq} = 1 \| 4 = \frac{4}{5}$

$\tau = \frac{4}{5} \times 2 = 1.6 \text{ sec}$

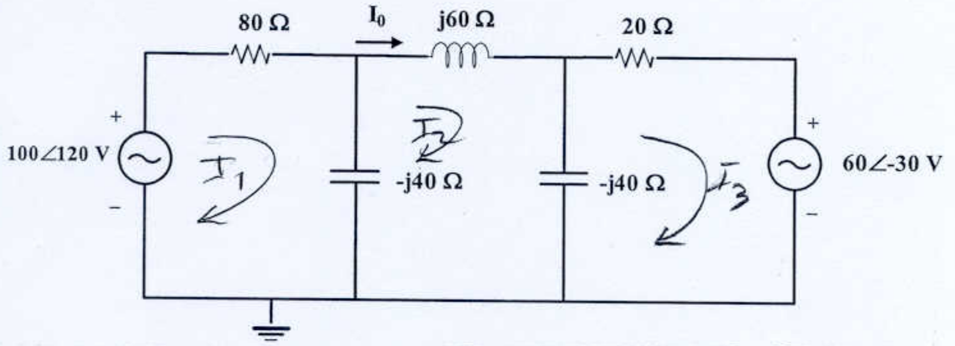
$V_o(t) = V_o(\infty) + [V_o(0^+) - V_o(\infty)]e^{-\frac{t}{\tau}}$

$V_o(t) = \frac{24}{5} + \left[5 - \frac{24}{5}\right]e^{-0.625t}\ \text{V}$

$V_o(t) = 4.8 + 0.2\, e^{-0.625t}$

Test A

(3) Use mesh analysis to find the phasor current $\mathbf{I_o}$ shown in the circuit of Figure 3. Express your answer in polar form.



mesh 1

$80(I_1) - j40(I_1 - I_2) = 100\angle 120$

$\boxed{(80 - j40)I_1 + j40 I_2 + 0 I_3 = 100\angle 120}$

mesh 2

$-j40(I_2 - I_1) + j60 I_2 - j40(I_2 - I_3) = 0$

$\boxed{j40 I_1 - j20 I_2 + j40 I_3 = 0}$

mesh 3

$-j40(I_3 - I_2) + 20 I_3 = -60\angle -30$

$\boxed{0 I_1 + j40 I_2 + (20 - j40) I_3 = -60\angle -30}$

$$\begin{bmatrix} 80-j40, & j40 & 0 \\ j40 & -j20 & j40 \\ 0 & j40 & 20-j40 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100\angle 120^\circ \\ 0 \\ -60\angle -30 \end{bmatrix}$$

$I_1 = 0.0412 + j0.5819$

$I_2 = 1.042 + j1.914 = \boxed{I_o = 2.1797\angle 61.43^\circ \text{ A}}$

$I_3 = 0.48 + j0.375$

Test A

(4) You are given the circuit of Figure 4. The switch has been in position 1 for a very long time. At t = 0, the switch is moved to position 2.

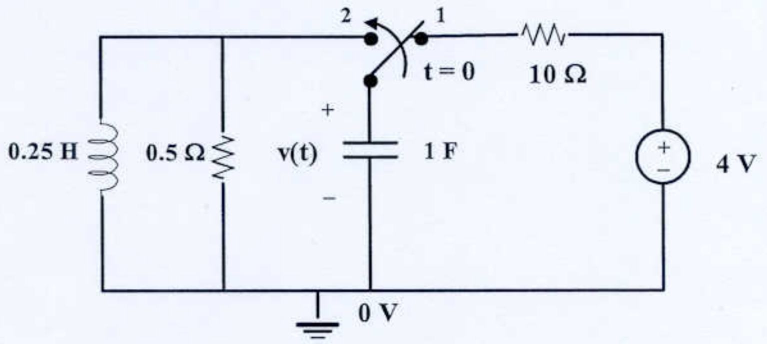


Figure 4: Circuit for Problem 4.

(a) Determine $v(0^+)$ *and* $\dfrac{dv(0^+)}{dt}$.

(b) Develop the differential equation that can be used to determine v(t). The equation is of the form

$$\frac{d^2v(t)}{dt^2}+K_1\frac{dv(t)}{dt}+K_2v(t)=K_3$$

Determine K_1, K_2, and K_3 using numerical values.

(c) Give the characteristic equation and the roots of the characteristic equation.

(d) Which if the following apply?

- (i) The response for v(t) is overdamped.
- (ii) The response for v(t) is underdamped.
- (iii) The response for v(t) is critically damped.
- (iv) Since this is a linear, time invariant, second order differential equation, none of the above apply.

(e) Determine ξ, ω_n *and* ω_d.

(f) Determine the solution for v(t) for $t \geq 0$.

(g) Approximately how long will it take (within 20%) before the response decays to 1% of its initial value?

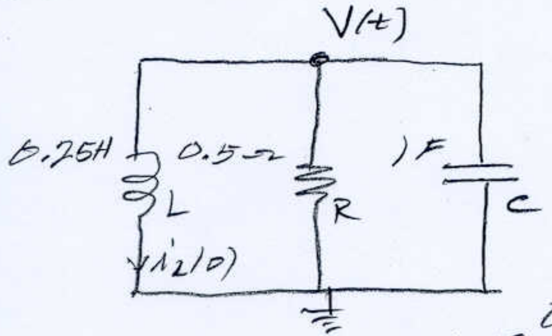
(a) Looking at the circuit we see the capacitor is charged to the source value;

$$\therefore \boxed{V(0^-) = V(0^+) = 4\text{ V}}$$

To get $\dfrac{dV(0^+)}{dt}$ we need to write an equation. (next page)

(4) cont

$t \geq 0$



$$\frac{V}{R} + C\frac{dV}{dt} + \cancelto{0}{\frac{1}{L}\int_0^t V(t)\,dt} + i_L(0^+) = 0 \qquad (A)$$

$i_L(0^+) = 0$

Evaluate the above at $t = 0^+$

$$C\frac{dV(0^+)}{dt} = -\frac{V(0^+)}{R}$$

$$\frac{dV(0^+)}{dt} = -\frac{4}{0.5 \times 1} = -8 \ V$$

so $\boxed{V(0^+) = 4V}$ $\boxed{\frac{dV(0^+)}{dt} = -8\ V}$

(b) Take the derivative of Eq (A)

$$\frac{1}{R}\frac{dV}{dt} + C\frac{d^2V}{dt^2} + \frac{V(t)}{L} = 0$$

OR

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{V(t)}{LC} = 0$$

OR

$$\frac{d^2V}{dt^2} + 2\frac{dV}{dt} + 4V(t) = 0 \qquad (B)$$

(4) cont

Compare Eq (B) to

$$\frac{d^2v}{dt^2} + k_1 \frac{dv}{dt} + k_2 v(t) = 0$$

we see

$\boxed{k_1 = 2 ;}$ $\boxed{k_2 = 4,}$ $\boxed{k_3 = 0}$

(c) The characteristic equation is

$$s^2 + 2s + 4 = 0$$

roots

$$(s+1+j1.732)(s+1-j1.732) = 0$$

$$s_1 = -1 - j1.732, \quad s_2 = -1 + j1.732$$

(d) ii underdamped

(e) $\omega_n^2 = 4$

$\boxed{\omega_n = 2}$

$2\zeta\omega_n = 2$

$\boxed{\zeta = \frac{1}{\omega_n} = 0.5}$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 2\sqrt{1-.25}$$

$\boxed{\omega_d = 1.732 \text{ rad/sec}}$

$\zeta\omega_n = 1$

(4) cont. 4.4

$$v(t) = e^{-\zeta \omega_n t}\left[B_1 \cos \omega_d t + B_2 \sin \omega_d t\right]$$

$$v(0^+) = \boxed{4 = B_1}$$

$$\frac{dv}{dt} = e^{-t}\left[-\omega_d B_1 \sin \omega_d t + \omega_d B_2 \cos \omega_d t\right] - e^{-t}\left[B_1 \cos \omega_d t + B_2 \sin \omega_d t\right]$$

with

$$\frac{dv(0^+)}{dt} = -8$$

$$-8 = 1.732 B_2 - B_1 = 1.732 B_2 - 4$$

$$B_2 = \frac{-4}{1.732} = -2.309$$

$$v(t) = e^{-t}\left[4 \cos 1.732 t - 2.309 \sin 1.732 t\right]$$

(g) $e^{-3} = 0.0497 \rightarrow 4.97\%$

$e^{-4} = 0.0183 \rightarrow 1.83\%$

$e^{-5} = .0067 \rightarrow 0.67\%$

use 5 time constants

$\boxed{\text{Ans} \doteq 5 \text{ sec}}$

(5) cont

(c)

$$\dot{I} = \frac{V_s}{Z} = \frac{100\angle 0}{\quad}$$

Test A

(5) You are given the AC circuit shown in Figure 5. The following phasor voltages are known.

$$V_S = 100\angle 0^\circ\ V; \quad V_R = 85.75\angle 30.96^\circ\ V; \quad V_L = 51.45\angle 120.96^\circ\ V$$

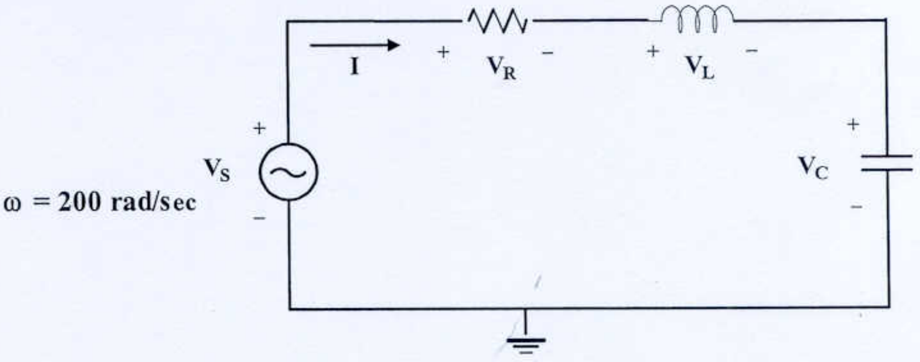
The frequency of the source is $\omega = 200$ rad/sec.

(a) Solve for the phasor voltage V_C. Express your answer in polar form.

(b) Draw the phasor diagram (close to scale) showing V_S, V_R, V_L, *and* V_C.

(c) Determine the phasor current **I** shown in the diagram. Express your answer in polar form.

(d) Determine the value of the capacitor C, expressed in μF.



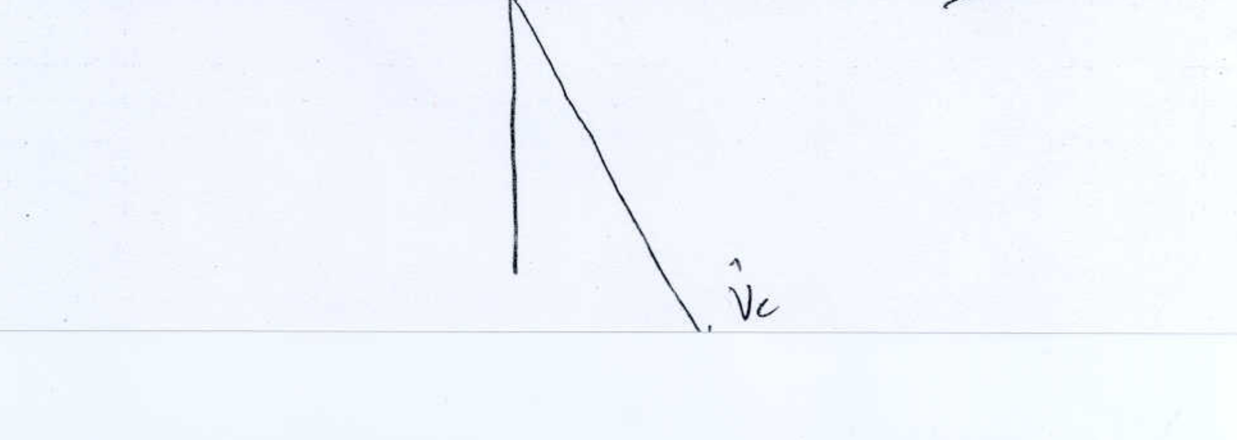
(a) $V_R + V_L + V_C = V_S$

$V_C = V_S - V_R - V_L$

$V_C = 100 - 85.75\angle 30.96 - 51.45\angle 120.96$

$\boxed{V_C = 102.9\angle -59^\circ}$

(b)



(5) cont. 5.2

We know

$$\vec{I} = \frac{V_R}{R} = \frac{85.75 \angle 30.96}{10}$$

$$\boxed{\vec{I} = 8.575 \angle 30.96^\circ \ A}$$

(d)

$$V_C = I \times \left(\frac{1}{j\omega C}\right)$$

$$102.9 \angle -59 = \frac{8.575 \angle 30.96}{j\,200\,C}$$

$$C = \frac{8.575 \angle 30.96}{(102.9 \angle -59)(j200)}$$

$$\boxed{C = 416.67 \mu F}$$