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- Test B

ECE 300
Test 3B
Spring 2007

(1) You are given the circuit of Figure 1. The make-before-break switch has been in position 1 for a very long time. At t = 0, the switch is moved to position 2

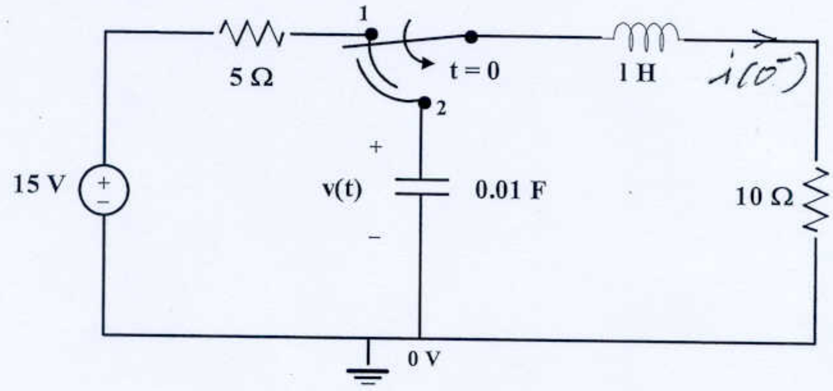


Figure 1: Circuit for Problem 1.

(a) Determine $v(0^+)$ *and* $\dfrac{dv(0^+)}{dt}$.

(b) Develop the differential equation that can be used to determine v(t). The equation is of the form

$$\frac{d^2v(t)}{dt^2} + K_1\frac{dv(t)}{dt} + K_2 v(t) = K_3$$

Determine K_1, K_2, and K_3 using numerical values.

(c) Give the characteristic equation and the roots of the characteristic equation.

(d) Which if the following apply?
 (i) The response for v(t) is overdamped.
 (ii) The response for v(t) is underdamped.
 (iii) The response for v(t) is critically damped.
 (iv) Since this is a linear, time invariant, second order differential equation, none of the above apply.

(e) Determine ξ, ω_n *and* ω_d.

(f) Determine the solution for v(t) for $t \geq 0$.

(g) Approximately how long will it take (within 20%) before the response decays to 1% of its initial value?

(a) Looking at the circuit,
$v(0^-) = V(0^+) = 0\ V$

1.2

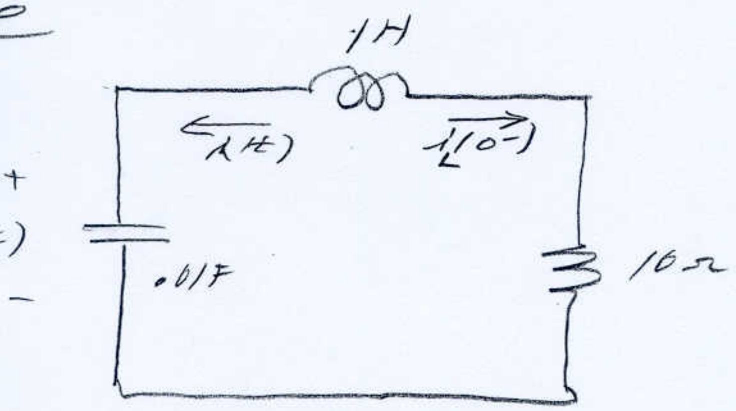
(1) cont

Looking at the circuit

$t < 0$

$i(0^-) = \frac{15}{15} = \underline{1\ A}$

$\underline{t > 0}$



$$L\frac{di}{dt} + V(t) + Ri = 0 \qquad (A)$$

$$i = C\frac{dV}{dt} \qquad (B)$$

$$RC\frac{dV}{dt} + LC\frac{d^2V}{dt^2} + V(t) = 0 \qquad (C)$$

From (B)

$i(0^-) = -i_L(0^-)$

$i(0^+) = -i_L(0^-) = -1\ A$

From (B)

$$\frac{dV(0^+)}{dt} = \frac{i(0^+)}{C} = \frac{-1}{.01} = -100\ V/sec$$

So $V(0) = 0$; $\frac{dV(0)}{dt} = -100\ V/sec$

(1) cont.

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v(t)}{LC} = 0$$

With numbers

$$\frac{d^2v}{dt^2} + 10\frac{dv}{dt} + 100\,v(t) = 0$$

$$s^2 + 10s + 100 = 0$$

$$(s + 5 + j8.66)(s + 5 - j8.66) = 0$$

(b) $$\frac{d^2v}{dt^2} + 10\frac{dv}{dt} + 100\,v(t) = 0$$

$K_1 = 10, \quad K_2 = 100, \quad K_3 = 0$

(c) characteristic eq:

$$s^2 + 10s + 100 = 0$$

$$s_1 = -5 - j8.66$$

$$s_2 = -5 + j8.66$$

(d) underdamped

(e) $\omega_n^2 = 100$

$$\boxed{\omega_n = 10}$$

$$2\zeta\omega_n = 10$$

$$\boxed{\zeta = 0.5}$$

$\zeta\omega_n = 5$ check

$\omega_d = 10\sqrt{1 - .5^2} = 8.66$ check

(1) cont

(f)

$$v(t) = e^{-\zeta\omega_n t}[B_1 \cos\omega_d t + B_2 \sin\omega_d t]$$

$$v(t) = e^{-5t}[B_1 \cos\omega_d t + B_2 \sin\omega_d t]$$

@ $t=0$; $v(0^+) = 0$

$$\boxed{0 = B_1}$$

$$v(t) = e^{-5t} B_2 \sin\omega_d t$$

$$\frac{dv}{dt} = e^{-5t}\omega_d B_2 \cos\omega_d t - 5e^{-5t} B_2 \sin\omega_d t$$

@ $t=0^+$

$$\frac{dv(0^+)}{dt} = -100 = \omega_d B_2$$

$$B_2 = \frac{-100}{8.66} = -11.55$$

$$v(t) = -e^{-5t}[11.55 \sin 8.66t] \text{ V}$$

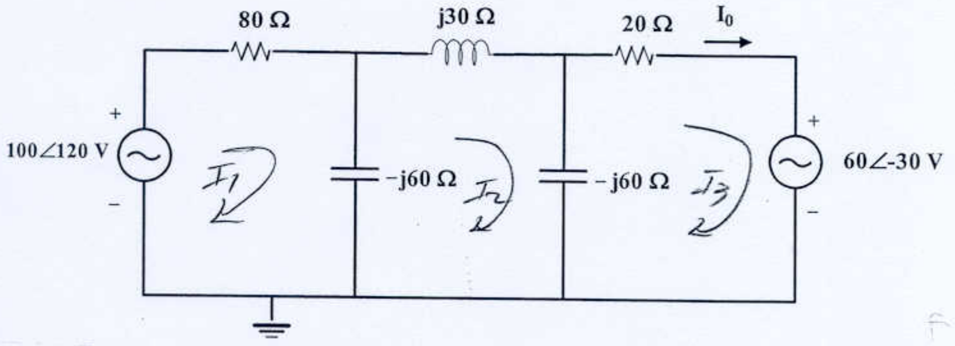
(g) e^{-4} $(t = \frac{4}{5}) = 0.183 = 1.83\%$

e^{-5} $(t = 1) = 0.0067 = 0.67\%$

$1^{\text{no}} > t > .8$ will take it to long than 1%

Test B

(2) Use mesh analysis to find the phasor current $\mathbf{I_o}$ shown in the circuit of Figure 2. Express your answer in polar form.



Need I_3

mesh 1

$$80I_1 - j60(I_1 - I_2) = 100\angle 120^\circ$$

$$\boxed{(80 - j60)I_1 + j60 I_2 + 0 I_3 = 100\angle 120^\circ}$$

mesh 2

$$-j60(I_2 - I_1) + j30 I_2 - j60(I_2 - I_3) = 0$$

$$\boxed{j60 I_1 - j90 I_2 + j60 I_3 = 0}$$

mesh 3

$$-j60(I_3 - I_2) + 20 I_3 = -60\angle -30$$

$$\boxed{0 I_1 + j60 I_2 + (20 - j60) I_3 = -60\angle -30}$$

$$\begin{bmatrix} 80-j60 & j60 & 0 \\ j60 & -j90 & j60 \\ 0 & j60 & 20-j60 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100\angle 120 \\ 0 \\ -60\angle -30 \end{bmatrix}$$

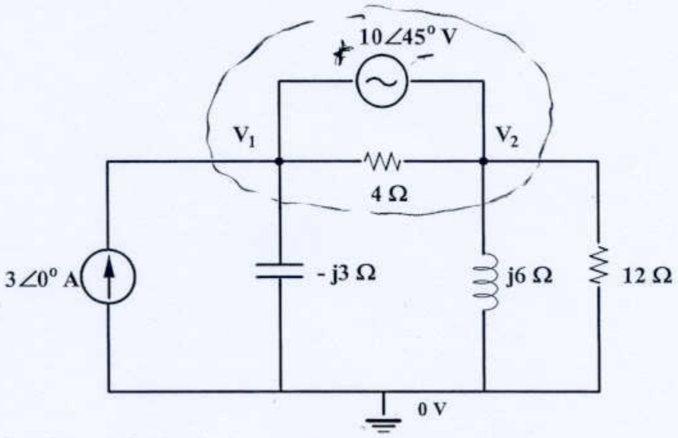
$I_1 = -0.356 + j1.464$

$I_2 = -0.864 + j1.823$

$I_3 = -0.9407 + j1.27 = \boxed{1.58\angle 126.5^\circ A = I_o}$

Test B

(3) You are given the circuit of Figure 3. Use Nodal Analysis to find the phasor voltages V_1 and V_2. Express your answers in polar form.



Super node

$$\frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12} = 3$$

$$j0.333V_1 - j0.1667V_2 + 0.08333V_2 = 3$$

$$j0.333V_1 + (0.08333 - j0.1667)V_2 = 3$$

Constraint

$$V_1 - 10\angle 45 - V_2 = 0$$

$$\begin{bmatrix} j0.333, & (.08333 - j0.1267) \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 10\angle 45 \end{bmatrix}$$

8.647 − j24.332

$$V_1 = 25.8\angle -70.43^\circ\ V \quad \checkmark$$ check

$$V_2 = 31.45\angle -87.13 \quad \checkmark$$ check

Test B

(4) You are given the AC circuit shown in Figure 4. The following phasor voltages are known.

$$V_S = 100\angle 0^o\ V; \quad V_R = 44.8\angle -63.43^o\ V; \quad V_L = 156.8\angle 26.57^o\ V$$

The source frequency is 300 rad/sec.

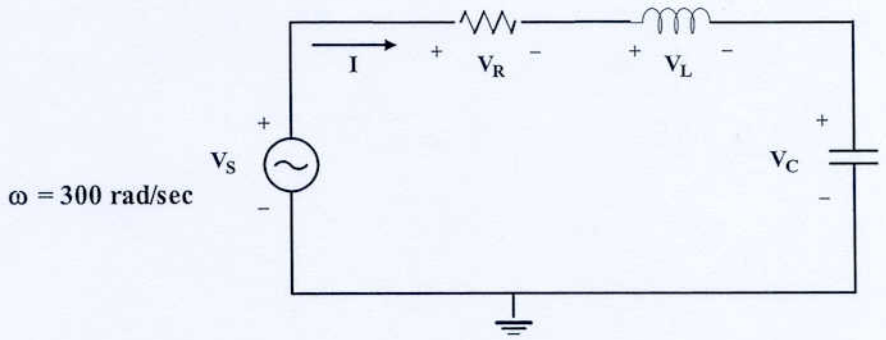


Figure 4: Circuit for Problem 4.

(a) Solve for the phasor voltage V_C. Express your answer in polar form.

(b) Draw the phasor diagram (close to scale) showing $V_S,\ V_R,\ V_L,\ and\ V_C$.

(c) Determine the phasor current **I** shown in the diagram. Express your answer in polar form.

(d) Determine the value of the capacitor C, expressed in μF.

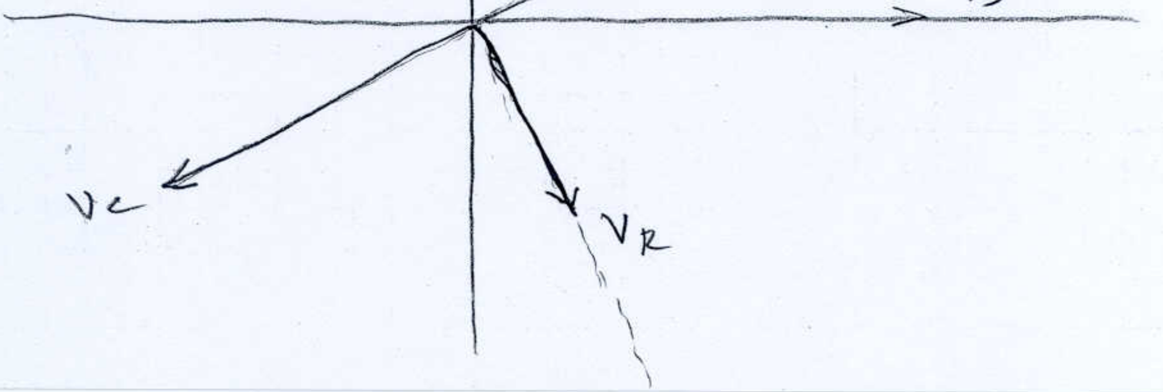
(a)

$$V_C = V_S - V_R - V_L$$

$$= 100 - 44.8\angle -63.43^\circ - 156.8\angle 26.57^\circ$$

$$V_C = 67.36\angle -153.5^\circ\ V$$

(b)



(4) cont. 4.2

(c)

$$I = \frac{V_R}{R} = \frac{44.8}{10} \angle -63.43$$

$$I = 4.48 \angle -63.43^\circ \ A$$

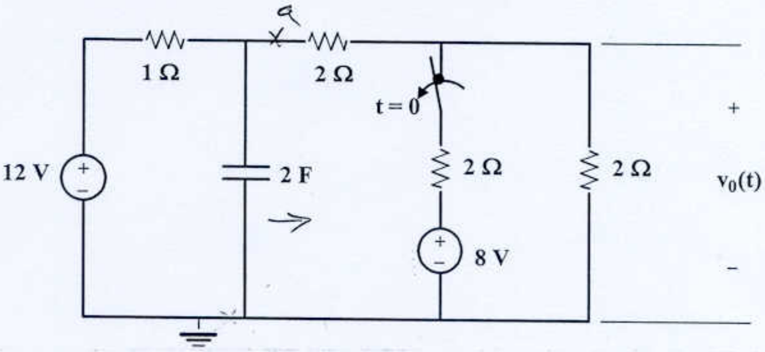
(d) $V_c = I \times \left(\frac{1}{j\omega C}\right)$

$$C = \frac{I}{V_c (j\omega)} = \frac{4.48 \angle -63.43}{67.36 \angle -153.5 \, (j300)}$$

$$\boxed{C = 22.17 \mu F}$$

Test B

(5) You are given the circuit shown in Figure 5. The switch has been closed for a very long time and is opened at t=0. Use the step-by-step method to find $v_o(t)$.

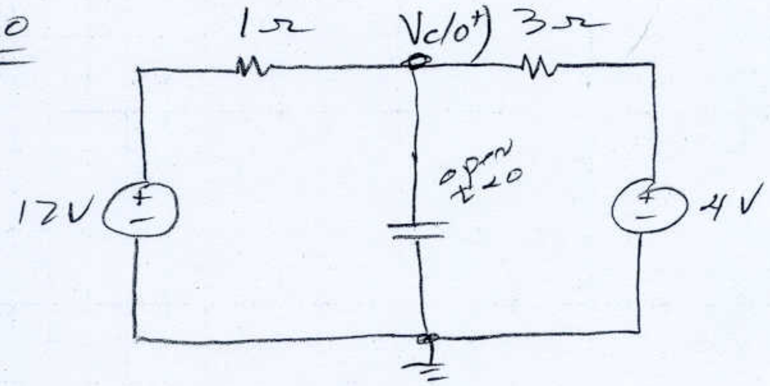


Get the Thevenin to the right of "a"

$R_{TH} = 2 + 2\|2 = 3\,\Omega$

$V_{TH} = \dfrac{8 \times 2}{4} = 4\,V$

$t<0$

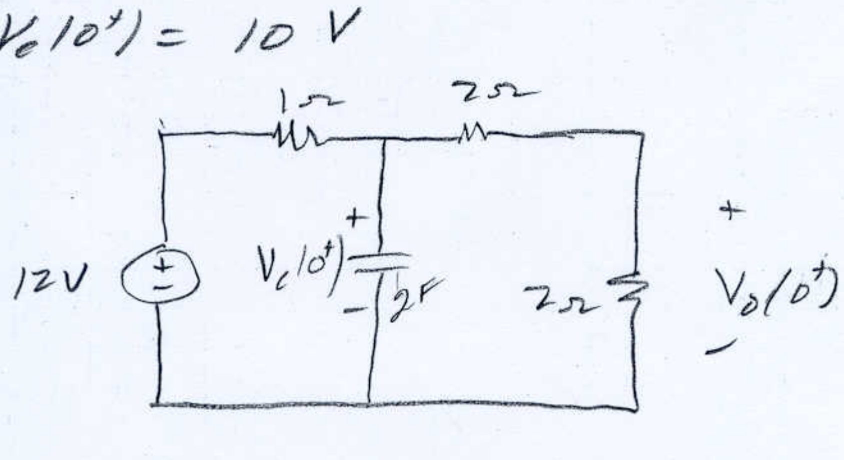


$$\frac{V_c(0^+)-12}{1} + \frac{V_c(0^+)-4}{3} = 0$$

$4V_c(0^+) = 40$

$V_c(0^+) = 10\ V$

$t>0$



(5) cont

$$V_o(0^+) = \frac{2 \times V_c(0^+)}{2+2} = 0.5\, V_c(0^+)$$

$$\boxed{V_o(0^+) = 0.5 \times 10 = 5\,V}$$

$$V_o(\infty) = \frac{12 \times 2}{2+2+1} = \frac{24}{5} = 4.8\,V$$

$$\boxed{V_o(\infty) = 4.8\,V}$$

$$R_{eq} = 1 \parallel 4 = \frac{4}{5} = 0.8$$

$$\tau = R_{eq}C = 0.8 \times 2 = 1.6$$

so

$$v(t) = V_o(\infty) + [V_o(0^+) - V_o(\infty)]e^{-\frac{t}{\tau}}$$

$$v(t) = 4.8 + [5 - 4.8]e^{-0.625t}\ V$$

$$\boxed{v(t) = 4.8 + 0.2e^{-0.625t}\ V}$$